UNIT-I &II

INFLUENCE LINES FOR DETERMINATE AND INDETERMINATE BEAMS

PART - A

1. **Where do you get rolling loads in practice**?

Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

2. Name the type of rolling loads for which the absolute maximum bending moment occurs at the midspan of a beam.

(i) Single concentrated load

(ii) udl longer than the span

(iii) udl shorter than the span

(iv) Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

3. What is meant by absolute maximum bending moment in a beam?

When a given load system moves from one end to the other end of a girder,depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these bending moments will usually occur near or at the midspan. The maximum of maximum bending moments is called the absolute maximum bending moment.

4. What is the absolute maximum bending moment due to a moving udl longer than the span of a simply supported beam?(**Apr/may-2018)**

When a simply supported beam is subjected to a moving udl longer than thespan, the absolute maximum bending moment occurs when the whole span is loaded.

$$M_{max} = wl^2/8$$

5. State the location of maximum shear force in a simple beam with any kind of loading.(My/june-2016)

In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

6.What is meant by maximum shear force diagram?

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the ‘design shear’ for each cross section.

7. What is meant by influence lines?

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

8. What are the uses of influence line diagrams?(Nov/dec-2017,2016,Apr/may-2015)

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

9. What do you understand by the term reversal of stresses?

In certain long trusses the web members can develop either tension or compression depending upon the position of live loads. This tendancy to change the nature of stresses is called reversal of stresses.

10. State Muller-Breslau principle.(Nov/dec-2016,Apr/may-2015)

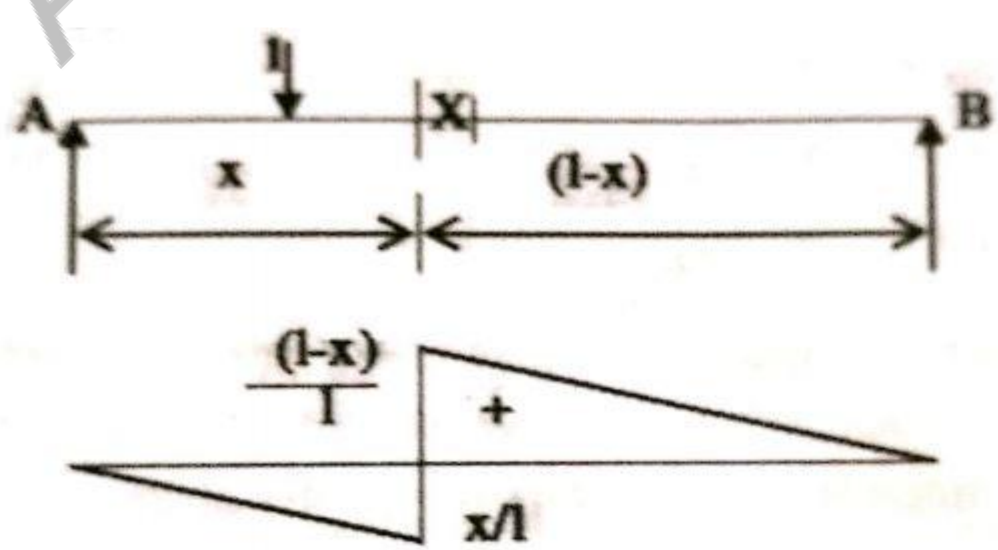
Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

(i) We remove from the structure the resistant to that force quantity and

(ii) We apply on the remaining structure a unit displacement corresponding to that force quantity.

The resulting displacements in the structure are the influence line ordinates sought.

11. **Sketch the Influence line for shear at D for the beam(.Nov/dec-2019,2015)**



12. Draw the influence line for shear to the left of B for the overhanging beam.(Nov/dec-2019,2015)

Figure 5.5(d) Beam with load between *B* and *C*

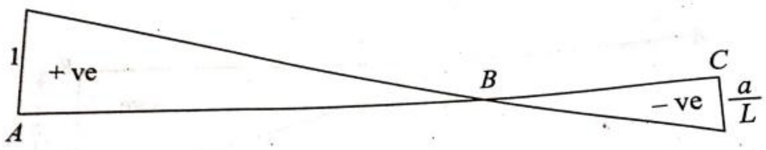
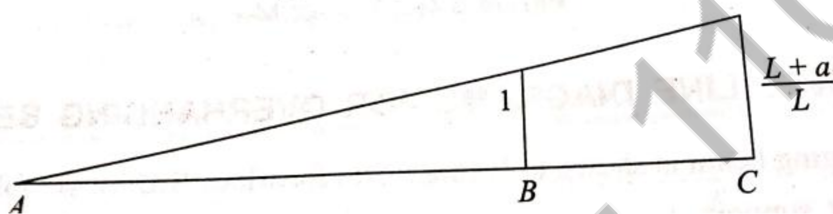


Figure 5.5(e) ILD for R_A

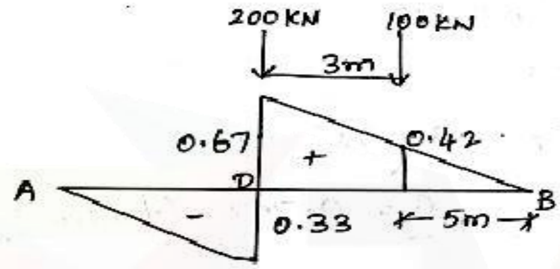


PART - B (16 marks)

1. Two point loads of 100 kN and 200 kN spaced 3 m apart cross a girder of span 12 m from left to right with the 100 kN leading. Draw the ILD for shear force and bending moment and find the values of maximum shear force and bending moment at a section 4 m from the left hand support. Also evaluate the absolute maximum bending moment due to the given loading system. (Apr/May 2012, Nov/Dec 2013)

Solution:

a.) Maximum positive shear force:



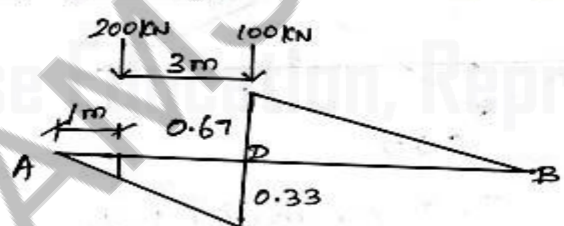
positive ordinate under 200 KN $= \frac{l-x}{l} = \frac{12-4}{12} = 0.67$

ordinate under 100 KN load $= \frac{0.67}{8} \times 5 = 0.42$

Maximum positive shear force $= (200 \times 0.67) + (100 \times 0.42)$

+ve SF = 176 kN.

b.) Negative shear force:-



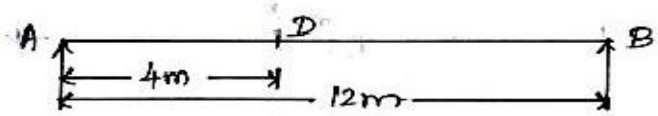
Negative ordinate under 100KN $= \frac{x}{l} = \frac{4}{12} = 0.33$

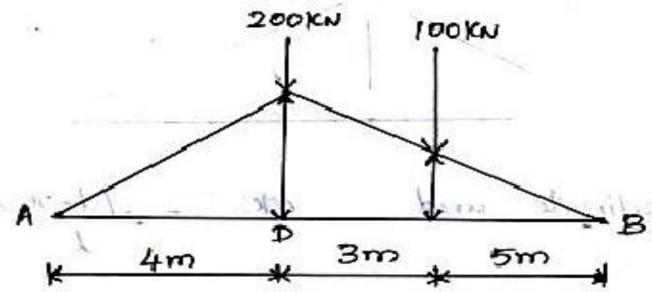
ordinate under 200KN load $= \frac{0.33}{4} \times 1 = 0.083$

Maximum Negative shear force $= (100 \times 0.33) + (200 \times 0.083)$

-ve SF = 49.6 kN

c.) Maximum Bending Moment :-





Maximum ordinate of ILD (200KN) $= \frac{x(l-x)}{l}$

$= \frac{4 \times (12-4)}{12}$

$= 2.67$

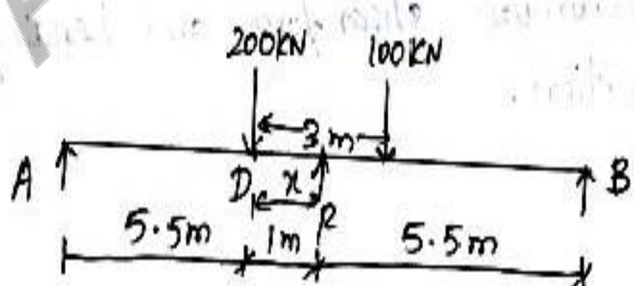
Maximum ordinate at 100 kN $= \frac{2.67}{8} \times 5 = 1.67$

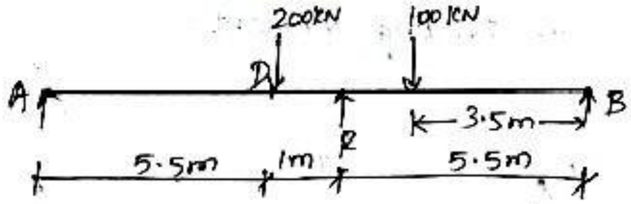
Maximum BM = Load × Ordinate

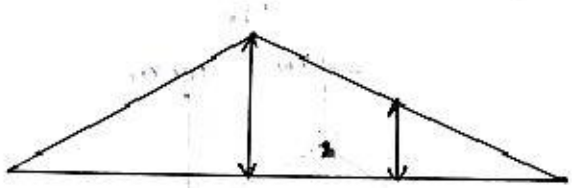
$= [(200 \times 2.67) + (100 \times 1.67)]$

Max. BM = 701 kN m

d.) Absolute Maximum bending moment :-







ordinate under 200 kN $= \frac{x(l-x)}{l} = \frac{5.5(12-5.5)}{12}$

$= 2.98$

ordinate under 100 kN $= \frac{2.98}{6.5} \times 3.5 = 1.6$

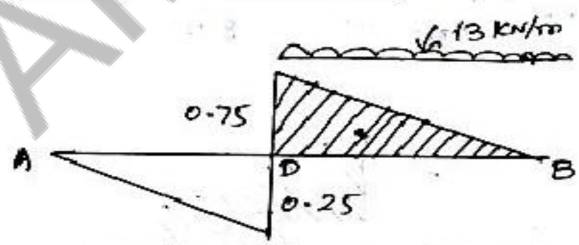
Absolute Max. BM $= (200 \times 2.98) + (100 \times 1.6)$

$= 756$ kNm

2. A simply supported beam has a span of 16 m is subjected to a UDL (dead load) of 5 kN/m and a UDL (live load) of 8 kN/m (longer than the span) traveling from left to right. Draw the ILD for shear force and bending moment at a section 4 m from the left end. Use these diagrams to determine the maximum shear force and bending moment at this section. (Apr/May 2018)

Solution:

step 1 : Maximum +ve shear force :-



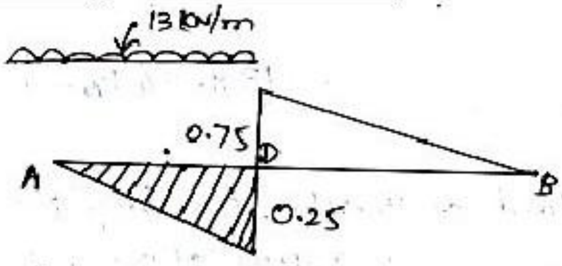
ILD ordinate for positive $= \frac{l-x}{l} = \frac{16-4}{16} = 0.75$

Maximum positive SF $=$ load $\times$ Area

$= 13 \times \left[\frac{1}{2} \times 12 \times 0.75\right]$

+ve SF $= 58.5$ kN .

Step 2: Maximum –ve shear force :-



ILD for negative ordinate $= x/l = \frac{4}{16} = 0.25$

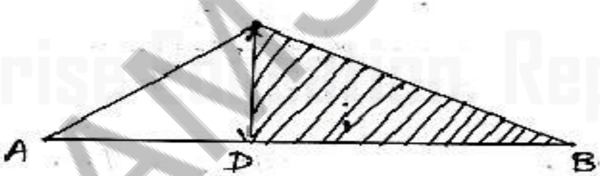
Maximum Negative SF $= 13 \times [1/2 \times 4 \times 0.25]$

–ve SF = 6.5 kN

Step 3: Maximum Bending moment :-

$$l/4 = \frac{16}{4} = 4\,m$$

Since the udl is acting longer than the span. Maximum bending moment occur right of D.



Maximum ordinate at D $= \frac{x(l-x)}{l} = \frac{4(12-4)}{12}$

$= 2.67$

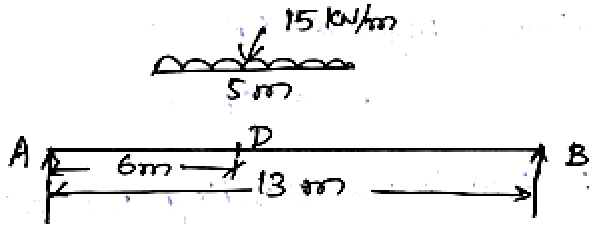
Maximum B.M = load × Area

$= 13 \times [1/2 \times (8 \times 2.67)]$

$$\boxed{\text{Max. B.M.} = 138.84\ kNm}$$

3. A live load of 15 kN/m, 5 m long moves on a girder simply supported on a span of 13 m. Find the maximum bending moment that can occur at a section 6 m from the left end. (AUC Nov/Dec 2012, May/June 2014)

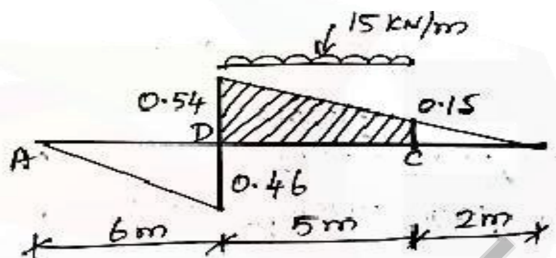
Solution:



Step 1: Maximum positive shear force :-

ILD for positive ordinate at D $= \frac{l-x}{l} = \frac{13-6}{13} = 0.54$

ordinate under at C $= \frac{0.54}{7} \times 2 = 0.15$

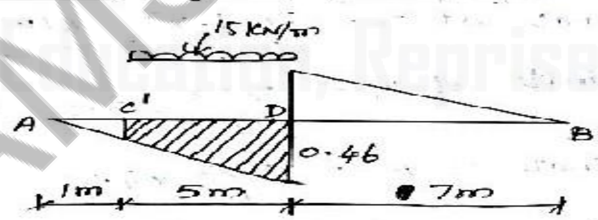


Maximum positive shear force = load x Area

$$= 15 \times \left[\left(\frac{0.54+0.15}{2}\right) \times 5\right]$$

$$= 25.87 \text{ KN}.$$

Step 2: Maximum Negative shear force :-



ILD for negative ordinate $= \frac{x}{l} = \frac{6}{13} = 0.46$

ordinate at C' $= \frac{0.46}{6} \times 1 = 0.077$.

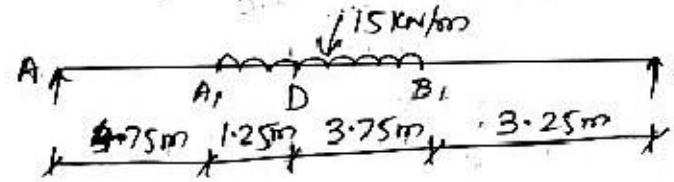
Maximum Negative SF $= 15 \times \left[\left(\frac{0.46+0.077}{2}\right) \times 5\right]$

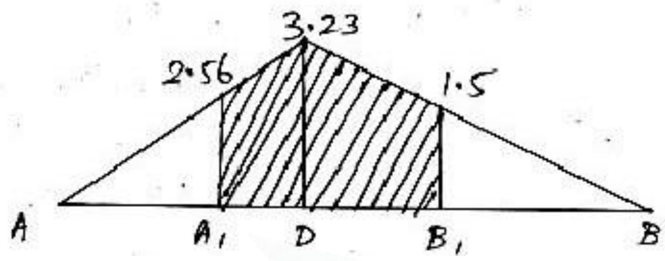
$$= 20.14 \text{ KN}$$

Step 3: Maximum Bending Moment :-

Maximum bending moment will occur at D and udl will be placed at $\frac{1}{4}$ distance.

$l/4 = 5/4 = 1.25\,m.$





$$\text{ordinate at } D = \frac{x(l-x)}{l} = \frac{6(13-6)}{13} = 3.23$$

$$\text{ordinate at } B_1 = \frac{3.23}{7} \times 3.25 = 1.5$$

$$\text{ordinate at } A_1 = \frac{3.23}{6} \times 4.75 = 2.56$$

$$\text{Maximum BM} = \text{load} \times \text{Area}$$

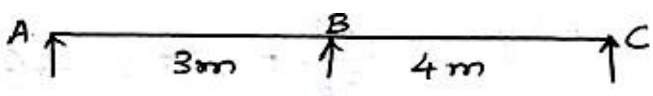
$$= 15 \times \left[(2.56 \times 1.25) + (\tfrac{1}{2} \times 0.67 \times 1.25) + (1.5 \times 3.75) + (\tfrac{1}{2} \times 1.73 \times 3.75) \right]$$

$$= 15 \times [12.48]$$

$$\boxed{\text{Max. BM} = 187.2\ \text{KNm}}$$

4. Draw the influence line for M_B of the continuous beam ABC simply supported at A & C using Muller Breslau's principle. AB = 3 m, BC = 4 m. EI is constant. (AUC Apr/May 2011)

Solution:

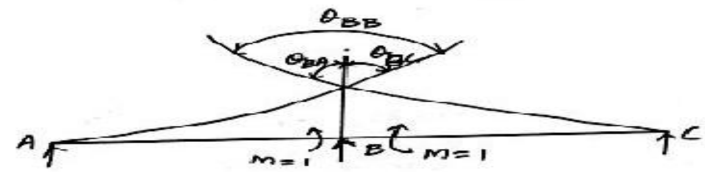


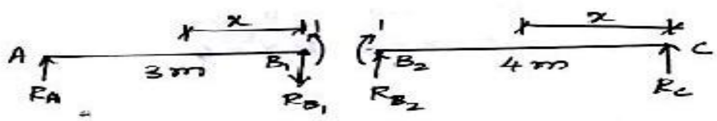
To get IL for M_B :-

i) Apply a unit BM at D.

ii) Determine the deflection y_{xB} any x and slope θ_B at B.

IL ordinate at any $x = \frac{y_{xB}}{\theta_B}$





Due to $M=1$ at B,

$$R_A \times 3 = 1$$
$$R_A = 1/3 = 0.333$$

$$R_{B_1} = 0.333 \downarrow$$
$$R_{B_2} = 0.333 \uparrow$$

Taking moments about B,

$$R_c \times 4 = 1$$
$$R_c = 1/4 = 0.25$$

The two regions AB and BC will be considered separately.

BM at any x is,

$$M_x = -EI\frac{d^2y}{dx^2} = 0.25x \;\vdots\; -0.333(x-4)$$

$$EI\frac{d^2y}{dx^2} = -0.25x + 0.333(x-4)$$

$$EI\frac{dy}{dx} = -0.25\frac{x^2}{2} + C_1 + 0.333\frac{(x-4)^2}{2} \rightarrow ①$$

$$EI\,y = -0.25\frac{x^3}{6} + 0.333\frac{(x-4)^3}{6} + C_1x + C_2 \rightarrow ②$$

The boundary conditions are

$$y = 0 \text{ at } x = 0 \;\&\; x = 4$$

$\therefore$ ② $\Rightarrow C_2 = 0$.

$$0 = -0.25 \times \frac{(4)^3}{6} + 4C_1 + 0$$
$$C_1 = 0.67$$

$$\therefore \quad EI\,y_{xB} = -0.25\frac{x^3}{6} + 0.67x \;\Big|\; + 0.333\frac{(x-4)^3}{6} \rightarrow (3)$$

$$(1) \Rightarrow \quad EI\frac{dy}{dx} = -0.25\frac{x^2}{2} + C_1 + \Big|\; 0.333\frac{(x-4)^2}{2}$$

At B, $x = 4$,

$$\theta_{BC} = \left(\frac{dy}{dx}\right)_{BC} = \frac{1}{EI}\left[-\frac{0.25\times(4)^2}{2} + 0.67 + \frac{0.333\times 0^2}{2}\right]$$

$$\theta_{BC} = \frac{-1.334}{EI}$$

$$(3) \Rightarrow \quad y_D = y_4 = \frac{1}{EI}\left[-0.25\times\frac{(4)^3}{6} + (0.67\times 4) + \frac{0.333(0)}{6}\right]$$

$$y_D = \frac{0.013}{EI}$$

$$(5) \Rightarrow \quad 0.013 = C_4$$

At $x = 3$; $y = 0$,

$$(5) \Rightarrow \quad 0 = \frac{0.333(3)^3}{6} - \frac{(3)^2}{2} + 3C_3 + 0.013$$

$$C_3 = 0.99 = 1.$$

$$(4) \Rightarrow \quad EI\frac{dy}{dx} = \frac{0.333x^2}{2} - x + 1$$

At $x = 0$;

$$\theta_{BA} = \left(\frac{dy}{dx}\right) = \frac{1}{EI}$$

$$\therefore \quad EI\,y = \frac{0.333x^3}{6} - \frac{x^2}{2} + x + 0.013 \rightarrow (6)$$

$$y_{xB} = \frac{1}{EI}\left[\frac{0.333x^3}{6} - \frac{x^2}{2} + x + 0.013\right] \rightarrow (7)$$

For the zone AB,

$$M_x = 1 - 0.333x$$

$$EI\frac{d^2y}{dx^2} = 0.333x - 1$$

$$EI\frac{dy}{dx} = \frac{0.333x^2}{2} - x + C_3 \rightarrow (4)$$

$$EI\,y = \frac{0.333x^3}{6} - \frac{x^2}{2} + C_3x + C_4 \rightarrow (5)$$

At $x = 0$; $y = \frac{0.013}{EI}$,

$$\theta_{BB} = \theta_{BA} - \theta_{BC}$$

$$= \frac{1}{EI} - \left(-\frac{1.33}{EI}\right) = \frac{1}{EI} + \frac{1.33}{EI}$$

$$\boxed{\theta_{BB} = \frac{2.33}{EI}}$$

For the region CB,

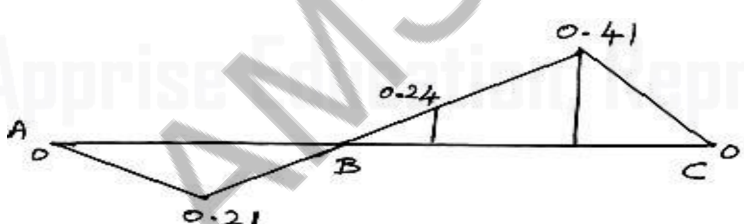
$$\text{IL ordinate for } M_B = \frac{y_{xB}}{\theta_{BB}} = \left[\frac{-\frac{0.25x^3}{6} + 0.67x \Big| + \frac{0.333(x-4)^3}{6}}{2.33}\right] \rightarrow (8)$$

For the region BA,

$$\text{IL ordinate for } M_B = \frac{y_{xB}}{\theta_{BB}} = \left[\frac{\frac{0.333x^3}{6} - \frac{x^2}{2} + x + 0.013}{2.33}\right] \rightarrow (9)$$

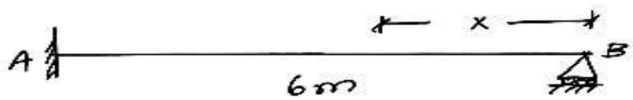
The influence line ordinates are tabulated,

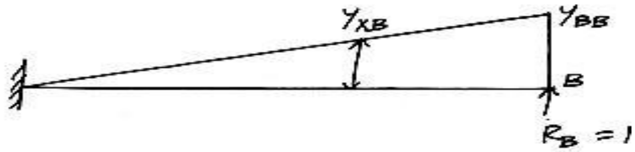
x from C	0	1.75	3.5	4	5.25	7
I.L.O	0	0.41	0.24	0	-0.21	0



IL.O for M_B

5. Draw the influence line diagram for the propped reaction of a propped cantilever beam having span 6 m. Take EI = constant. (Nov/dec- 2019,2016)
Solution:





a) To generate the ILD for R_B :-

i) Remove the restraint due to R_B (remove support B)

ii) Apply a unit displacement (upward) at B.

When $R_B = 1$, then Y_{XB} is the displacement at section x due to unit load applied at B,

$$M_x = -EI\frac{d^2y}{dx^2} = R_B \cdot x = 1x$$

$$EI\frac{d^2y}{dx^2} = -x$$

$$EI\frac{dy}{dx} = -\frac{x^2}{2} + C_1 \longrightarrow (1)$$

$$EI\, y = -\frac{x^3}{6} + C_1 x + C_2 \longrightarrow (2)$$

At $x = 6$,

$$y = 0\,;\ \frac{dy}{dx} = 0$$

(1) & (2) we get,

$$(1) \Rightarrow \quad 0 = -\frac{x^2}{2} + C_1$$

$$C_1 = \frac{x^2}{2} = \frac{(6)^2}{2} = 18$$

$$\boxed{C_1 = 18}$$

$$(2) \Rightarrow \quad 0 = -\frac{x^3}{6} + C_1 x + C_2$$

$$C_2 = \frac{x^3}{6} - C_1 x = \frac{(6)^3}{6} - (18 \times 6)$$

$$\boxed{C_2 = -72}$$

Hence,

$$② \Rightarrow \quad y_{xB} = \frac{1}{EI}\left[-\frac{x^3}{6} + 18x - 72\right] \rightarrow ③$$

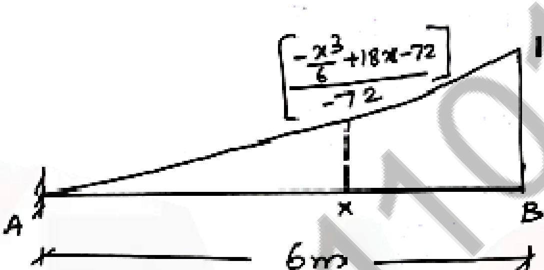
$$② \Rightarrow \quad y_{BB}\ (\text{at } x = 0) = \left[0 + C_1(0) + C_2\right]\frac{1}{EI}$$

$$y_{BB} = -\frac{72}{EI} \rightarrow ④$$

$$\text{IL ordinate for } R_B \text{ at } x = \frac{y_{xB}}{y_{BB}}$$

$$= \frac{\frac{1}{EI}\left[-\frac{x^3}{6} + 18x - 72\right]}{-\frac{72}{EI}}$$

$$R_B = \frac{-\frac{x^3}{6} + 18x - 72}{-72}$$

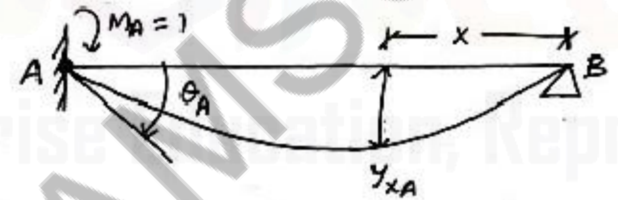


ILD for R_B

b.) ILD for M_A :-

i.) Introduce a hinge at A

ii.) Apply a unit rotation at A.



Due to $M_A = 1$, $\Rightarrow R_B \times 12 = 1 \Rightarrow R_B = \frac{1}{6}$

$$M_x = -EI\frac{d^2y}{dx^2} = \frac{x}{6}$$

$$EI\frac{d^2y}{dx^2} = -\frac{x}{6}$$

$$EI\frac{dy}{dx} = -\frac{x^2}{12} + C_1 \rightarrow ⑤$$

$$EI\,y = -\frac{x^3}{36} + C_1x + C_2 \rightarrow ⑥$$

At $x=0$; $y=0$ $\Rightarrow$ $\boxed{C_2=0}$

At $x=6$; $y=0$ $\Rightarrow$ $0=-6+6C_1$

$\boxed{C_1=1}$

(6) $\Rightarrow$ $$y_{xA} = \frac{1}{EI}\left(-\frac{x^3}{36}+x\right)$$

$$\frac{dy}{dx} = \theta_{xA} = \frac{1}{EI}\left(-\frac{x^2}{12}+1\right)$$

At $x=6$,

$$\theta_A = -\frac{2}{EI}$$

when we divide y_{xA} by θ_A,

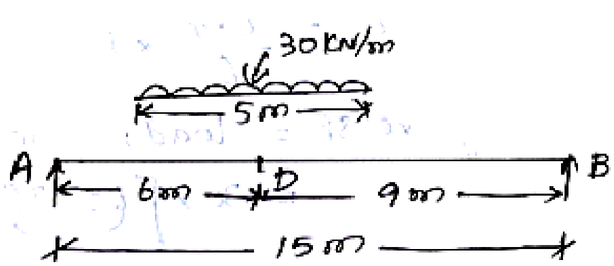
$$\text{IL ordinate at } x \text{ for } M_A = \frac{\frac{1}{EI}\left(-\frac{x^3}{36}+x\right)}{-2/EI}$$

$$M_A = \left(\frac{x^3}{72}-\frac{x}{2}\right)$$

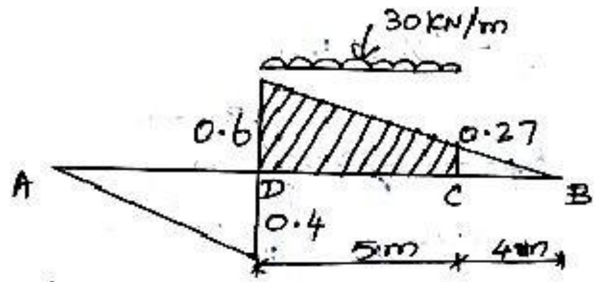
ILD for M_A

6. A simply supported beam has a span of 15 m and subjected to an UDL of 30 kN/m, 5 m long travelling from left to right. Draw the ILD for shear force and bending moment at a section 6 m from the left end. Use these diagrams for calculating the maximum BM and SF at this section. (Apr/may 2017,2018)

Solution:



Step 1: Maximum positive SF :-



ordinate at D $= \frac{l-x}{l} = \frac{15-6}{15} = 0.6$

ordinate at C $= \frac{0.6}{9} \times 4 = 0.27$

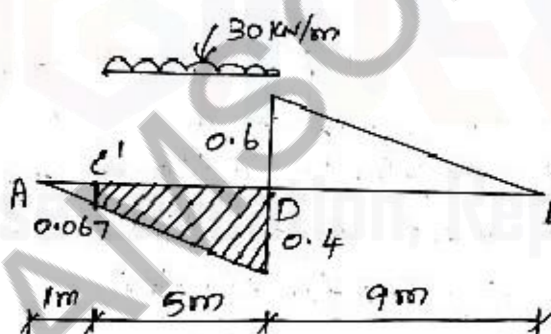
Maximum positive SF = load x Area

$= 30 \times \left[\left(\frac{0.6+0.27}{2}\right) \times 5\right]$

$= 65.25$ KN.

Step 2: Maximum Negative SF :

ordinate at D $= x/l = \frac{6}{15} = 0.4$



ordinate at c' $= \frac{0.4}{6} \times 1 = 0.067$

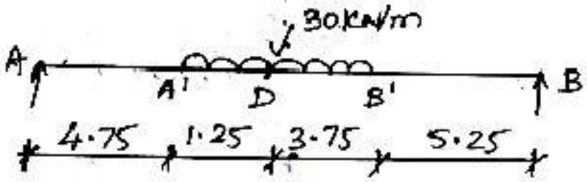
Maximum Negative SF = Load x Area

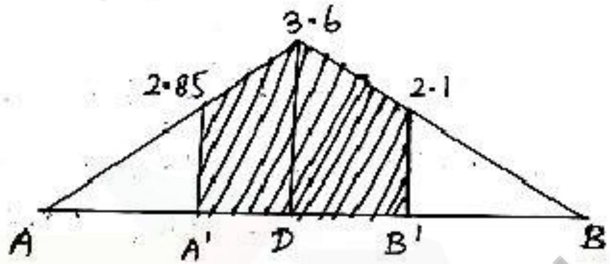
$= 30 \times \left[\left(\frac{0.4+0.067}{2}\right) \times 5\right]$

$= 35$ KN

Step 3: Maximum Bending Moment :-

$\frac{l}{4} = \frac{5}{4} = 1.25\,m$ from left of D.





ordinate at $D = x\left(\frac{l-x}{l}\right) = 6\left(\frac{15-6}{15}\right) = 3.6$

ordinate at $B' = \frac{3.6}{9} \times 5.25 = 2.1$

ordinate at $A' = \frac{3.6}{6} \times 4.75 = 2.85$

Maximum BM = load x Area

$= 30\left\{(2.85 \times 1.25) + (\tfrac{1}{2} \times 0.75 \times 1.25) + (2.1 \times 3.75) + (\tfrac{1}{2} \times 1.5 \times 3.75)\right\}$

$= 30 \times 14.72$

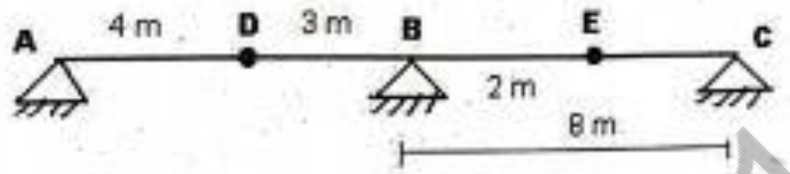
$\boxed{\text{Max. BM} = 441.6\ \text{KNm}}$

7.A continuous beam ABC is simply is simply resting on supports A and C, continuous over the support B and has an internal hinge D at 4m from A and E at middle of span BC. The span AB is 7m and the span BC is 8m.Draw Influence lines for

i. **Reactions at A,B and C**

ii. **Shear to the right of B**

iii. **Bending moment at E.(Nov/dec-2019,2016Apr/may-2017,2016)**

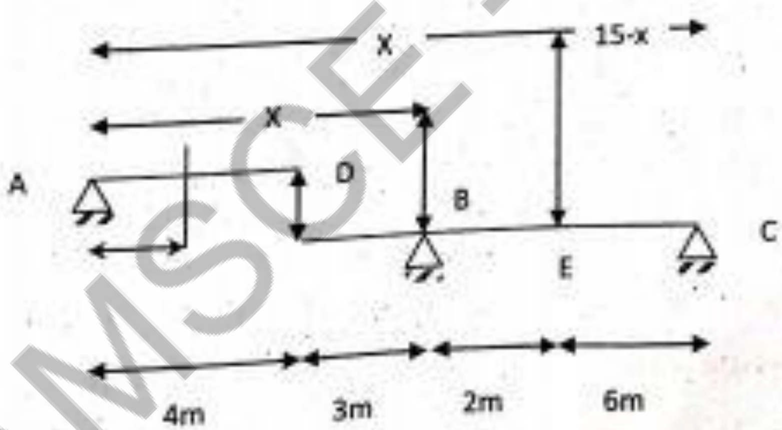


Solution

a) Influence lines for R_A, R_B, and R_C:

i) IL for R_A:

Due to hinge at D, AD will behave as a s/s beam at D, Whose it acts upward at D on AD, downward of DBC. Any load on DBC will have no effect on AD.



When a unit load is on AD,

$$R_A = \frac{4-x}{4}$$

At x = 0; R_A = 1,

At x = 4; R_A = 0

When x > 4; R_A = 0.

i) IL for R_B

When a unit load is on AD,

$$R_A = 1 - \frac{x}{4}$$

$$R_D = 1 - R_A = \frac{x}{4}$$

Taking moment about C,

$$R_B \times 8 - R_D \times 11 = 0$$

$$8R_B - \frac{x \times 11}{4} = 0$$

$$R_B = \frac{11x}{4 \times 8} = \frac{11x}{32}$$

When $x = 0$; $R_B = 0$

At $x = 4$; $R_B = 1.375$

When the load is on DBC, the reaction at D is zero Taking moment about C,

$$R_B \times 8 - 1(15 - x) = 0$$

$$R_B = \frac{15 - x}{8}$$

When $x = 4$; $R_B = 1.375$

$x = 7$; $R_B = 1$

$x = 15$; $R_B = 0$

ii) I_L for R_C:

When the unit load is on AD; load at $D = \frac{x}{4}(\downarrow)$

Taking moment about B,

$$-\frac{x}{4} \times 3 - R_C \times 8 = 0$$

$$R_C = \frac{-3x}{32}$$

When $x = 0$; $R_B = 0$.

$x = 4$; $R_B = -0.375$

When the load is over DBC, $R_D = 0$.

Taking moments about B,

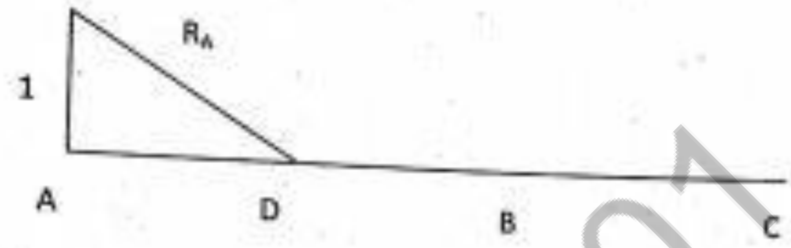
$$-1(7-x)R_c \times 8 = 0$$

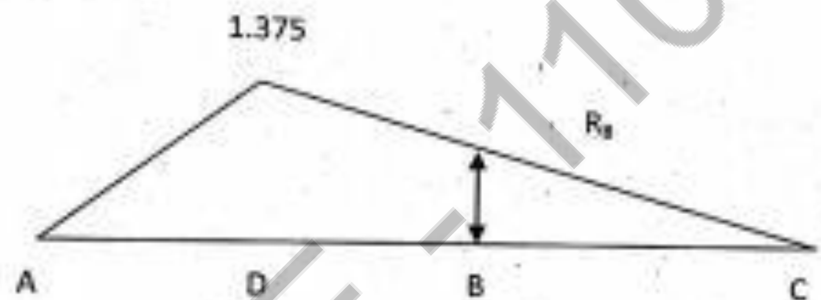
$$R_c = \frac{x-7}{8}$$

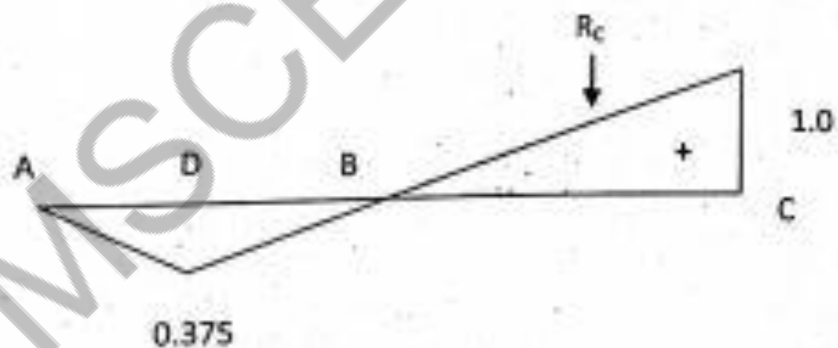
$$x = 4;\ R_c = -0.375$$

$$x = 7:\ R_c = 0$$

$$x = 15;\ R_c = 1$$







b) IDL for shear to the right of B (F_B);

When the load is on AD, F_B = -R_C

$$R_C = \frac{-3x}{32};\ F_B = \frac{-3x}{32}$$

$$x = 0;\ \ F_B = 0$$

$$x = 4;\ \ F_B = 0.375$$

When the load is over DB, F_B = -R_C

$$R_C = \frac{x-7}{8}; \; F_B = \frac{7-x}{8}$$

$x = 4; \; F_B = 0.375$

$x = 7; \; F_B = 0$

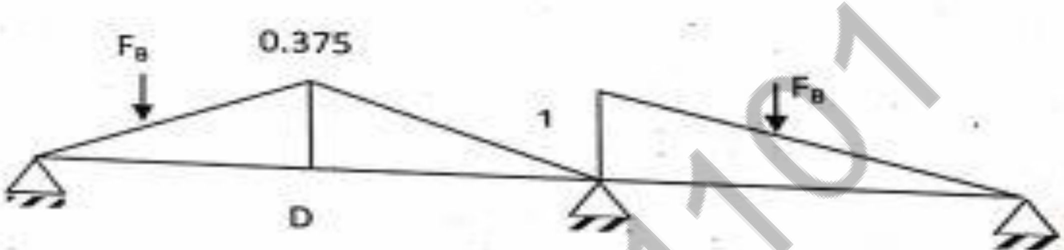
When the load is over BC, $F_B = R_B$

$$R_B \times 8 - 1(15 - x) = 0$$

$$F_B = R_B = \frac{15-x}{8}$$

$x = 7; \; F_B = 1$

$x = 15; F_B = 0$



c) ILD for BM at E (M_E)

When unit load is on AD,

$$M_E = R_C \times 6 = \frac{-3x}{32} \times 6$$

$$M_E = \frac{-18x}{32}$$

$x = 0; \; M_E = 0$

$x = 4; \; M_E = -2.25$

When the load is between E & C

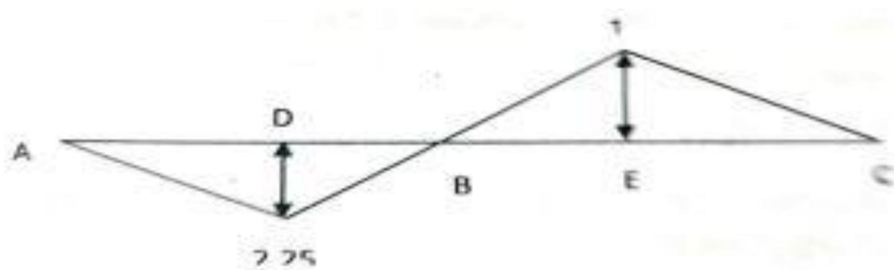
$$M_E = R_C \times 6 - 1(x - 9)$$

$$R_C = \frac{x-7}{8}$$

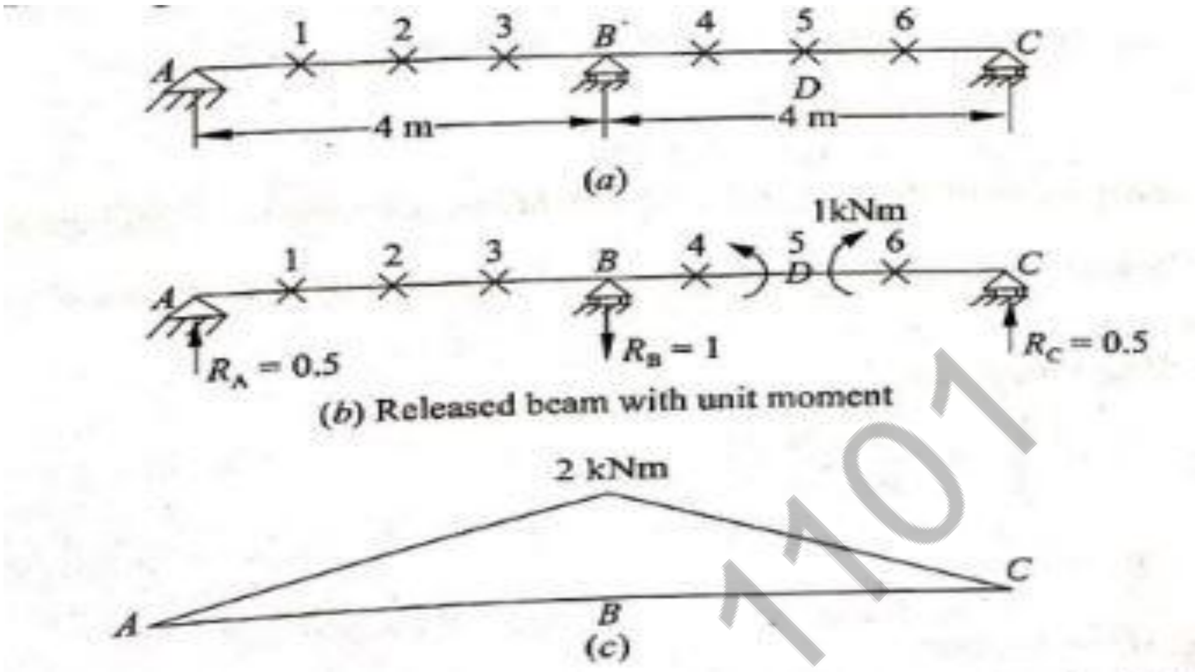
$$M_E = (x-7)\frac{6}{8} - (x-9)$$

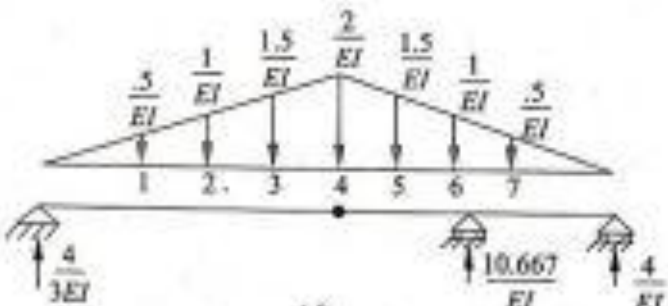
$x = 9\text{m}; \; M_E = 1.5$

$x = 15; \; M_E = 0$

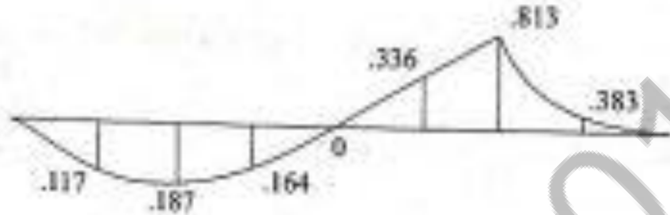


8. Using Muller Breslau Principle, draw Influence line for bending moment at mid span BC i.e(D),Compute the ordinates at 1m interval.(Nov/dec-201,20157)





(*d*)



(*e*)

Figure 5.9(*d*): Conjugate beam. **(*e*)** ILD for moment at *D*.

Solution Restraint for moment at mid-span of *BC* is removed by introducing hinges and unit moment is applied as shown in Figure 5.9(*b*). According to Müller-Breslau principle, the deflected shape of this released structure represents ILD for moment at that point.

In a released structure (Figure 5.9(*b*)),

$$R_C \times 2 = 1$$

$$\therefore \quad R_C = 0.5 \text{ kN}$$

Taking moment about *A*, we get,

$$R_B \times 4 - R_C \times 8 = 0$$

$$R_B = 2R_C = 2 \times 0.5 = 1 \text{ kN}\downarrow$$

$$\Sigma V = 0, \text{ gives}$$

$$R_A = 1 - 0.5 = 0.5 \text{ kN}$$

Therefore, bending moment diagram for released beam is as shown in Figure 5.9(*c*).

The conjugate beam for the released beam and $\frac{M}{EI}$ loading on it are as shown in Figure 5.9(*d*). In conjugate beam,

$$\Sigma M_B = 0, \text{ gives}$$

$$R'_A \times 4 = \frac{1}{2} \times 4 \times \frac{2}{EI} \times \frac{4}{3}$$

$$R'_A = \frac{4}{3EI}$$

$$\Sigma M_C = 0, \text{ gives}$$

$$R'_A \times 8 + R'_D \times 2 - \frac{1}{2} \times 8 \times \frac{2}{EI} \times 4 = 0$$

$$\frac{4}{3EI}\times 8+R'_D\times 2-\frac{1}{2}\times 8\times\frac{2}{EI}\times 4=0$$

$$R'_D=\frac{10.667}{EI}$$

$\Sigma V=0$, gives

$$R'_A+R'_B+R'_C=\frac{1}{2}\times 8\times\frac{2}{EI}=\frac{8}{EI}$$

$$\therefore \quad R'_C=\left(\frac{8}{EI}\right)-\left(\frac{4}{3EI}\right)-\left(\frac{10.669}{EI}\right)$$

$$=-\frac{4}{EI}=\frac{4}{EI}\downarrow$$

Calculating moments from left end for portion AB, we get,

$$\delta_A=M_A=0$$

$$\delta_1=M_1=\left(\frac{4}{3EI}\times 1\right)-\left(\frac{1}{2}\times 1\times\frac{0.5}{4EI}\times\frac{1}{3}\right)=\frac{1.25}{EI}$$

$$\delta_2=M_2=\left(\frac{4}{3EI}\times 2\right)-\left(\frac{1}{2}\times 2\times\frac{1}{EI}\times\frac{2}{3}\right)=\frac{2}{EI}$$

$$\delta_3=M_3=\left(\frac{4}{3EI}\times 3\right)-\left(\frac{1}{2}\times 3\times\frac{1.5}{EI}\times 1\right)=\frac{1.75}{EI}$$

$$\delta_4=M_B=0$$

For portion BC, calculating moments in conjugate beam from end C, we get,

$$\delta_C=M_C=0$$

$$\delta_5=M_5=-\left(\frac{4}{EI}\times 1\right)-\left(\frac{1}{2}\times 1\times\frac{0.5}{EI}\times\frac{1}{3}\right)=-\frac{4.083}{EI}$$

$$\delta_D=M_D=-\left(\frac{4}{EI}\times 2\right)-\left(\frac{1}{2}\times 2\times\frac{1}{EI}\times\frac{2}{3}\right)=-\frac{8.667}{EI}$$

$$\delta_4=M_4=-\left(\frac{4}{EI}\times 3\right)-\left(\frac{1}{2}\times\frac{1.5}{EI}\times 3\times 1\right)+\left(\frac{10.667}{EI}\times 1\right)=-\frac{3.583}{EI}$$

The total rotation at hinge D in a released structure

$$=\text{Reaction at support } D \text{ in conjugate beam}=\frac{10.667}{EI}$$

Hence, values of the influence line ordinates at each of the above sections is computed by dividing each deflection by $\alpha=\frac{10.667}{EI}$. Hence ILD for M_D is as shown in Figure 5.9(*e*).

9. Draw the ILD for the foces in members U_2L_2 and U_2L_3 of the truss shown in fig.(Nov/dec-2017,2015)

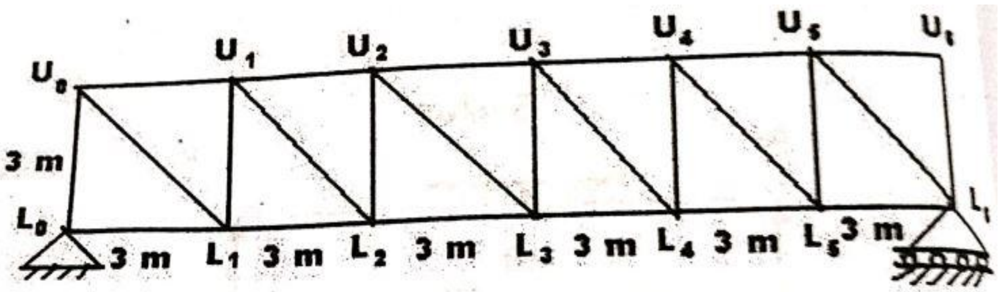
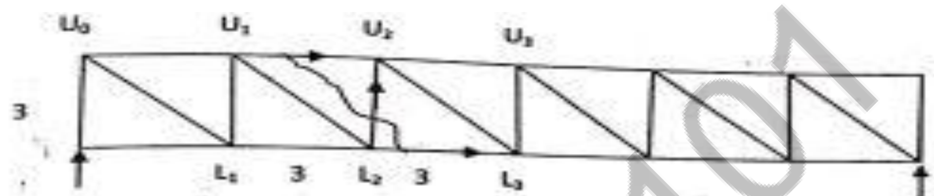


Fig.



The nature of the shear force in the panel L_2, L_3, change as the load moves from L_2 to L_3

When unit load is at L_2, shear force is negative and force in U_2, L_2 is tension. When it is at L_3, the force in U_2 L_2 is compression.

To find IL 0 at L, and at L_3 and join them to get the IL for shear in the panel L_2,L_3.

Unit load is at L_2, shear in the $L_2L_3 = -R_B$

$R_A + R_B = 1$

Taking moment about B

$R_A \times 18 - 1 \times 12 = 0$

$R_A = \frac{12}{18} = 0.63$

$R_B = 0.33$

Shear in panel at L_2, $L_3 = -R_B = -0.33$

When the load is at L_3,

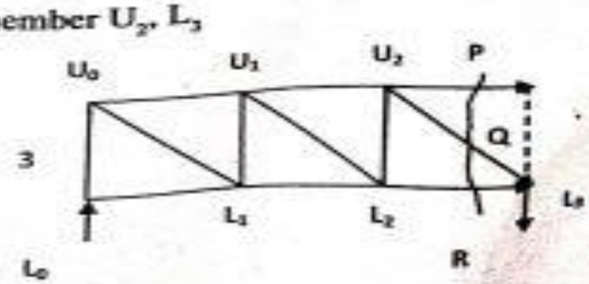
$R_A + R_B = 1$

$R_A \times 18 - 1 \times 9 = 0$

$R_A = \frac{9}{18} = 0.5$

ILO at A and B or L_a & L_b are zero

b) ILD for the member U_2, L_3



i) Unit load at L_2

Taking moment about B

$R_A \times 18 - 1 \times 12 = 0$

$R_A = 0.67$

Since $\sum V = 0$

$$R_A - 1 - Q\cos 45^\circ = 0$$

$$Q = \frac{0.67-1}{\cos 45^\circ} = 0.47(\text{Comp})$$

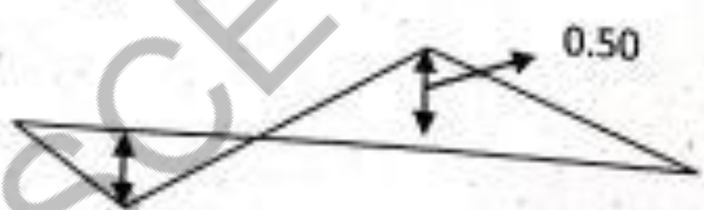
ii) Unit load at L_3;

$R_A = 0.5$

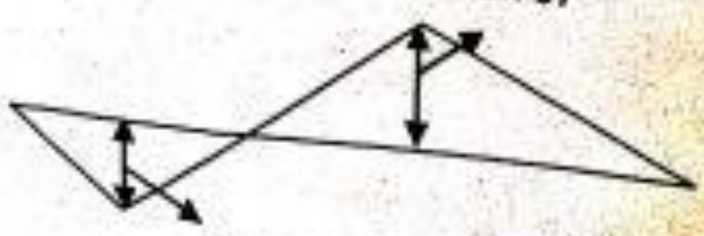
Since $\sum V = 0$

$R_A - Q\cos 45^\circ = 0$

$Q = 0.707(\text{Torsion})$



ILD for member U_3L_2



IDL for member U_2,

10. A train of 5 wheel loads crosses a simply supported beam of span 22.5m. Using influence lines calculate maximum posistive and negative shear forces at mid span and absolute maximum bending moment anywhere in the span.(Nov/dec-2017,May/june-2016)

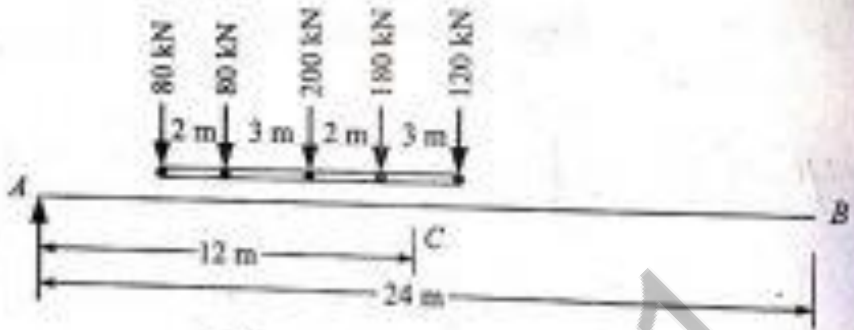


Figure 5.22(a) Example 5.6

Solution The ILD for shear force at centre of span is as shown in Figure 5.22(*b*). For maximum negative shear force the leading load should be on the section. The maximum ILD ordinate for shear force at *C*

$$= \frac{12}{24} = 0.5$$

∴ Maximum negative S.F. at mid-span

$$= 120 \times 0.5 + \left[180\times\frac{9}{12} + 200\times\frac{7}{12} + 80\times\frac{4}{12} + 80\times\frac{2}{12}\right] \times 0.5$$

$$= 205.833 \text{ kN}$$

For maximum shear force at *C*, the trailing load should be at *C* (Refer Figure 5.2

Maximum shear force at *D*

$$= 80\times 0.5 + \left[80\times\frac{10}{12} + 200\times\frac{7}{12} + 180\times\frac{5}{12} + 120\times\frac{2}{12}\right]\times 0.5$$

$$= 179.167 \text{ kN}$$

When leading 80 kN load is on *C*,

Positive S.F. at *C*

$$= \left[-80\times\frac{10}{12} + 80\times 1 + 200\times\frac{9}{12} + 180\times\frac{7}{12} + 120\times\frac{4}{12}\right]0.5$$

$$= 154.167 \text{ kN}$$

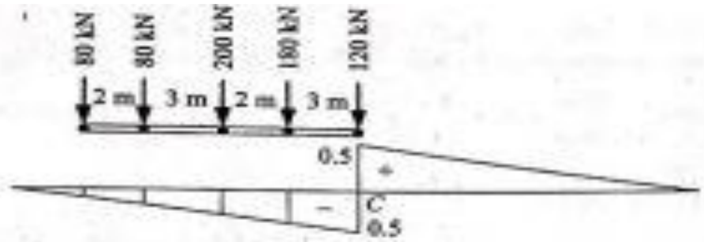


Figure 5.22(b) ILD for S.F. at mid-span with loading for maximum –ve S.F.

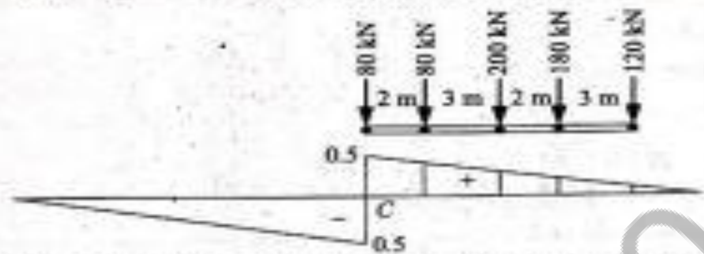


Figure 5.22(c) ILD for S.F. at mid-span with loading for maximum +ve S.F.

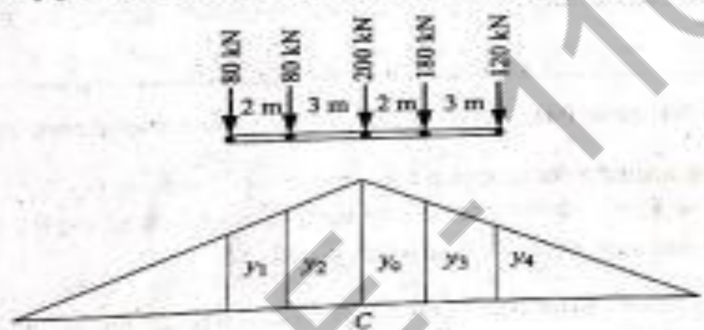


Figure 5.22(d)

Hence, maximum shear force at mid-span $C = 179.167$ kN

Let C.G. of loads from leading wheel load be at a distance x.

Then,
$$x = \frac{180\times3+200\times5+80\times8\times80\times10}{120+180+200+80+80}$$
$$= 4.5152 \text{ m}$$

This is nearer to 200 kN load. Hence, maximum moment is likely to occur under this load.

Its distance from CG
$$= 5-4.5152 = 0.4848 \text{ m}$$

Therefore, its position from end A
$$= 12-\frac{0.4848}{2} = 11.7576 \text{ m}$$

Therefore, ILD ordinate under this load is
$$y_c = \frac{z(L-z)}{L} = \frac{11.7576(24-11.7576)}{24}$$
$$= 5.9975$$

Referring to Figure 5.22(d).

Therefore, absolute maximum $BM = 80y_1 + 80y_2 + 200y_c + 180y_3 + 120y_4$
$$= \left[80\times\frac{6.7576}{11.7576} + 80\times\frac{8.7576}{11.7576} + 200 + 180\times\frac{10.2424}{12.2424} + 120\times\frac{7.2424}{12.2424}\right]$$
$= $ Since, $y_c = 5.99$
$= 3157.64$ kNm

UNIT-III ARCHES

PART - A

1. What is an arch? Explain.

An arch is defined as a curved girder, having convexity upwards and supported at its ends.The supports must effectively arrest displacements in the vertical and horizontal directions. Only then there will be arch action.

2. State Eddy's theorem.

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the centre line of the actual arch."

3. What is the degree of static indeterminacy of a three hinged parabolic arch?

For a three hinged parabolic arch, the degree of static indeterminancy is zero. It is statically determinate.

5. Explain with the aid of a sketch, the normal thrust and radial shear in an arch rib.

Let us take a section X of an arch. (fig (a)). Let θ be the inclination of the tangent at X. If H is the horizontal thrust and V the vertical shear at X, from the free body of the RHS of the arch, it is clear that V and H will have normal and radial components given by,

$N = H \cos\theta + V \sin\theta$

$R = V \cos\theta - H \sin\theta$

4. Which of the two arches, viz. circular and parabolic is preferable to carry a uniformly distributed load? Why?

Parabolic arches are preferably to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every Section of the arch will be zero. The arch will be under pure compression which will be economical.

5. What is the difference between the basic action of an arch and a suspension cable?

An arch is essentially a compression member which can also take bending moments and shears. Bending moments and shears will be absent if the arch is parabolic and the loading uniformly distributed. A cable can take only tension. A suspension bridge will therefore have a cable and a stiffening girder.

The girder will take the bending moment and shears in the bridge and the cable, only tension. Because of the thrusts in the cables and arches, the bending moments are considerably reduced.If the load on the girder is uniform, the bridge will have only cable tension and no bending moment on the girder.

6. Under what conditions will the bending moment in an arch be zero throughout.(Apr/may-2018)

The bending moment in an arch throughout the span will be zero, if (i) the arch is parabolic and (ii) the arch carries uniformly distributed load throughout the span.

7. Indicate the positions of a moving point load for maximum negative and positive bending moments in a three hinged arch.

Considering a three hinged parabolic arch of span 'l' and subjected to a moving point load W, the position of the point load for a. Maximum negative bending moment is 0.25l from end supports. b. Maximum positive bending moment is 0.211l from end supports.

8. Draw the influence line for radial shear at a section of a three hinged arch.

Radial shear is given by Fx = H sinθ - V cosθ,

Where θ is the inclination of tangent at X. l sinθ l – x cosθ 4r

9. Distinguish between two hinged and three hinged arches.

Two hinged arches	**Three hinged arches**
Statically indeterminate	Statically determinate
Might develop temperature stresses	Increase in temperature causes increase in central rise. No stresses.
Structurally more efficient	Easy to analyse. But in costruction, the central hinge may involve additional expenditure.
Will develop stresses due to sinking of supports	Since this is determinate, no stresses due to support sinking.

10. Explain the effect of yielding of support in the case of an arch.

Yielding of supports has no effect in the case of a 3 hinged arch which is determinate. These displacements must be taken into account when we analyse 2 hinged or fixed arches under

∂U = ∆H instead of zero ∂H

∂U = ∆VA instead of zero ∂VA

Here U is the strain energy of the arch and ∂H and ∆VA are the displacements due to yielding of supports.

11. Write the formula to calculate the change in rise in three hinged arch if there is a rise in temperature.

Change in rise = $l^2 + 4r^2$ α T 4r

Where l = span length of the arch r = central rise of the arch α = coefficient of thermal expansion

T = change in temperature

12. In a parabolic arch with two hinges how will you calculate the slope of the arch at any point?

Slope of parabolic arch = $\theta = \tan\text{-}1\ 4r\ (l - 2x)\ l2$

Where θ = Slope at any point x (or) inclination of tangent at x.

l = span length of the arch, r = central rise of the arch

13. How will you calculate the horizontal thrust in a two hinged parabolic arch if there is a rise in temperature?

Horizontal thrust = $l\ \alpha\ TEI\ /y^2dx$

Where l = span length of the arch

y = rise of the arch at any point x

α = coefficient of thermal expansion

T = change in temperature

E = Young's Modulus of the material of the arch

I = Moment of inertia

14. What are the types of arches according to the support conditions.

i. Three hinged arch

ii. Two hinged arch

iii. Single hinged arch

iv. Fixed arch (or) hingeless arch

15. What are the types of arches according to their shapes.

i. Curved arch

ii. Parabolic arch iii. Elliptical arch iv. Polygonal arch

16. what is the value of horizontal thrust when it is subjected to an UDL over the right half of the three hinged parabolic arch?(Nov/dec-2019,2016)

$$H = wl^2/16y_c$$

17.Name any two methods for analysis of two hinged arches.(Nov/dec-2019,2016)

Eddy's theorem

Double Integration method

18.Define Rib shortening.(Apr/may-2018,2015)

If the magnitude is high it may result in the change in the length of the arch, due to corresponding strain, which can be found out by the Hooke's law. The final result will be the shortening of the arch. This effect is known as the Rib shortening.

19. What are the advantages of three hinged circular arches?(Nov/dec-2017,2015)

a. The bending moments and shears operative over cross sections of three hinged arches are significantly smaller as compared to the subsequent stresses in a simple beam covering the equivalent span and bearing the same load. So, three hinged arches are inexpensive relating to ordinary beams, specifically for large span structures.

b. Calculations are very simple as compared to other type of arches.

c. No bending moments are generated at the abutments and the crown since hinges cannot withstand moments.

d. Differential settlements of the supports do not impact stresses, since the pines or hinges allow the arch to assume the slightly different shape consequent upon settlement.

20. Name the stress resultants induced in the arch section?(Apr/may-2017)

Bending moment

Shear Force

21.Name the two methods for analysis of fixed arches (Apr/may-2017)

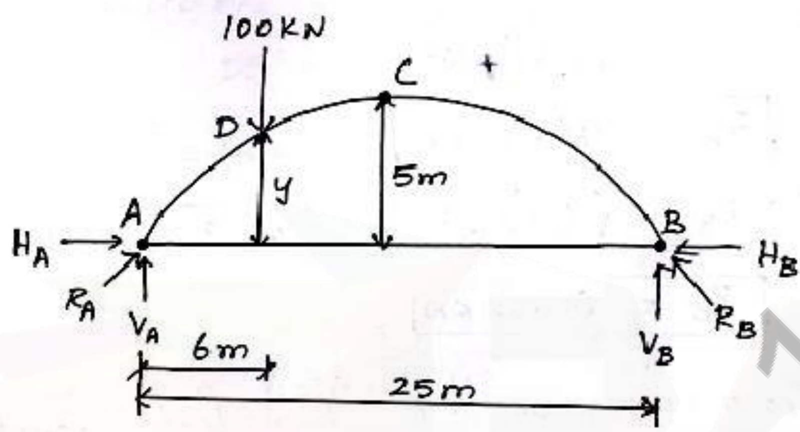
- Castiglino's theorem
- Elastic centre method
- Column analogy method.

PART - B

1. A circular (three hinged) arch of span 25 m with a central rise of 5 m is hinged at the crown and the end supports. It carries a point load of 100 kN at 6 m from the left support. Calculate

i. The reaction at the supports and

ii. Moment at 5 m from the left support. (Apr/May 2017,Nov/dec-2017,2015)
Solution:



Step 1: Vertical reactions and Horizontal thrust (H):-

i) Vertical reactions V_A and V_B:-

Taking moment about A,

$$(100 \times 6) - V_B(25) = 0$$

$$\boxed{V_B = 24\ KN}$$

$\Sigma V = 0$,

$$V_A + V_B = 100$$

$$\therefore V_A + 24 = 100$$

$$\boxed{V_A = 76\ KN}$$

ii) Horizontal thrust (H):-

Taking moment about C,

$$-V_B \times 12.5 + (H \times 5) = 0$$

$$H = \frac{24 \times 12.5}{5}$$

$$\boxed{H = 60\ KN}$$

Step 2 : Reactions R_A and R_B :-

$$R_A = \sqrt{V_A^2 + H^2}$$

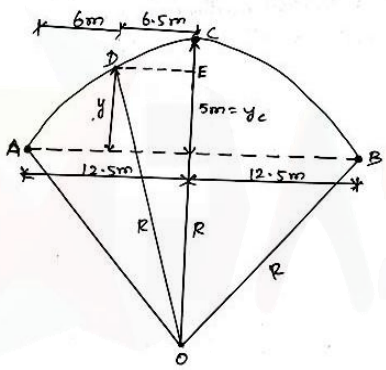
$$= \sqrt{(76)^2 + (60)^2}$$

$$\boxed{R_A = 96.83\ KN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{(24)^2 + (60)^2}$$

$$\boxed{R_B = 64.62\ KN}$$

Step 3 : Moment at 5m from left support :-



To find R :-

$$(2R - y_c)\, y_c = (l/2)^2$$

$$(2R - 5)\,5 = (12.5)^2$$

$$10R - 25 = 156.25$$

$$\boxed{R = 18.125\,m}$$

To find y :-

In ΔODE,

$$OD^2 = DE^2 + OE^2$$

$$(R)^2 = (6.5)^2 + (R - y_c + y)^2$$

$$(18.125)^2 = (6.5)^2 + (18.125 - 5 + y)^2$$

$$328.52 = 42.25 + (13.125 + y)^2$$

$$286.27 = (13.125 + y)^2$$

Taking square root on both sides,

$$16.92 = 13.125 + y$$

$$\boxed{y = 3.795 \text{ m}}$$

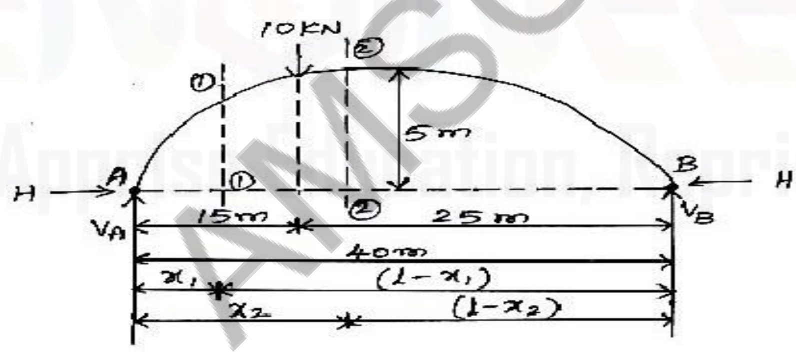
$$(BM)_D = V_A \times 6 - H \times 3.795$$

$$= (76 \times 6) - (60 \times 3.795)$$

$$\boxed{(BM)_D = 228.3 \text{ kNm.}}$$

2. A parabolic two hinged arch has a span of 40 m and a rise of 5 m. a concentrated load 10 kN acts at 15 m from the left support. The second moment of area varies as the secant of the inclination of the arch axis. Calculate the horizontal thrust and reactions at the hinge. Also calculate maximum bending moment at the section. (Nov/dec- 2019,Apr/may-2018,2016)

Solution:



Step 1: Vertical Reactions :-

$\Sigma V = 0$

$$V_A + V_B = 10$$

Taking moment about A,

$$-V_B \times 40 + (10 \times 15) = 0$$

$$\boxed{V_B = 3.75 \text{ kN}}$$

$$\therefore \; V_A = 10 - 3.75$$

$$\boxed{V_A = 6.25 \text{ kN}}$$

Step 2: Horizontal Thrust (H):-

$$H = \frac{\int_0^l \mu y \, dx}{\int_0^l y^2 dx}$$

$$H = \frac{\int_0^{15} \mu_1 y \, dx + \int_{15}^{40} \mu_2 y \, dx}{\int_0^{40} y^2 dx} = \frac{Nr(1) + Nr(2)}{Dr}$$

where, μ_1 = Beam bending moment in AX.

μ_2 = Beam bending moment in XB.

i) Denominator:-

$$y = \frac{4y_c}{l^2} x(l-x) = \frac{4 \times 5}{(40)^2}(40x - x^2)$$

$$y = 0.5x - 0.0125x^2$$

$$\therefore \; Dr = \int_0^{40} y^2 dx$$

$$= \int_0^{40} (0.5x - 0.0125x^2)^2 dx$$

$$= \int_0^{40} \left(0.25x^2 + 1.56 \times 10^{-4} x^4 - 0.0125 x^3\right) dx$$

$$= \left[\frac{0.25x^3}{3} + \frac{1.56 \times 10^{-4} x^5}{5} - \frac{0.0125 x^4}{4}\right]_0^{40}$$

$$= \left[(5333.33 + 3194.88 - 8000) - 0\right]$$

$$\boxed{Dr = 528.21}$$

ii) Numerator (1) :-

$$Nr(1) = \int_0^{15} \mu_1 y\, dx$$

here, $\mu_1 = V_A x_1 = 6.25 x_1 = 6.25x$

$$Nr(1) = \int_0^{15} 6.25x(0.5x - 0.0125x^2)\, dx$$

$$= \int_0^{15} (3.125x^2 - 0.078x^3)\, dx$$

$$= \left[\frac{3.125x^3}{3} - \frac{0.078x^4}{4}\right]_0^{15}$$

$$= [(3515.63 - 987.18) - 0]$$

$$\boxed{Nr(1) = 2528.45}$$

iii) Numerator (2) :-

$$Nr(2) = \int_{15}^{40} \mu_2 y\, dx$$

here, $\mu_2 = V_A \times x_2 - 10(x_2 - 15)$

$$= 6.25x - 10x + 150$$

$$\mu_2 = 150 - 3.75x$$

$$Nr(2) = \int_{15}^{40} (150 - 3.75x)(0.5x - 0.0125x^2)\, dx$$

$$= \int_{15}^{40} (75x - 1.875x^2 - 1.875x^2 + 0.047x^3)\, dx$$

$$= \int_{15}^{40} (75x - 3.75x^2 + 0.047x^3)\, dx$$

$$= \left[\frac{75x^2}{2} - \frac{3.75x^3}{3} + \frac{0.047x^4}{4}\right]_{15}^{40}$$

$$= [(60000 - 80000 + 30080) - (8437.5 - 4218.75 + 594.84)]$$

$$\boxed{N_r(2) = 5266.41}$$

$$\therefore \quad H = \frac{N_r(1) + N_r(2)}{D_r} = \left(\frac{2528.45 + 5266.41}{528.21}\right)$$

$$\boxed{H = 14.76\ kN}$$

Step 3 : Reactions at A and B :-

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{(6.25)^2 + (14.76)^2}$$

$$\boxed{R_A = 16.03\ kN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{(3.75)^2 + (14.76)^2}$$

$$\boxed{R_B = 15.22\ kN}$$

Step 4 : Maximum Bending Moment :-

$$M_x = V_A \times 15 - H \times y$$

here, $y = \frac{4y_c}{l^2} x(l - x) = \frac{4 \times 5}{(40)^2} \times 15(40 - 15)$

$$\boxed{y = 4.68\ m}$$

$$\therefore \quad M_x = (6.25 \times 15) - (14.76 \times 4.68)$$

$$\boxed{M_{max} = 24.67\ kNm}$$

3. A symmetrical three hinged circular arch has a span of 13 m and a rise to the central hinge of 3 m. it carries a vertical load of 15 kN at 3 m from the left hand end. Find

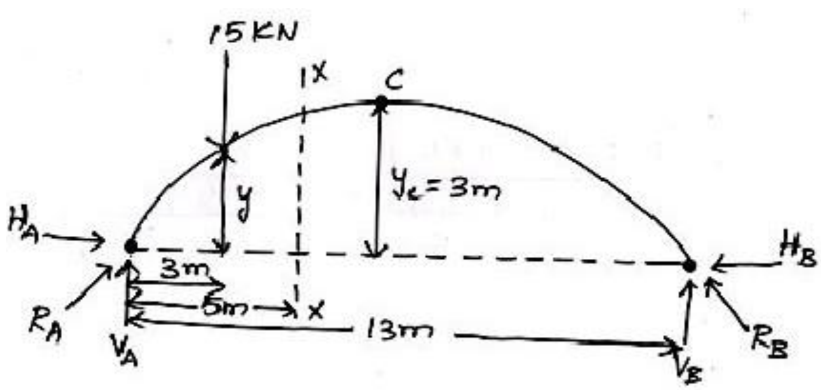
i. The reactions at the supports,

ii. Magnitude of the thrust at the springing,

iii. Bending moment at 5 m from the left hand hinge and

iv. The maximum positive and negative bending moment. (Nov/Dec 2012)

Solution:



Step 1: Vertical Reactions :-

$\Sigma V = 0.$

$V_A + V_B = 15$

Taking moment about A,

$-V_B \times 13 + (15 \times 3) = 0$

$\boxed{V_B = 3.46\ KN}$

$V_A + 3.46 = 15$

$\boxed{V_A = 11.54\ KN}$

Step 2: Horizontal thrust :-

Taking moment about C,

$V_A \times 6.5 - H_A \times y_c - 15 \times 3.5 = 0$

$11.54 \times 6.5 - 3 H_A - 15 \times 3.5 = 0$

$\boxed{H_A = 7.5\ KN}\ (\rightarrow)$

$\Sigma H = 0$

$H_A = -H_B$

$\boxed{H_B = 7.5\ KN}\ (\leftarrow)$

Step 3: Resultant Reactions :-

$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(11.54)^2 + (7.5)^2}$

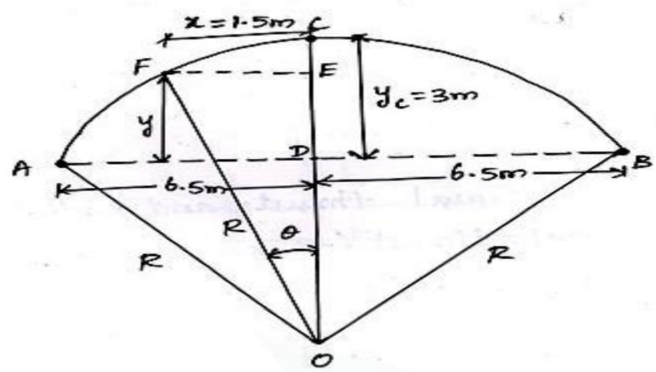
$\boxed{R_A = 13.76\ KN}$

$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(3.46)^2 + (7.5)^2}$

$\boxed{R_B = 8.25\ KN}$

Step 4 : Bending Moment at 5m from left support.

In the Bending moment at $x = 5m$ from the left support, we find the radius and y value by using the formula.



To find Radius(R):-

$$(2R - y_c)\,y_c = \left(\frac{l}{2}\right)^2$$

$$(2R - 3)\times 3 = \left(\frac{13}{2}\right)^2$$

$$\boxed{R = 8.54m}$$

To find y :-

In Δ OFE,

$$R^2 = x^2 + (R - y_c + y)^2$$

$$R^2 = (1.5)^2 + (R - 3 + y)^2$$

$$(8.54)^2 = (1.5)^2 + (8.54 - 3 + y)^2$$

$$70.68 = (5.54 + y)^2$$

$$8.41 = 5.54 + y$$

$\boxed{y = 2.86\,m}$ at $x = 1.5m$ from centre.

$$B.M = V_A \times 5 - H_A(y) - 15 \times 2$$

$$= (11.54 \times 5) - (7.5 \times 2.86) - (15 \times 2)$$

$$\boxed{B.M = 6.25\ KNm}$$

step 5 : Normal thrust and Radial shear :-

i) Normal thrust (at $x = 5m$ from A).

$$N_x = V \sin\theta + H \cos\theta$$

here,

$$\theta = \tan^{-1}\left(\frac{FE}{OE}\right)$$

$$= \tan^{-1}\left(\frac{1.5}{8.4}\right)$$

$$\boxed{\theta = 10^\circ\ 7'}$$

V = Net vertical shear force at $x = 5m$ from A.

$$= V_A - 15$$

$$= 11.54 - 15$$

$$\boxed{V = -3.46\ KN}$$

$$N_x = (-3.46 \times \sin(10^\circ\ 7')) + (7.5 \times \cos(10^\circ\ 7'))$$

$$\boxed{N_x = 6.77\ KN}$$

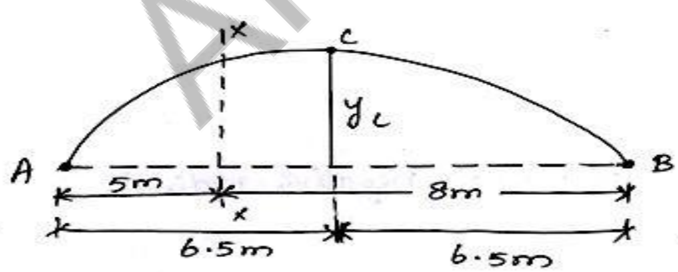
ii) Radial shear :-

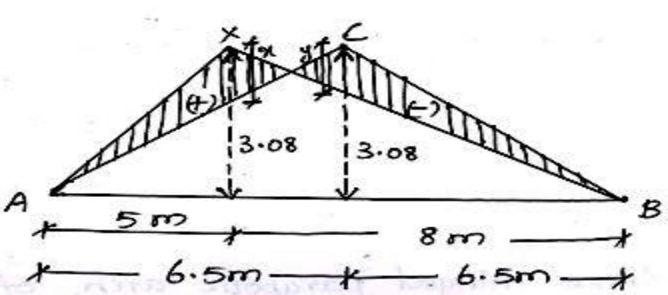
$$R_x = V \cos\theta - H \sin\theta$$

$$= (-3.46 \times \cos(10^\circ\ 7')) - (7.5 \times \sin(10^\circ\ 7'))$$

$$\boxed{R_x = -4.72\ KN}$$

step 6 : Maximum Bending Moment :-





Maximum positive (sagging) ordinate, $= \dfrac{x(l-x)}{l}$

$$= \frac{5(13-5)}{13}$$

$$= 3.08$$

Net Maximum positive ordinate, $x = 3.08 - \dfrac{3.08}{6.5} \times 5$

$$x = 0.71$$

Maximum positive moment = load x ordinate

$$= 15 \times 0.71$$

$$= 10.65 \text{ KNm}$$

Maximum Negative (hogging) ordinate $= \dfrac{x(l-x)}{l}$

$$= \frac{5(13-5)}{13}$$

$$= 3.08$$

Net maximum negative ordinate $= 3.08 - \dfrac{3.08}{8} \times 6.5$

$$= 0.58$$

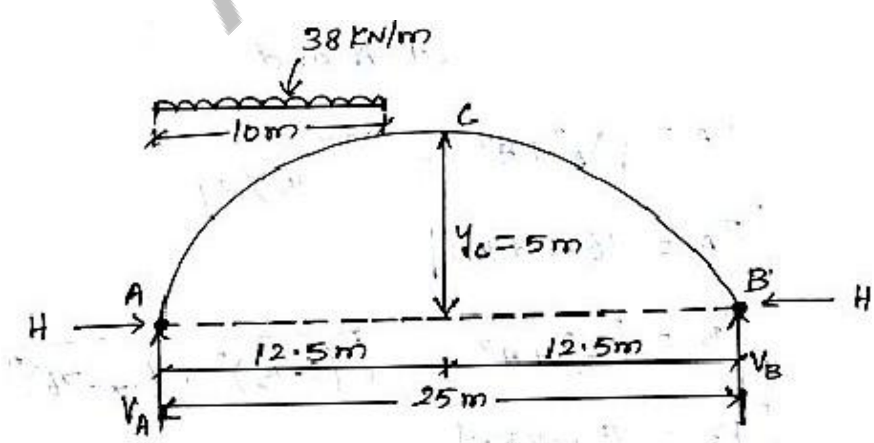
Maximum Negative moment = Load x ordinate

$$= 15 \times 0.58$$

$$= 8.7 \text{ KNm}.$$

4. A two hinged parabolic arch of span 25 m and rise 5 m carries an udl of 38 kN/m covering a distance of 10 m from left end. Find the horizontal thrust, the reactions at the hinges and the maximum negative moment. (Nov/Dec 2012, 2013, May/June 2014,2015)

Solution:



step 1: Vertical Reactions:-

$\Sigma V = 0$

$V_A + V_B = 38 \times 10 = 380$

Taking moment about A,

$-V_B \times 25 + 38 \times \frac{(10)^2}{2} = 0$

$\boxed{V_B = 76\ kN}$

$\therefore V_A = 380 - 76$

$\boxed{V_A = 304\ kN}$

step 2: Horizontal thrust (H):-

$$H = \frac{\int_0^l (\mu_1 y + \mu_2 y)\,dx}{\int_0^l y^2\,dx}$$

$$y = \frac{4y_c}{l^2}(lx - x^2) = \frac{4 \times 5}{(25)^2}(25x - x^2)$$

$$y = 0.8x - 0.032x^2$$

i) Denominator:-

$$Dr = \int_0^{25} y^2\,dx = \int_0^{25} (0.8x - 0.032x^2)\,dx$$

$$= \int_0^{25} (0.64x^2 + 1.02 \times 10^{-3} x^4 - 0.051x^3)\,dx$$

$$= \left[\frac{0.64x^3}{3} + \frac{1.02 \times 10^{-3} x^5}{5} - \frac{0.051x^4}{4}\right]_0^{25}$$

$$= [(3333.33 + 1992.18 - 4980.46) - 0]$$

$\boxed{Dr = 345.05}$

ii) Numerator (1) :- (loaded portion)

$$M_1 = V_A \times x - \frac{38 \times x^2}{2} = 304x - 19x^2$$

$$Nr(1) = \int_0^{10} M_1\, y\, dx = \int_0^{10} (304x - 19x^2) \times (0.8x - 0.032x^2)\, dx$$

$$= \int_0^{10} (243.2x^2 - 9.728x^3 - 15.2x^3 + 0.608x^4)\, dx$$

$$= \int_0^{10} (243.2x^2 - 24.93x^3 + 0.608x^4)\, dx$$

$$= \left[\frac{243.2x^3}{3} - \frac{24.93x^4}{4} + \frac{0.608x^5}{5}\right]_0^{10}$$

$$= \left[(81066.67 - 62325 + 12160) - 0\right]$$

$$\boxed{Nr(1) = 30901.67}$$

iii) Numerator (2) :-

$$M_2 = V_B \times (l - x) = 76(25 - x) = 1900 - 76x$$

$$Nr(2) = \int_{10}^{25} M_2\, y\, dx$$

$$= \int_{10}^{25} (1900 - 76x)(0.8x - 0.032x)\, dx$$

$$= \int_{10}^{25} (1520x - 60.8x^2 - 60.8x^2 + 2.43x^3)\, dx$$

$$= \left[\frac{1520x^2}{2} - \frac{121.6x^3}{3} + \frac{2.43x^4}{4}\right]_{10}^{25}$$

$$= \left[(475000 - 633333.33 + 237304.68) - (76000 - 40533.33 + 6075)\right]$$

$$= 78971.35 - 41541.67$$

$$\boxed{Nr(2) = 37429.68}$$

$$H = \frac{N_r(1) + N_r(2)}{D_r}$$

$$= \left(\frac{30901.67 + 37429.68}{345.05}\right)$$

$$\boxed{H = 198.03 \text{ KN}}$$

Step 3 : Resultant Reactions :-

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{(304)^2 + (198.03)^2}$$

$$\boxed{R_A = 362.81 \text{KN}}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{(76)^2 + (198.03)^2}$$

$$\boxed{R_B = 212.11 \text{ KN}}$$

Step 4 : Maximum Bending Moment :-

The maximum negative bending moment occurs at right of the span.

$$x = 3/4\, l = 3/4 \times 25 = 18.75 \text{m from A.}$$

$$l - x = 25 - 18.75 = 6.25 \text{m from B.}$$

$$B.M = V_B \times 6.25 - H \times y_x$$

here, $y = \frac{4 y_c}{l^2} x(l-x)$

$$= \frac{4 \times 5}{(25)^2} \times 18.75 \times 6.25$$

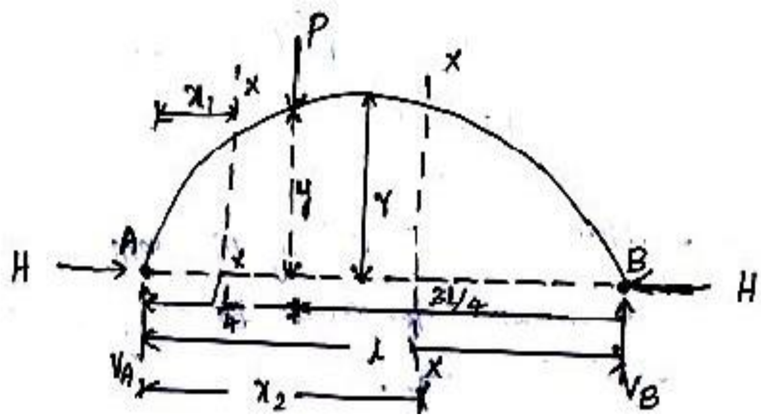
$$y = 3.75 \text{m.}$$

$$\therefore\ M_x = (76 \times 6.25) - (198.03 \times 3.75)$$

$$\boxed{M_x = -267.61 \text{KNm.}}$$

5. Derive the expression for horizontal thrust in a two hinged parabolic arch carrying a point load P at distance one fourth spans from left support. Assume I =Io sec θ. (AUC Apr/May 2011)

Solution:



Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = P$$

Taking moment about A,

$$-V_B \times l + P\frac{l}{4} = 0$$

$$\boxed{V_B = P/4}$$

$$V_A = P - P/4$$

$$\boxed{V_A = 3P/4}$$

Step 2: Horizontal Thrust (H) :-

$$H = \frac{\int M y\,dx}{\int y^2 dx} = \frac{\int_0^{l/4} M_1 y\,dx + \int_{l/4}^{l} M_2 y\,dx}{\int_0^l y^2 dx}$$

$$y = \frac{4y_c}{l^2} x(l-x) = \frac{4r}{l^2}(lx - x^2).$$

i) Denominator :-

$$Dr = \int_0^l y^2 dx = \int_0^l \left[\frac{4r}{l^2}(lx - x^2)\right]^2 dx$$

$$= \int_0^l \frac{16r^2}{l^4}(l^2x^2+x^4-2lx^3)\,dx$$

$$= \frac{16r^2}{l^4}\int_0^l (l^2x^2+x^4-2lx^3)\,dx$$

$$= \frac{16r^2}{l^4}\left[\frac{l^2x^3}{3}+\frac{x^5}{5}-\frac{2lx^4}{4}\right]_0^l$$

$$= \frac{16r^2}{l^4}\left[\left(\frac{l^5}{3}+\frac{l^5}{5}-\frac{l^5}{4}\right)-0\right]$$

$$= \frac{16r^2l^5}{l^4}\left[\frac{1}{3}+\frac{1}{5}-\frac{1}{4}\right]$$

$$= 16r^2l\,(0.283)$$

$$\boxed{Dr = 4.528\,r^2l}$$

ii) Numerator (1):-

$$Nr(1) = \int_0^{l/4} \mu_1 y\,dx$$

here, $\mu_1 = V_A \times x_1 = \frac{3Px}{4}$

$$Nr(1) = \int_0^{l/4} \frac{3Px}{4}\left[\frac{4r}{l^2}(lx-x^2)\right]dx$$

$$= \frac{3Pr}{l^2}\int_0^{l/4}(lx^2-x^3)\,dx$$

$$= \frac{3Pr}{l^2}\left[\frac{lx^3}{3}-\frac{x^4}{4}\right]_0^{l/4}$$

$$= \frac{3Pr}{l^2}\left[\frac{l}{3}(l/4)^3-\frac{1}{4}(l/4)^4\right]$$

$$= \frac{3Pr}{l^2}\left[\frac{l^4}{192}-\frac{l^4}{1024}\right]$$

$$= \frac{3Prl^4}{l^2}\left(\frac{1}{192}-\frac{1}{1024}\right)$$

$$\boxed{Nr(1) = 0.0126\,Prl^2}$$

iii) Numerator (2) :-

$$N_r(2) = \int_{l/4}^{l} M_2\, y\, dx$$

here, $M_2 = V_A \times x_2 - P(x_2 - l/4)$

$$= \frac{3Px}{4} - Px + Pl/4$$

$$M_2 = \frac{Pl}{4} - \frac{Px}{4}$$

$$N_r(2) = \int_{l/4}^{l} \left(\frac{Pl}{4} - \frac{Px}{4}\right)\left(\frac{4r}{l^2}(lx - x^2)\right) dx$$

$$= \frac{4r}{l^2}\int_{l/4}^{l} \left(\frac{Pl^2x}{4} - \frac{Plx^2}{4} - \frac{Plx^2}{4} + \frac{Px^3}{4}\right) dx$$

$$= \frac{4r}{l^2}\int_{l/4}^{l} \left(\frac{Pl^2x}{4} - \frac{Plx^2}{2} + \frac{Px^3}{4}\right) dx$$

$$= \frac{4r}{l^2}\left[\frac{Pl^2x^2}{8} - \frac{Plx^3}{6} + \frac{Px^4}{16}\right]_{l/4}^{l}$$

$$= \frac{4r}{l^2}\left[\left(\frac{Pl^4}{8} - \frac{Pl^4}{6} + \frac{Pl^4}{16}\right) - \left(\frac{Pl^4}{128} - \frac{Pl^4}{384} + \frac{Pl^4}{4096}\right)\right]$$

$$= \frac{4r}{l^2}\left[\frac{Pl^4}{48} - \frac{67Pl^4}{12288}\right]$$

$$= \frac{4rPl^4}{l^2}(0.015)$$

$$\boxed{N_r(2) = 0.06\, Prl^2}$$

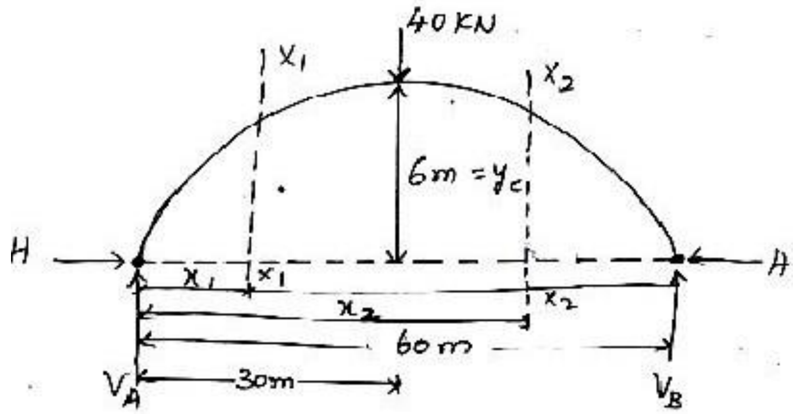
$$\therefore\ H = \frac{N_r(1) + N_r(2)}{D_r} = \frac{0.0126\, Prl^2 + 0.06\, Prl^2}{4.528\, r^2 l}$$

$$= \frac{0.0726\, Prl^2}{4.528\, r^2 l}$$

$$\boxed{H = \frac{0.016\, Pl}{r}}$$

7. A parabolic two hinged arch of span 60 m and central rise of 6 m is subjected to a crown load of 40 kN. Allowing rib shortening and temperature rise of 20^o C, determine horizontal thrust, H. IC = 6x10^5 cm^4, AC = 1000 cm^2, E = 1x10^4 MPa, α = 11x10^{-6} /^{o} C, I = IC sec θ. (Apr/may 2017)

Solution:



Horizontal thrust, $H = H_1 + H_2$

here, H_1 = horizontal thrust under the load.

H_2 = horizontal thrust due to temperature rise

$$H_2 = \frac{l\alpha T EI}{\int_0^l y^2 ds}$$

$$H_1 = \frac{\int \mu y \, dx}{\int y^2 dx}$$

Step 1: Vertical Reactions:-

$$\Sigma V = 0$$

$$V_A + V_B = 40$$

Taking moment about A,

$$-V_B \times 60 + (40 \times 30) = 0$$

$$\boxed{V_B = 20 \text{ KN}}$$

$$V_A = 40 - 20$$

$$\boxed{V_A = 20 \text{ KN}}$$

Step 2: Horizontal thrust (H_1):-

$$H_1 = \frac{\int \mu y\,dx}{\int y^2 dx} = \frac{\int_0^{30} \mu_1 y\,dx + \int_{30}^{60} \mu_2 y\,dx}{\int_0^{60} y^2 dx}$$

$$H_1 = \frac{N_r(1) + N_r(2)}{D_r}$$

i) Denominator:-

$$D_r = \int_0^{60} y^2 dx$$

here, $y = \frac{4y_c}{l^2} x(l-x) = \frac{4\times 6}{(60)^2}(lx - x^2)$

$$= \frac{24}{(60)^2}(60x - x^2)$$

$$\boxed{y = 0.4x - 0.0067x^2}$$

$$D_r = \int_0^{60} (0.4x - 0.0067x^2)^2 dx$$

$$= \int_0^{60} (0.16x^2 + 4.48\times10^{-5}x^4 - 0.0054x^3)\,dx$$

$$= \left[\frac{0.16x^3}{3} + \frac{4.48\times10^{-5}x^5}{5} - \frac{0.0054x^4}{4}\right]_0^{6}$$

$$= [(11520 + 6967.29 - 17496) - 0]$$

$$\boxed{D_r = 991.29}$$

ii) Numerator (1):-

$$N_r(1) = \int_0^{30} \mu_1 y\,dx = \int_0^{30} (20x)(0.4x - 0.0067x^2)\,dx$$

$$= \int_0^{30} (8x^2 - 0.134x^3)\,dx$$

$$= \left(\frac{8x^3}{3} - \frac{0.134x^4}{4}\right)_0^{30}$$

$$= [(72000 - 27135) - 0]$$

$$\boxed{N_r(1) = 44865}$$

iii) Numerator(2) :-

$$N_r(2) = \int_{30}^{60} M_2 \, y \, dx$$

here, $M_2 = 20x - 40x + 1200$

$M_2 = 1200 - 20x$

$$N_r(2) = \int_{30}^{60} (1200-20x)(0.4x - 0.0067x^2)\,dx$$

$$= \int_{30}^{60} (480x - 8.04x^2 - 8x^2 + 0.134x^3)\,dx$$

$$= \int_{30}^{60} (480x - 16.04x^2 + 0.134x^3)\,dx$$

$$= \left[\frac{480x^2}{2} - \frac{16.04x^3}{3} + \frac{0.134x^4}{4}\right]_{30}^{60}$$

$$= \left[(864000 - 1154880 + 434160) - (216000 - 144360 + 27135)\right]$$

$$\boxed{N_r(2) = 44505}$$

$$\therefore H_1 = \frac{N_r(1) + N_r(2)}{D_r}$$

$$= \left(\frac{44865 + 44505}{991.29}\right)$$

$$\therefore \boxed{H_1 = 90.16 \text{ kN}}$$

Step 3: Increased Horizontal thrust :-

$$H_2 = \frac{l \alpha T E I}{\int_0^l y^2 ds}$$

here,

$l = 60\,m$;

$\alpha = 11 \times 10^{-6} /^\circ c$

$T = 20^\circ c$.

$E = 1 \times 10^4 MPa = 1 \times 10^{10} N/m^2$.

$I = I_c \sec\theta$

$I_c = 6 \times 10^5\, cm^4 = 6 \times 10^5 \times 10^{-8} = 0.006\, m^4$

$$y = \frac{4r}{l^2}(lx - x^2)$$

$$\theta = \frac{dy}{dx} = \frac{4r}{l^2}(l - 2x)$$

$$= \frac{4 \times 6}{(60)^2}(60 - 2(30))$$

$$\theta = 0.$$

$$\therefore \quad I = I_c \sec\theta$$

$$= \frac{0.006}{\cos(0)}$$

$$I = 0.006\, m^4$$

$$y = \frac{4y_c}{l^2}(lx - x^2)$$

$$= \frac{4 \times 6}{(60)^2}(60x - x^2)$$

$$y = 0.4x - 0.0067x^2$$

$$H_2 = \frac{l \alpha T E I}{\int_0^{60} y^2 dx}$$

here,

$$Dr = \int_0^{60} y^2 dx = \int_0^{60} (0.4x - 0.0067x^2)^2 dx$$

$$= \int_0^{60} (0.16x^2 + 4.489\times10^{-5}x^4 - 5.36\times10^{-3}x^3) dx$$

$$= \left[\frac{0.16x^3}{3} + \frac{4.489\times10^{-5}x^5}{5} - \frac{5.36\times10^{-3}x^4}{4} \right]_0^{60}$$

$$= \left[(11520 + 6981.29 - 17366.4) - 0 \right]$$

$$\boxed{Dr = 1134.89}$$

$$\therefore H_2 = \left(\frac{60\times11\times10^{-6}\times20\times1\times10^{10}\times0.006}{1134.89} \right)$$

$$H_2 = 697.86\ N$$

$$\boxed{H_2 = 0.697\,KN}$$

$$H = H_1 + H_2 = 90.16 + 0.697$$

$$\boxed{H = 90.857\,KN}$$

8. A Symmetrical three hinged circular arch has s span of 13 m and rise to the central hinge of 3.5 m as shown if Fig. Q. No. 13(a). It carries a vertical load of 15 kN at 3m from the left hand end. Find:

i. **The magnitude of horizontal thrust at supports**
ii. **The reaction at the suppots**
iii. **Bending moment of 5 m from the left hand hinge and**
iv. **The maximum positive and negative moment. (Nov/dec-2015)**

Step 1: Vertical Reactions :-

$\Sigma V = 0.$

$V_A + V_B = 15$

Taking moment about A,

$-V_B \times 13 + (15 \times 3) = 0$

$\boxed{V_B = 3.46 \text{ KN}}$

$V_A + 3.46 = 15$

$\boxed{V_A = 11.54 \text{ KN}}$

Step 2: Horizontal thrust :-

Taking moment about C,

$V_A \times 6.5 - H_A \times y_c - 15 \times 3.5 = 0$

$11.54 \times 6.5 - 3 H_A - 15 \times 3.5 = 0$

$\boxed{H_A = 7.5 \text{ KN}} (\rightarrow)$

$\Sigma H = 0$

$H_A = -H_B$

$\boxed{H_B = 7.5 \text{ KN}} (\leftarrow)$

Step 3: Resultant Reactions :-

$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(11.54)^2 + (7.5)^2}$

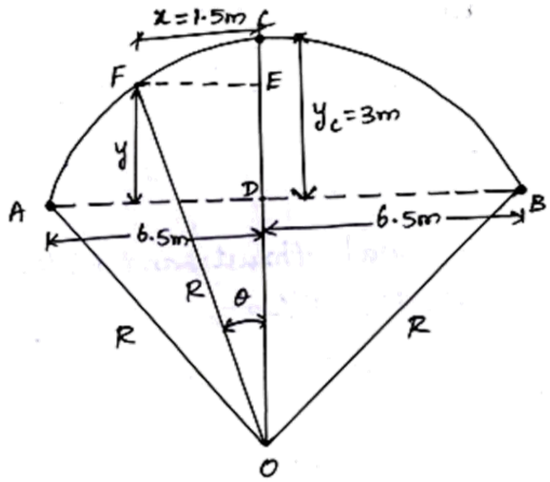
$\boxed{R_A = 13.76 \text{ KN}}$

$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(3.46)^2 + (7.5)^2}$

$\boxed{R_B = 8.25 \text{ KN}}$

Step 4: Bending Moment at 5m from left support.

In the Bending moment at $x = 5$m from the left support, we find the radius and y value by using the formula.



To find Radius(R):-

$$(2R - y_c)\, y_c = \left(\frac{l}{2}\right)^2$$

$$(2R - 3) \times 3 = \left(\frac{13}{2}\right)^2$$

$$\boxed{R = 8.54m}$$

To find y :-

In Δ OFE,

$$R^2 = x^2 + (R - y_c + y)^2$$

$$R^2 = (1.5)^2 + (R - 3 + y)^2$$

$$(8.54)^2 = (1.5)^2 + (8.54 - 3 + y)^2$$

$$70.68 = (5.54 + y)^2$$

$$8.41 = 5.54 + y$$

$\boxed{y = 2.86\,m}$ at $x = 1.5m$ from centre.

$$B.M = V_A \times 5 - H_A(y) - 15 \times 2$$

$$= (11.54 \times 5) - (7.5 \times 2.86) - (15 \times 2)$$

$$\boxed{B.M = 6.25\ KNm}$$

Step 5 : Normal thrust and Radial shear :-

i) Normal thrust (at $x = 5m$ from A).

$$N_x = V \sin\theta + H \cos\theta$$

here,

$$\theta = \tan^{-1}\left(\frac{FE}{OE}\right)$$

$$= \tan^{-1}\left(\frac{1.5}{8.4}\right)$$

$$\boxed{\theta = 10^\circ\ 7'}$$

V = Net vertical shear force at $x = 5m$ from A.

$$= V_A - 15$$

$$= 11.54 - 15$$

$$\boxed{V = -3.46\ KN}$$

$$N_x = (-3.46 \times \sin(10^\circ\ 7')) + (7.5 \times \cos(10^\circ\ 7'))$$

$$\boxed{N_x = 6.77\ KN}$$

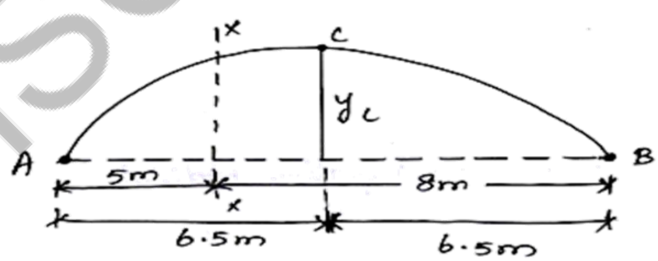
ii) Radial shear :-

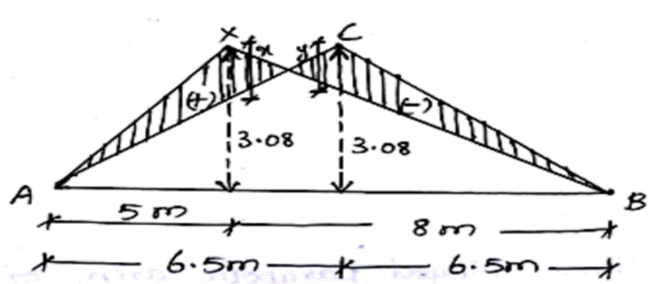
$$R_x = V \cos\theta - H \sin\theta$$

$$= (-3.46 \times \cos(10^\circ\ 7')) - (7.5 \times \sin(10^\circ\ 7'))$$

$$\boxed{R_x = -4.72\ KN}$$

Step 6 : Maximum Bending Moment :-





Maximum positive (sagging) ordinate, $= \frac{x(l-x)}{l}$

$= \frac{5(13-5)}{13}$

$= 3.08$

Net Maximum positive ordinate, $x = 3.08 - \frac{3.08}{6.5} \times 5$

$x = 0.71$

Maximum positive moment = Load x ordinate

$= 15 \times 0.71$

$= 10.65$ KNm

Maximum Negative (hogging) ordinate $= \frac{x(l-x)}{l}$

$= \frac{5(13-5)}{13}$

$= 3.08$

Net maximum negative ordinate $= 3.08 - \frac{3.08}{8} \times 6.5$

$= 0.58$

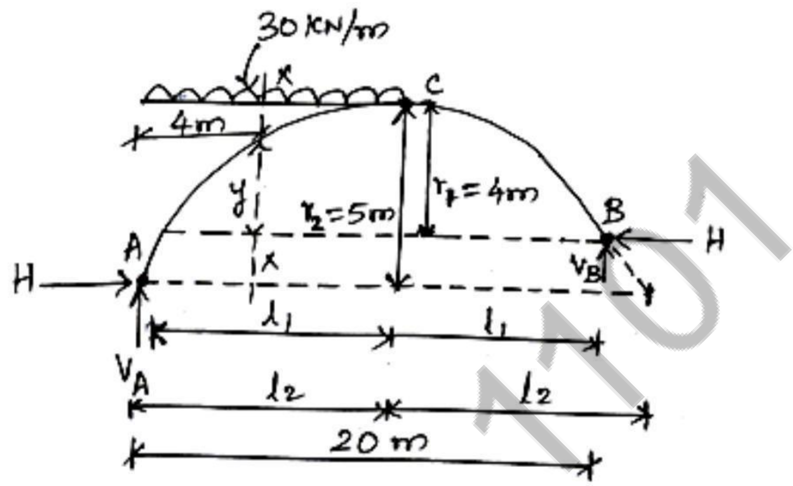
Maximum Negative moment = Load x ordinate

$= 15 \times 0.58$

$= 8.7$ KNm.

9.A parabolic 3 hinged arch shown in Fig. 13(b) carries loads as indicated determine.

i. **Resultant reactions at the end supports**
ii. **Bending moment,radial shear and normal thrust at D, 4m from A.(Nov/dec-2016)**



Step 1 : To find l_1 and l_2 :-

$$\frac{l_1}{l_2} = \sqrt{\frac{r_1}{r_2}}$$

$$\frac{l_1}{20-l_1} = \sqrt{\frac{4}{5}} = 0.89$$

$$l_1 = 0.89\,(20-l_1)$$

$$l_1 = 17.8 - 0.89\,l_1$$

$$1.89\,l_1 = 17.8$$

$$\boxed{l_1 = 9.42\text{ m}}$$

$$l_2 = 20 - l_1 = 20 - 9.42$$

$$\boxed{l_2 = 10.58\text{ m}}$$

Step 2 : Vertical and Horizontal Reactions :-

Considering the portion to left of c and taking moment,

$$V_A \times 10.58 - H \times 5 - 30 \times \frac{(10.58)^2}{2} = 0$$

$$10.58\,V_A - 5H = 1679 \rightarrow (1)$$

Taking moment about right of C,

$$-V_B \times 9.42 + H \times 4 = 0$$

$$-9.42 V_B + 4H = 0 \rightarrow (2)$$

By $\Sigma V = 0$,

$$V_A + V_B = 30 \times 10 = 300$$

$$V_B = 300 - V_A \rightarrow (3)$$

Sub (3) in (2),

$$-9.42(300 - V_A) + 4H = 0$$

$$-2826 + 9.42 V_A + 4H = 0$$

$$+9.42 V_A + 4H = 2826 \rightarrow (4)$$

By solving eqn. (1) and (4),

$$\boxed{V_A = 233.12 \text{ KN}}$$

$$\boxed{H = 157.49 \text{ KN}}$$

(3) $\Rightarrow$ $\boxed{V_B = 66.88 \text{ KN}}$

Step 3: Bending Moment at $x = 4m$ from A :-

$$BM_x = V_A \times 4 - H \times y - 30 \times \frac{(4)^2}{2}$$

here, $y = \frac{4y_c}{l^2} x (l - x)$

$$= \frac{4 \times 5}{(2 \times 10.58)^2} \times 4((2 \times 10.58) - 4)$$

$$\boxed{y = 3.05 m}$$

$$M_x = (233.12 \times 4) - (157.49 \times 3.05) - 240$$

$$\boxed{M_x = 212.14 \text{ KNm}}$$

Step 4: Normal thrust and Radial shear :-

i) Normal thrust :-

$$N = V \sin\theta + H \cos\theta.$$

here,

V = Net vertical shear force at $x = 4m$ from A,

$= V_A - 30 \times 4$

$= 233.12 - 120$

$$\boxed{V = 113.12 \text{ KN}}$$

$H = 157.49$ KN

$$\theta = \tan^{-1}\left(\frac{4y_c}{(21.16)^2} \times (21.16 - 2 \times 4)\right)$$

$$= \tan^{-1}\left(\frac{4 \times 5}{(21.16)^2} \times (21.16 - 8)\right)$$

$$= \tan^{-1}(0.588)$$

$$\boxed{\theta = 30^\circ 27'}$$

$$N = 113.12 \times \sin(30^\circ 27') + 157.49 \times \cos(30^\circ 27')$$

$$= 57.33 + 135.77$$

$$\boxed{N = 193.1 \text{ KN}}$$

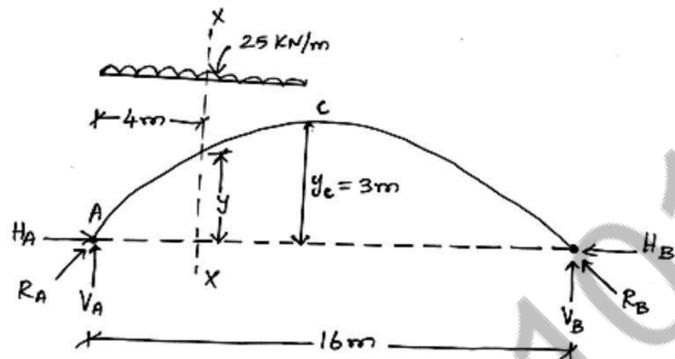
ii) Radial shear :-

$$R = V\cos\theta - H\sin\theta$$

$$= 113.12 \times \cos(30^\circ 27') - 157.49 \times \sin(30^\circ 27')$$

$$\boxed{R = 17.7 \text{ KN}}$$

10. A Uniformly distributed load of 25 kN/m covers the left span of a three hinged symmetrical parabolic arch of span 16 m and central rise 3 m. Determine the horizontal thrust and also the bending moment, shearing force and normal thrust at the loaded quarter span. (Nov/dec-2019,2017,Apr/may-2016,2018)



Step 1: Vertical Reactions V_A and V_B :-

$\Sigma V = 0$.

$$V_A + V_B = (25 \times 8) = 200 \rightarrow (1)$$

Taking moment about A,

$$-V_B \times 16 + 25 \times \left(\frac{8^2}{2}\right) = 0$$

$$\boxed{V_B = 50\ KN}$$

(1) $\Rightarrow$

$$V_A + V_B = 200$$

$$V_A + 50 = 200$$

$$V_A = 200 - 50$$

$$\boxed{V_A = 150\ KN}$$

Step 2: Horizontal Reactions :-

Taking moment about C,

$$V_A \times 8 - 25 \times \frac{8^2}{2} - H_A \times 3 = 0$$

$$(150 \times 8) - \left(25 \times \frac{8^2}{2}\right) - 3H_A = 0$$

$$\boxed{H_A = 133.33\ KN}$$

$$\therefore \boxed{H_B = 133.33\ KN}$$

Step 3 : Resultant Reactions at A and B :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(150)^2 + (133.33)^2}$$

$$\boxed{R_A = 200.69\,kN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(50)^2 + (133.33)^2}$$

$$\boxed{R_B = 142.39\,kN}$$

Step 4 : Bending Moment at $x = 4m$ from A ;-

$$BM = V_A \times 4 - 25 \times \frac{4^2}{2} - H_A \times y$$

To find y :-

For parabolic arches,

$$y = \frac{4y_c}{l^2} x(l-x)$$

$$= \frac{4 \times 3}{(16)^2} \times 4(16-4)$$

$$\boxed{y = 2.25\,m}$$

$$B.M = (150 \times 4) - \left(25 \times \frac{4^2}{2}\right) - (133.33 \times 2.25)$$

$$\boxed{B.M = 100\,kNm}$$

Step 5 : Radial shear force at $x = 4m$ from A :-

$$S.F ; R_x = V_x \cos\theta - H \sin\theta$$

here, V = Net vertical shear force at $x = 4m$ from A.

$$= V_A - w \times 4 = 150 - (25 \times 4)$$

$$\boxed{V = 50\,kN}$$

$$H = \text{Horizontal shear force} = 133.33\,kN$$

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right]$$

$$= \tan^{-1}\left(\frac{4 \times 3}{(16)^2}(16 - 2(4))\right)$$

$$= \tan^{-1}(0.375)$$

$$\boxed{\theta = 20^\circ 33'}$$

$R = 50 \times \cos(20^\circ 33') - 133.33 \times \sin(20^\circ 33')$

$\boxed{R = 0.016 \quad KN}$

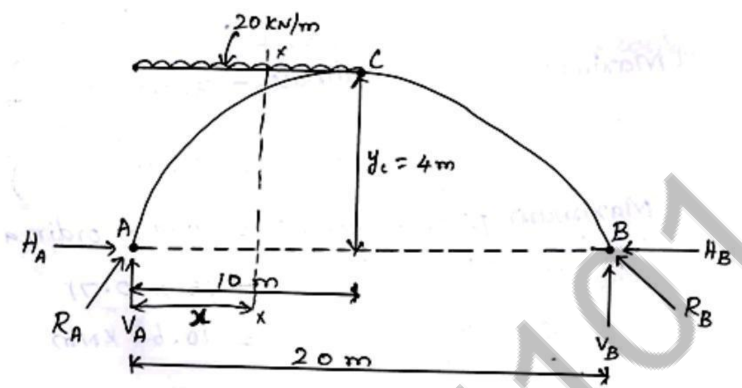
Step 6 : Normal thrust at $x = 4m$ from A :-

Normal thrust, $N_x = V_x \sin\theta + H \cos\theta$

$= 50 \times \sin(20^\circ 33') + 133.33 \times \cos(20^\circ 33')$

$\boxed{P_N = 142.39 \quad KN}$

11. A three hinged parabolic arch of span 20 m and rise 4m is subjected to a uniformly distributed load of 20 kN/m over a entire left span. Find the horizontal thrust and maximum bending moment.(Nov/dec-2015)



Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = 200$$

Taking moment about A,

$$-V_B \times 20 + \left(20 \times \frac{(10)^2}{2}\right) = 0$$

$$\boxed{V_B = 50 \text{ KN}}$$

$$V_A + 50 = 200$$

$$\boxed{V_A = 150 \text{ KN}}$$

Step 2 : Horizontal Reactions :-

$$\Sigma H = 0$$

$$H_A + H_B = 0$$

$$H_A = -H_B$$

Taking moment about c,

$$V_A \times 10 - 20 \times \frac{(10)^2}{2} - H_A \times y_c = 0$$

$$(150 \times 10) - 1000 - 4H_A = 0$$

$$\boxed{H_A = 125\,KN}$$

$$\boxed{H_B = -125\,KN} \boxed{= 125\,KN}$$

step 3 : Resultant Reactions :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(150)^2 + (125)^2}$$

$$\boxed{R_A = 195.25\,KN}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(50)^2 + (125)^2}$$

$$\boxed{R_B = 134.63\,KN}$$

step 4 : Maximum Bending Moment :-

$$BM_{xx} = V_A \times x - \frac{20 \times x^2}{2} - H_A \times y$$

here,

$$y = \frac{4 y_c}{l^2} x (l - x)$$

$$y = \frac{4 \times 4}{(20)^2} x (20x - x^2)$$

$$y = 0.8x - 0.04x^2$$

$$BM = 150x - 10x^2 - 125(0.8x - 0.04x^2)$$

$$= 150x - 10x^2 - 100x + 5x^2$$

$$BM = 50x - 5x^2$$

Differentiate with respect to x.

$$\frac{dM}{dx} = 50 - 10x$$

$$0 = 50 - 10x$$

$$\boxed{x = 5m}$$

$$\therefore \quad BM = 50x - 5x^2 = (50 \times 5) - 5 \times (5)^2$$

$$\boxed{BM = 125\ KNm}$$

12. Find the reaction components at the supports of a symmetrical parabolic fixed arch 20 m span and 3 m central rise when it is subjected to a uniformly distributed load of 2 kN/m over the left half span.(Nov/dec-2016)

Horizontal thrust, $H = H_1 + H_2$

here, H_1 = horizontal thrust under the load.

H_2 = horizontal thrust due to temperature rise

$$H_2 = \frac{l \alpha T EI}{\int_0^l y^2 ds}$$

$$H_1 = \frac{\int \mu y \, dx}{\int y^2 dx}$$

Step 1 : Vertical Reactions :-

$\Sigma V = 0$

$$V_A + V_B = 40$$

Taking moment about A,

$$-V_B \times 60 + (40 \times 30) = 0$$

$$\boxed{V_B = 20 \text{ kN}}$$

$$V_A = 40 - 20$$

$$\boxed{V_A = 20 \text{ kN}}$$

Step 2 : Horizontal thrust (H_1) :-

$$H_1 = \frac{\int \mu y \, dx}{\int y^2 dx} = \frac{\int_0^{30} \mu_1 y \, dx + \int_{30}^{60} \mu_2 y \, dx}{\int_0^{60} y^2 dx}$$

$$H_1 = \frac{N_r(1) + N_r(2)}{D_r}$$

i) Denominator :-

$$D_r = \int_0^{60} y^2 dx$$

here, $y = \frac{4 y_c}{l^2} x(l - x) = \frac{4 \times 6}{(60)^2} (lx - x^2)$

$$= \frac{24}{(60)^2} (60x - x^2)$$

$$\boxed{y = 0.4x - 0.0067x^2}$$

$$Dr = \int_0^{60} (0.4x - 0.0067x^2)^2 dx$$

$$= \int_0^{60} (0.16x^2 + 4.48\times10^{-5} x^4 - 0.0054x^3) dx$$

$$= \left[\frac{0.16x^3}{3} + \frac{4.48\times10^{-5} x^5}{5} - \frac{0.0054x^4}{4} \right]_0^6$$

$$= [(11520 + 6967.29 - 17496) - 0]$$

$$\boxed{Dr = 991.29}$$

ii) Numerator (1) :-

$$Nr(1) = \int_0^{30} M_1 y\, dx = \int_0^{30} (20x)(0.4x - 0.0067x^2) dx$$

$$= \int_0^{30} (8x^2 - 0.134x^3) dx$$

$$= \left(\frac{8x^3}{3} - \frac{0.134x^4}{4} \right)_0^{30}$$

$$= [(72000 - 27135) - 0]$$

$$\boxed{Nr(1) = 44865}$$

iii) Numerator(2) :-

$$Nr(2) = \int_{30}^{60} M_2\, y\, dx$$

here, $M_2 = 20x - 40x + 1200$

$M_2 = 1200 - 20x$

$$Nr(2) = \int_{30}^{60} (1200 - 20x)(0.4x - 0.0067x^2) dx$$

$$= \int_{30}^{60} (480x - 8.04x^2 - 8x^2 + 0.134x^3) dx$$

$$= \int_{30}^{60} (480x - 16.04x^2 + 0.134x^3) dx$$

$$= \left[\frac{480x^2}{2} - \frac{16.04x^3}{3} + \frac{0.134x^4}{4}\right]_{30}^{60}$$

$$= \left[(864000 - 1154880 + 434160) - (216000 - 144360 + 27135)\right]$$

$$\boxed{N_r(2) = 44505}$$

$$\therefore H_1 = \frac{N_r(1) + N_r(2)}{Dr}$$

$$= \left(\frac{44865 + 44505}{991.28}\right)$$

$$\therefore \boxed{H_1 = 90.16 \text{ KN}}$$

Step 3: Increased Horizontal thrust :-

$$H_2 = \frac{l \alpha T E I}{\int_0^l y^2 ds}$$

here,

$l = 60 m$;

$\alpha = 11 \times 10^{-6} /°C$

$T = 20°C$.

$E = 1 \times 10^4 MPa = 1 \times 10^{10} N/m^2$.

$I = I_c \sec\theta$

$I_c = 6 \times 10^5 cm^4 = 6 \times 10^5 \times 10^{-8} = 0.006 m^4$.

$$y = \frac{4r}{l^2}(lx - x^2)$$

$$\theta = \frac{dy}{dx} = \frac{4r}{l^2}(l - 2x)$$

$$= \frac{4 \times 6}{(60)^2}(60 - 2(30))$$

$$\theta = 0.$$

$$\therefore I = I_c \sec\theta$$

$$= \frac{0.006}{\cos(0)}$$

$$I = 0.006 m^4$$

$$y = \frac{4y_c}{l^2}(lx - x^2)$$

$$= \frac{4 \times 6}{(60)^2}(60x - x^2)$$

$$y = 0.4x - 0.0067x^2$$

$$H_2 = \frac{l \alpha T E I}{\int_0^{60} y^2 \, dx}$$

here,

$$Dr = \int_0^{60} y^2 dx = \int_0^{60} (0.4x - 0.0067x^2)^2 dx$$

$$= \int_0^{60} (0.16x^2 + 4.489 \times 10^{-5} x^4 - 5.36 \times 10^{-3} x^3) dx$$

$$= \left[\frac{0.16x^3}{3} + \frac{4.489 \times 10^{-5} x^5}{5} - \frac{5.36 \times 10^{-3} x^4}{4} \right]_0^{60}$$

$$= \left[(11520 + 6981.29 - 17366.4) - 0 \right]$$

$$\boxed{Dr = 1134.89}$$

$$\therefore H_2 = \left(\frac{60 \times 11 \times 10^{-6} \times 20 \times 1 \times 10^{10} \times 0.006}{1134.89} \right)$$

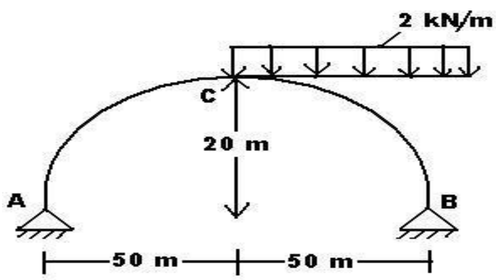
$$H_2 = 697.86 \text{ N}$$

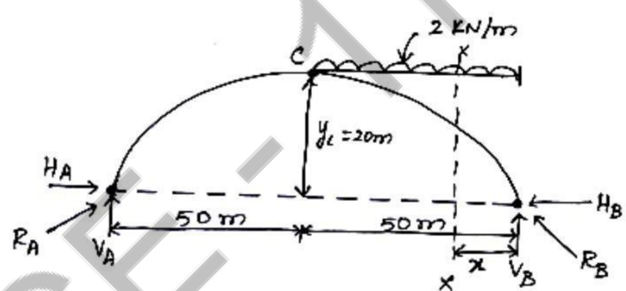
$$\boxed{H_2 = 0.697 \text{ KN}}$$

$$H = H_1 + H_2 = 90.16 + 0.697$$

$$\boxed{H = 90.857 \text{ KN}}$$

13.A three hinged parabolic arch of span 100m and rise 20m carries a uniformly distributed load of 2KN/m length on the right half as shown in the figure. Determine the maximum bending moment in the arch.





Step 1: Vertical Reactions :-

$$\Sigma V = 0$$

$$V_A + V_B = 100$$

Taking moment about B,

$$V_A \times 100 - \frac{2 \times (50)^2}{2} = 0$$

$$\boxed{V_A = 25 \text{ KN}}$$

$$V_B = 100 - 25$$

$$\boxed{V_B = 75 \text{ KN}}$$

Step 2 : Horizontal Reactions :-

$$\Sigma H = 0$$

$$H_A = H_B$$

Taking moment about C,

$$V_B \times 50 - 2 \times \frac{(50)^2}{2} - H_B \times y_c = 0$$

$$(75 \times 50) - (50)^2 - 20 H_B = 0$$

$$\boxed{H_B = 62.5 \text{ KN}}$$

$$\boxed{H_A = 62.5 \text{ KN}}$$

Step 3 : Resultant Reactions :-

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(25)^2 + (62.5)^2}$$

$$\boxed{R_A = 67.31 \text{ KN}}$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(75)^2 + (62.5)^2}$$

$$\boxed{R_B = 97.63 \text{ KN}}$$

Step 4 : Maximum Bending moment :-

The maximum bending moment will occur at a section xx at 'x' distance from B.

$$BM = -V_B \times x + \frac{2 \times x^2}{2} + H_B \times y$$

here,

$$y = \frac{4 y_c}{l^2} x (l - x)$$

$$= \frac{4 \times 20}{(100)^2} \times x (100 - x)$$

$$y = 0.8x - 0.008x^2$$

$$M_x = -75x + x^2 + 62.5 (0.8x - 0.008x^2)$$

$$= -75x + x^2 + 50x - 0.5x^2$$

$$M_x = -25x + 0.5x^2$$

For maximum bending moment, $\frac{dM_x}{dx} = 0$.

Differentiate with respect to x,

$$\frac{dM_x}{dx} = -25 + x$$

$$0 = -25 + x$$

$$\boxed{x = 25m}$$

$$\therefore M_x = -25x + 0.5x^2$$

$$= (-25 \times 25) + (0.5 \times (25)^2)$$

$$\boxed{M_x = -312.5 \text{ kNm}}$$

14. A two hinged parabolic arch of span L and rise R carries a UDL of w/m run over the left hand half of the span. The moment of inertia of the arch rib varies as the secant of the slope of the rib axis.

a. Obtain the expression for the horizontal thrust H.

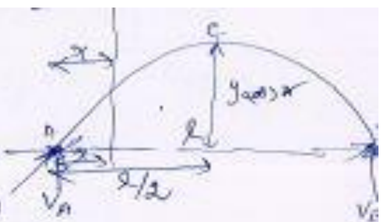
b. Calculate the horizontal thrust and bending moment at quarter span point on the right half of the span if l=20m, r = 4m and w=20kN/m.(Nov/dec-2017)

1) Vertical reactions.

$\Sigma M_B = 0$

$$V_A(l) - w\left(\frac{l}{2}\right)\left(\frac{l}{4} + \frac{l}{2}\right) = 0$$

$$\boxed{V_A = \frac{3}{8} wl}$$



$$V_B = \text{Total load} - V_A$$

$$= \frac{wl}{2} - \frac{3}{8} wl$$

$$\boxed{V_B = \frac{wl}{8}}$$

2) Horizontal Thrust. $H = \dfrac{\int \mu y \, dx}{\int y^2 \, dx}$

$$\int y^2 dx = \int_0^l \left[\frac{4r}{l^2}(x)(l-x)\right]^2 dx$$

$$= \int_0^l \frac{16r^2}{l^4} x^2 (l^2 + x^2 - 2lx) \, dx$$

$$= \frac{16r^2}{l^4}\int_0^l l^2x^2 + x^4 - 2lx^3\,dx$$

$$= \frac{16r^2}{l^4}\left[\frac{l^2x^3}{3} + \frac{x^5}{5} - \frac{2lx^4}{4}\right]_0^l$$

$$= \frac{16r^2}{l^4}\left[\frac{l^5}{3} + \frac{x^5}{5} - \frac{2l^5}{4_2}\right]$$

$$= \frac{16r^2}{l^4}\left[\frac{10l^5 + 6l^5 - 15l^5}{30}\right]$$

$$= \frac{\overset{8}{\cancel{16}}r^2}{\cancel{l^4}} \times \frac{l^5}{\cancel{30}_{15}} \Rightarrow \frac{8r^2l}{15}$$

<u>Numerator (1)</u>

$$M_1 = V_A x - wx\left(\frac{x}{2}\right)$$

$$= \frac{3}{8}wlx - \frac{wx^2}{2}$$

$$Nr(1) = \int_0^{l/2} M_1 y\,dx$$

$$= \int_0^{l/2}\left[\frac{3}{8}wlx - \frac{wx^2}{2}\right]\left[\frac{4r}{l^2}\,x(l-x)\right]dx$$

$$= \frac{4r}{l^2}\int_0^{l/2}\left[\frac{3}{8}wl^2x^2 - \frac{3}{8}wlx^3 - \frac{wlx^3}{2} + \frac{wx^4}{2}\right]dx.$$

$$= \frac{4r}{l^2}\left[\frac{3}{8}wl^2\frac{x^3}{3} - \frac{3}{8}wl\frac{x^4}{4} - \frac{wlx^4}{2\times4} + \frac{wx^5}{10}\right]_0^{l/2} dx$$

$$= \frac{4r}{l^2}\left[\frac{wl^2}{8}\times\frac{l^3}{8} - \frac{3}{32}\frac{wl^4}{16} - \frac{wl}{8}\times\frac{l^4}{16} + \frac{w}{10}\frac{l^5}{32}\right]$$

$$= \frac{4r}{l^2} \times w l^5 \left[\frac{1}{64} - \frac{3}{512} - \frac{1}{128} + \frac{1}{320}\right]$$

$$= 4wr l^3 \left[\frac{40 - 15 - 20 + 8}{2560}\right]$$

$$N_r(1) = \frac{6.5 w l^3 r}{320}$$

$$N_r(2) \Rightarrow \mu_2 = V_B(l - x)$$

$$= \frac{wl}{8}(l - x)$$

$$N_r(2) = \int_{l/2}^{l} \mu_2 y$$

$$= \int_{l/2}^{l} \frac{wl}{8}(l - x) \frac{4r}{l^2} x(l - x)\, dx$$

$$= \frac{wl}{\cancel{8}_2} \times \frac{4r}{l^2} \int_{l/2}^{l} x(l - x)^2 dx$$

$$= \frac{wr}{2l} \int_{l/2}^{l} x(l^2 + x^2 - 2lx)\, dx$$

$$= \frac{wr}{2l} \left[\frac{l^2 x^2}{2} + \frac{x^4}{4} - \frac{2 l x^3}{3}\right]_{l/2}^{l}$$

$$= \frac{wr l^4}{2l} \left[\frac{3}{8} + \frac{15}{64} - \frac{14}{24}\right]$$

$$Nr(2) = \frac{5wrl^3}{384}$$

$$Nr = Nr(1) + Nr(2)$$

$$= \frac{6.5wrl^3}{320} + \frac{5wrl^3}{384}$$

$$\boxed{Nr = \frac{wrl^3}{30}}$$

$$H = \frac{Nr}{Dr} = \frac{\frac{wrl^3}{30}}{\frac{8}{15}r^2l} = \frac{wl^2}{16r}$$

when $l = 20m$, $r = 4m$ & $w = 20kN/M$

$$H = \frac{20 \times 20^2}{16 \times 4} = 125 KN$$

$$V_A = \frac{3}{8}wl \Rightarrow \frac{3}{8} \times 20 \times 20 = 150 kN$$

$$V_B = \frac{wl}{8} = \frac{20 \times 20}{8} = 50 kN$$

Rise $y = \frac{4r}{l^2} x(l - x)$ $\qquad x = \frac{3}{4}l$

$$= \frac{4 \times 4}{20^2}\left[\frac{3}{4}\left(20 - \frac{3}{4}(20)\right)\right]$$

$$= 3m$$

B.M @ right quarter span $= V_B(5) - Hy_c$

$= 50(5) - 125(3)$

$= \underline{-125 kNm}$

UNIT 5 - PLASTIC ANALYSIS OF STRUCTURES

PART - A (2 marks)

1. **What is shape factor?** **(Apr/May 2016) (Nov/Dec 2019,2017)**

   The shape factor is defined as the ratio of the plastic moment of a section to the yield moment of the section.

2. **State upper bound theorem.** **(Apr/May 2018) (Nov/Dec2016)**

   Upper bound theorem states that "A load computed on the basis of an assumed mechanism is always greater than or equal to the true ultimate load".

3. **Define plastic modulus.** **(Nov/Dec 2017)**

   The plastic modulus of a section is the first moment of the area above and below the equal area axis. It is the resisting modulus of a fully plasticized section.

   $$Z_p = \frac{A}{2}(y_1 + y_2)$$

4. **What are meant by load factor and collapse load?(Nov/Dec 2018,2017 & May/June 2016)**

   Load factor:

   Load factor is defined as the ratio of collapse load to working load.

   $$\text{Load factor, } \lambda = \frac{\text{collapse load}}{\text{working load}} = \frac{W_C}{W}$$

   Collapse load:

   The load that causes the (n + 1) the hinge to form a mechanism is called collapse load where n is the degree of statically indeterminacy. Once the structure becomes a mechanism.

5. **Define plastic hinge with an example.(Nov/dec-2019,Apr/may-2018)**

   When a section attains full plastic moment Mp, it acts as hinge which is called a plastic hinge. It is defined as the yielded zone due to bending at which large rotations can occur with a constant value of plastic moment Mp.

6. **What is difference between plastic hinge and mechanical hinge?**

   Plastic hinges modify the behavior of structures in the same way as mechanical hinges. The only difference is that plastic hinges permit rotation with a constant resisting moment equal to the plastic moment Mp. At mechanical hinges, the resisting moment is equal to zero.

7. **List out the assumptions made for plastic analysis.(Apr/may-2016)**

   The assumptions for plastic analysis are:

   - Plane transverse sections remain plane and normal to the longitudinal axis before and after bending.

- Effect of shear is neglected.
- The material is homogeneous and isotropic both in the elastic and plastic state.
- Modulus of elasticity has the same value both in tension and compression.
- There is no resultant axial force in the beam.
- The cross-section of the beam is symmetrical about an axis through its centroid and parallel to the plane of bending.

8. List out the shape factors for the following sections.(Apr/may-2017)

- Rectangular section, S = 1.5
- Triangular section, S = 2.346
- Circular section, S = 1.697
- Diamond section, S = 2

9. Mention the section having maximum shape factor.

The section having maximum shape factor is a triangular section, S = 2.345.

10. State lower bound theory.(Nov/dec-2018)

Lower bound theory states that the collapse load is determined by assuming suitable moment distribution diagram. The moment distribution diagram is drawn in such a way that the conditions of equilibrium are satisfied.

11. What are the different types of mechanisms?

The different types of mechanisms are:

- Beam mechanism
- Column mechanism
- Panel or sway mechanism
- Cable mechanism
- Combined or composite mechanism

12. Mention the types of frames.

Frames are broadly of two types:

- Symmetric frames
- Un-symmetric frames

13. What are symmetric frames and how they analyzed?

Symmetric frames are frames having the same support conditions, lengths and loading conditions on the columns and beams of the frame. Symmetric frames can be analyzed by:

- Beam mechanism
- Column mechanism

14. What are unsymmetrical frames and how are they analyzed?

Un-symmetric frames have different support conditions, lengths and loading conditions on its columns and beams. These frames can be analyzed by:

- Beam mechanism
- Column mechanism
- Panel or sway mechanism
- Combined mechanism

15. How is the shape factor of a hollow circular section related to the shape factor of a ordinary circular section?

The shape factor of a hollow circular section = A factor K x shape factor of ordinary circular section. SF of hollow circular section = SF of circular section x $\{(1 - c^3)/ (1 - c^4)\}$

16. Give the governing equation for bending.

The governing equation for bending is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

Where M = Bending moment
I = Moment of inertia
σ = Stress
y = C.G. distance

17. Give the theorems for determining the collapse load.

The two theorems for the determination of collapse load are:

- Static Method [Lower bound Theorem]
- Kinematic Method [Upper bound Theorem]

18. What is a mechanism?

When a n-degree indeterminate structure develops n plastic hinges, it becomes determinate and the formation of an additional hinge will reduce the structure to a mechanism. Once a structure becomes a mechanism, it will collapse.

19. What are the assumptions made in fully plastic moment of a section?

- Plane traverse sections remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.
- Modulus of elasticity has the same value in tension and compression.
- The material is homogeneous and isotropic in both the elastic and plastic state.
- There is no resultant axial force on the beam. i.e., total compression = total tension.
- The cross-section of the beam is symmetrical about an axis through its centroid parallel to the plane of bending.
- Longitudinal fibres are free to expand and contract without affecting the fibres in the lateral dimension.

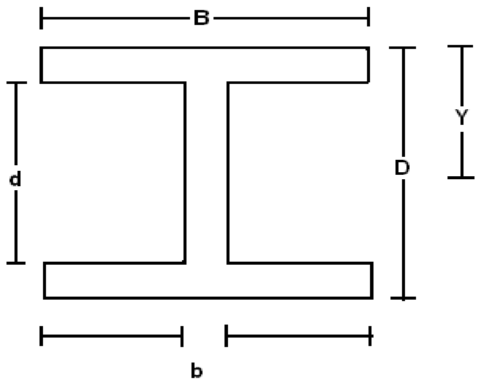
20. What are the limitations of load factor concept?

- The analysis procedure does not give us any clue if at a load W_u / load factor the structure behaves well.
- The stresses are within limit, so we have to check the stresses at crucial points by conventional elastic method.
- This is a peculiar and unrealistic assumption.
- The assumption of monotonic increase in loading is a simplistic.

PART - B

1. Derive the shape factor for I section and circular section. **(Apr/May 2011)**

Isection:



Shape factor, $S = \frac{Z_p}{Z} = \frac{\text{Plastic modulus}}{\text{Elastic modulus}}$

Elastic modulus (Z) :

$$Z = \frac{I}{Y}$$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$Y = \frac{D}{2}$$

$$Z = \frac{\left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)}{\left(\frac{D}{2}\right)} = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right) x \ \frac{2}{D}$$

$$Z = \frac{BD^3 - bd^3}{6D}$$

Plastic modulus (Z_p) :

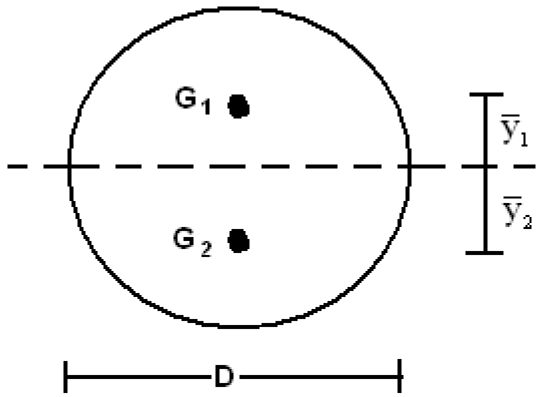
$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

$$A = 2(b_1 d_1) + b_2 d_2$$

$$\bar{y}_1 = \bar{y}_2 = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$S = \frac{Z_p}{Z} = \frac{\frac{A}{2}(\bar{y}_1 + \bar{y}_2)}{\left(\frac{BD^3 - bd^3}{6D}\right)}$$

Circular Section:



Shape factor, $S = \dfrac{Z_p}{Z} = \dfrac{\text{Plastic modulus}}{\text{Elastic modulus}}$

Elastic modulus (Z) :

$$Z = \frac{I}{y} = \frac{\left(\dfrac{\pi D^4}{64}\right)}{\dfrac{D}{2}}$$

$$Z = \frac{\pi D^3}{32}$$

Plastic modulus (Z_p) :

$$Z_p = \frac{A}{2}\left(\bar{y}_1 + \bar{y}_2\right)$$

$$A = \frac{\pi D^2}{4}$$

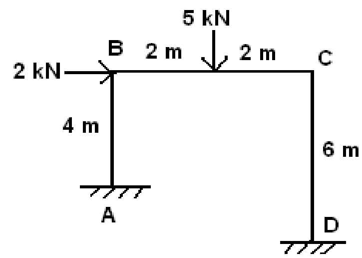
$$\bar{y}_1 = \bar{y}_2 = \frac{4r}{3\pi} = \frac{2D}{3\pi}$$

$$Z_P = \frac{\pi D^2}{4 \times 2}\left(\frac{2D}{3\pi} + \frac{2D}{3\pi}\right) = \frac{\pi D^2}{8} \times \frac{4D}{3\pi} = \frac{D^3}{6}$$

$$S = \frac{Z_P}{Z} = \frac{\left(\dfrac{D^3}{6}\right)}{\left(\dfrac{\pi D^3}{32}\right)} = \frac{D^3}{6} \times \frac{32}{\pi D^3} = \frac{32}{6\pi}$$

$$S = 1.697$$

2. **Find the fully plastic moment required for the frame shown in figure, if all the members have same value of M_P.(Nov/dec-2019,2018,2017,2016, Apr/May 2016)**



Solution:

Step 1: Degree of indeterminacy:

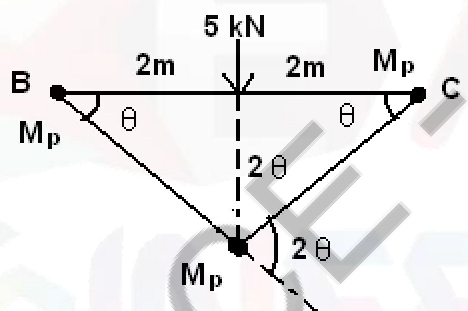
Degree of indeterminacy = (No. of closed loops x 3) – No. of releases

= (1 x 3) – 0 = 3

No. of possible plastic hinges = 5

No. of independent mechanisms = 5 – 3 = 2

Step 2: Beam Mechanism:



EWD = $5\,(2\,\theta) = 10\,\theta$

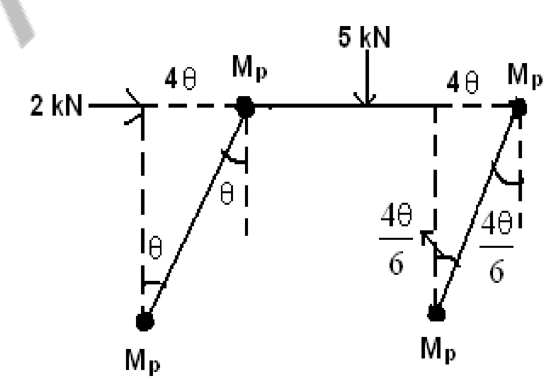
IWD = $M_p\,\theta + 2\,M_p\,\theta + M_p\,\theta = 4\,M_p\,\theta$

EWD = IWD

$10\,\theta = 4\,M_p\,\theta$

$M_p = 2.5$ kN.m

Step 3: Sway Mechanism:



$$EWD = (2 \times 4\theta) = 8\theta$$

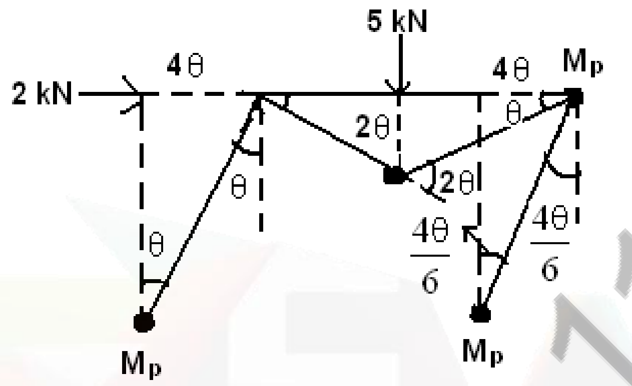
$$IWD = M_p\ \theta + M_p\,\theta + M_p\left(\frac{4\theta}{6}\right) + M_p\left(\frac{4\theta}{6}\right) = 3.33\,M_p\,\theta$$

$$EWD = IWD$$

$$8\theta = 3.33\ M_p\,\theta$$

$$M_p = 2.4\ \text{kN.m}$$

Step 4: Combined Mechanism:



$$EWD = (2 \times 4\theta) + (5 \times 2\,\theta) = 18\theta$$

$$IWD = M_p\,\theta + M_p(2\theta) + M_p\left(\theta + \frac{4\theta}{6}\right) + M_p\left(\frac{4\theta}{6}\right) = 5.33\ M_p\,\theta$$

$$EWD = IWD$$

$$18\theta = 5.33\ M_p\,\theta$$

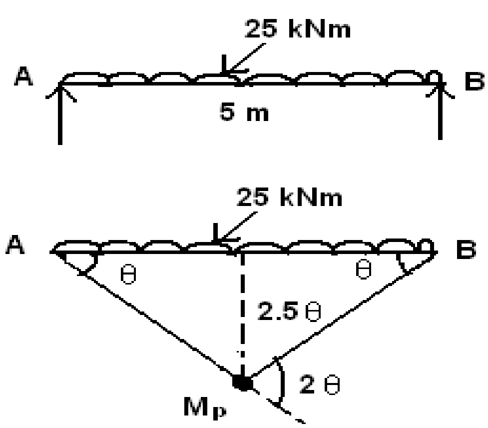
$$M_p = 3.38\ \text{kN.m}$$

The fully plastic moment, M_P = 3.38 kNm.

3. A simply supported beam of span 5 m is to be designed for an udl of 25 kN/m. Design a suitable I section using plastic theory, assuming yield stress in steel as $f_y = 250\ N/mm^2$.

(AUC Nov/Dec 2011)

Solution:



IWD = 0 + M_P (2θ) + 0 = 2 M_Pθ

EWD = Load intensity X area of triangle under the load

$$= 25 \times \left(\frac{1}{2} \times 5 \times 2.5\theta\right)$$

$$= 156.25\theta$$

IWD = EWD

2 M_Pθ = 156.25θ

M_P = 78.125 kNm

W.K.T.,

$M_P = \sigma_y \times Z_P$

$$Z_P = \frac{M_P}{\sigma_y} = \frac{78.125 \times 10^6}{250} = 3.12 \times 10^5 \text{ mm}^3$$

Assuming the shape factor for I - section as 1.15

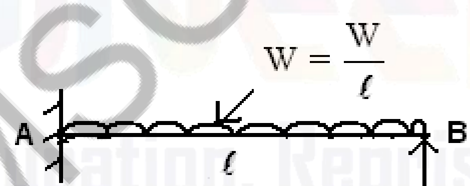
$$S = \frac{Z_P}{Z}$$

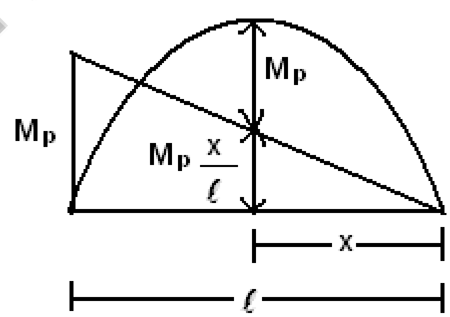
$$Z = \frac{Z_P}{S} = \frac{3.12 \times 10^5}{1.15} = 271.74 \times 10^3 \text{ mm}^3.$$

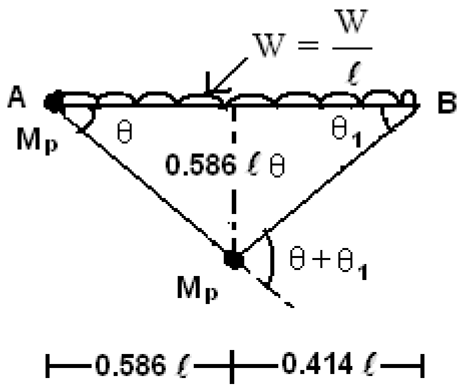
Adopt ISLB 250 @ 279 N / m (from steel table)

4. Analyse a propped cantilever of length 'L' and subjected to udl of w/m length for the entire span and find the collapse load.(Nov/Dec 2017,2019)

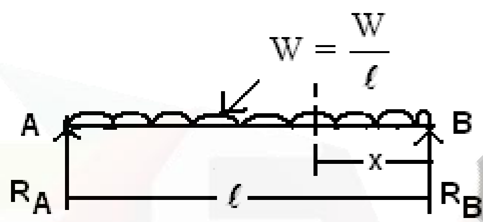
Solution:







Consider the moment at A as redundant and that it reaches M_P. the second hinge will form where the net positive BM is maximum.



$$\sum V = 0$$

$$R_A + R_B = W_C$$

$$R_A = R_B = \frac{W_C}{2}$$

$$M_x = \frac{W_C X}{2} - \frac{W_C X^2}{2\ell}$$

$$M_P + \frac{M_P X}{\ell} = \frac{W_C X}{2} - \frac{W_C X^2}{2\ell}$$

$$M_P\left(1 + \frac{X}{\ell}\right) = \frac{W_C X}{2}\left(1 - \frac{X}{\ell}\right)$$

$$M_P\left(\frac{\ell + X}{\ell}\right) = \frac{W_C X}{2}\left(\frac{\ell - X}{\ell}\right)$$

$$M_P = \frac{W_C X}{2}\left(\frac{\ell - X}{\ell + X}\right) = \frac{W_C}{2}\left(\frac{\ell X - X^2}{\ell + X}\right)$$

For M_P to be maximum, $\frac{dM_P}{dx} = 0$

$$\frac{dM_P}{dx} = \frac{W_C}{2}\left[\frac{(\ell + x)(\ell - 2x) - (\ell x - x^2)(1)}{(\ell + x)^2}\right] = 0$$

$$(\ell + x)(\ell - 2x) - (\ell x - x^2) = 0$$

$$\ell^2 - 2\ell x + x\ell - 2x^2 - \ell x + x^2 = 0$$

$$\ell^2 - 2\ell x - x^2 = 0$$

$$x^2 + 2\ell x - \ell^2 = 0$$

$$x = \frac{-2\ell \pm \sqrt{8\ell^2}}{2}$$

$$x = 0.414\ell$$

Mechanism :

$$0.586\,\ell\theta = 0.414\,\ell\theta_1$$

$$\theta_1 = 1.4155\theta$$

$$\theta + \theta_1 = \theta + 1.4155\theta = 2.4155\theta$$

$$EWD = \frac{W_C}{\ell} \text{ x } \frac{1}{2} \text{ x } \ell \text{ x } 0.586\,\ell\theta = 0.293 W_C\,\ell\theta$$

$$IWD = M_P\,\theta + M_P\,(2.4155\,\theta) + 0 = 3.4155 M_P\theta$$

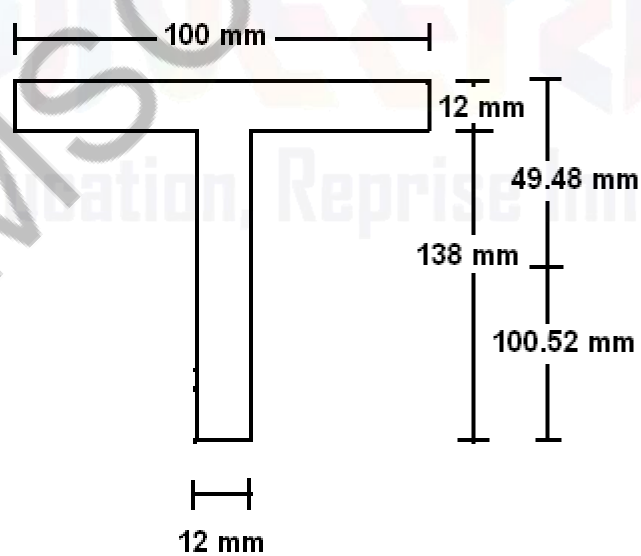
$$EWD = IWD$$

$$0.293\,W_C\ell\theta = 3.4155\,M_P\theta$$

$$W_C = \frac{11.66\,M_P}{\ell}$$

5. Determine the shape factor of a T-section beam of flange dimension 100 x 12 mm and web dimension 138 x 12 mm thick.(May/June 2019,2018,Nov/dec 2016)

Solution:



Shape factor, $S = \frac{Z_p}{Z} = \frac{\text{Plastic modulus}}{\text{Elastic modulus}}$

i) Elastic modulus (Z_e):

$$\bar{y}_t = \frac{(100 \times 12 \times 6) + (12 \times 138 \times 81)}{(100 \times 12) + (12 \times 138)} = 49.48\text{mm}$$

$$\bar{y}_b = 150 - 49.48 = 100.52 \text{ mm}$$

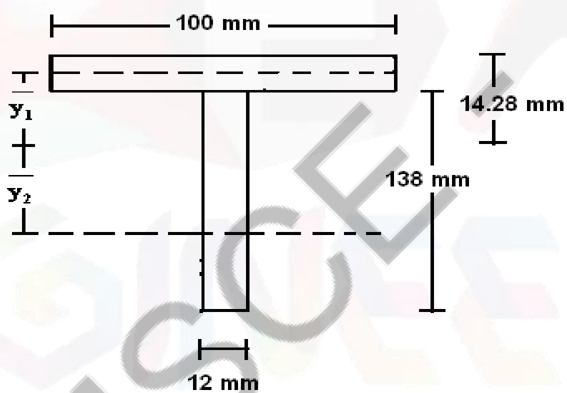
$$I_{xx} = \left[\frac{b_1 d_1^3}{12} + A_1 h_1^2\right] + \left[\frac{b_2 d_2^3}{12} + A_2 h_2^2\right]$$

$$= \left[\frac{100 \times 12^3}{12} + (100 \times 12 \times 43.48)^2\right] + \left[\frac{12 \times 138^3}{12} + (10 \times 138 \times 31.52)^2\right]$$

$$I_{xx} = 6.27 \times 10^6 \text{ mm}^4$$

$$Z_e = \frac{I}{y_{max}} = \frac{6.27 \times 10^6}{100.52} = 62375.65 \text{ mm}^3$$

ii) Plastic modulus:



Equal area axis,

$$\frac{A}{2} = \text{width of the flange X h}$$

$$\frac{2856}{2} = 100 \text{ h}$$

$$h = 14.28 \text{ mm (from top)}$$

$$\bar{y}_1 = \frac{(100 \times 12 \times (6 + 2.28)) + (12 \times 135.72 \times 67.86)}{(100 \times 12) + (12 \times 135.72)} = 42.58\text{mm}$$

$$\bar{y}_2 = \frac{107.42}{2} = 53.71 \text{ mm}$$

$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{2856}{2}(42.58 + 53.71)$$

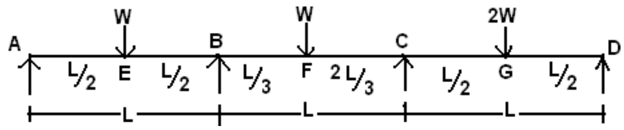
$$Z_p = 137502.12 \text{ mm}^3$$

Shape factor,

$$S = \frac{Z_P}{Z} = \frac{137502.12}{62375.65}$$

$$S = 2.20$$

6. Determine the collapse load 'W' for a three span continuous beam of constant plastic moment 'M_P' loaded as shown in figure. (Nov/dec-2018)



Solution:

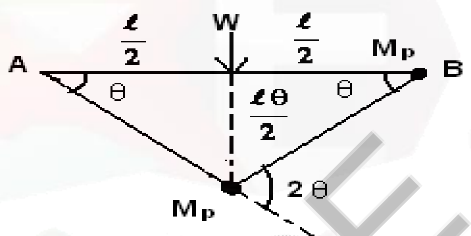
Step 1: Degree of indeterminacy:

Degree of indeterminacy = 4 – 2 = 2

No. of possible plastic hinges = 5

No. of independent mechanisms = 5 – 2 = 3

Step 2: Mechanism (1):



$$EWD = W \times \frac{\ell\theta}{2} = \frac{W\ell\theta}{2}$$

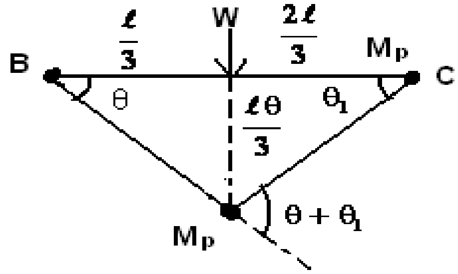
$$IWD = M_P(2\theta) + M_P\theta = 3M_P\theta$$

$$IWD = EWD$$

$$3M_P\theta = \frac{W\ell\theta}{2}$$

$$W_C = \frac{6M_P}{\ell}$$

Step 3: Mechanism (2):



$$\frac{\ell\theta}{3} = \frac{2\ell\theta_1}{3}$$

$$\theta_1 = \frac{\theta}{2}$$

$$\theta + \theta_1 = \theta + \frac{\theta}{2} = \frac{3\theta}{2}$$

$$\text{EWD} = W \times \frac{\ell\theta}{3} = \frac{W\ell\theta}{3}$$

$$\text{IWD} = M_P\theta + M_P(\theta + \theta_1) + M_P\theta_1$$

$$= M_P\theta + \frac{3M_P\theta}{2} + \frac{M_P\theta}{2}$$

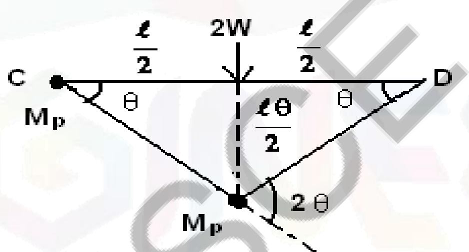
$$= 3M_P\theta$$

$$\text{IWD} = \text{EWD}$$

$$3M_P\theta = \frac{W\ell\theta}{3}$$

$$W_C = \frac{9M_P}{\ell}$$

Step 4: Mechanism (3):



$$\text{EWD} = 2W \times \frac{\ell\theta}{2} = W\ell\theta$$

$$\text{IWD} = M_P\theta + M_P(2\theta) = 3M_P\theta$$

$$\text{IWD} = \text{EWD}$$

$$3M_P\theta = W\ell\theta$$

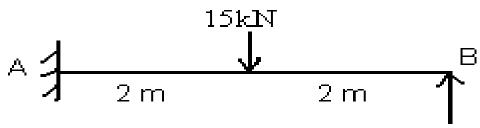
$$W_C = \frac{3M_P}{\ell}$$

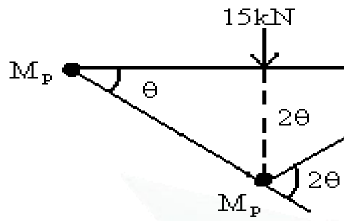
The collapse load $W_C = \frac{3M_P}{\ell}$ and the beam will fail.

7. **A uniform beam of span 4 m and fully plastic moment M_P is simply supported at one end and rigidly clamped at other end. A concentrated load of 15 kN may be applied anywhere within the span. Find the smallest value of M_P such that collapse would first occur when the load is in its most unfavourable position. (AUC May/June 2013)**

Solution:

i) When the load is at centre:

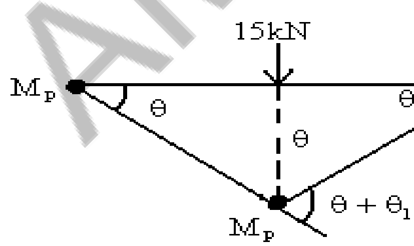




Degree of indeterminacy = 4 – 3 = 1
No. of possible plastic hinges = 2
No. of independent mechanisms = 2 – 1 = 1

$$\text{EWD} = 15\,(2\theta) = 30\,\theta$$
$$\text{IWD} = M_P\,\theta + M_P\,(2\theta) = 3M_P\,\theta$$
$$\text{IWD} = \text{EWD}$$
$$3M_P\,\theta = 30\,\theta$$
$$M_P = 10 \text{ kNm}$$

ii) When the load is at unfavourable position:



$$1 \text{ x } \theta = 3 \text{ x } \theta_1$$
$$\theta_1 = \frac{\theta}{3}$$

$$EWD = 15\theta$$

$$IWD = M_P\,\theta + M_P(\theta + \theta_1) = M_P\,\theta + M_P\left(\theta + \frac{\theta}{3}\right)$$

$$= \frac{7}{3} M_P\,\theta$$

$$IWD = EWD$$

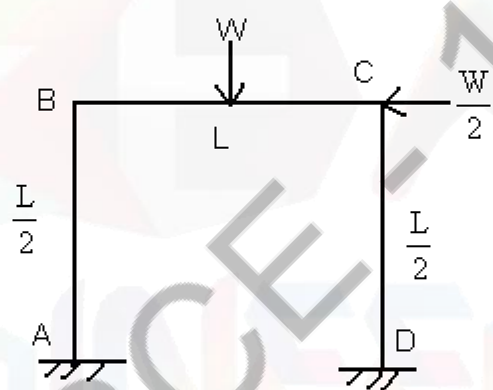
$$\frac{7}{3} M_P\,\theta = 15\theta$$

$$M_P = 6.43\ \text{kNm}$$

The smallest value of M_P is 6.43 kNm.

8. A rectangular portal frame of span L and $L/2$ is fixed to the ground at both ends and has a uniform section throughout with its fully plastic moment of resistance equal to M_y. It is loaded with a point load W at centre of span as well as a horizontal force $W/2$ at its top right corner. Calculate the value of W at collapse of the frame. (AUC May/June 2013)

Solution:



Step 1: Degree of indeterminacy:

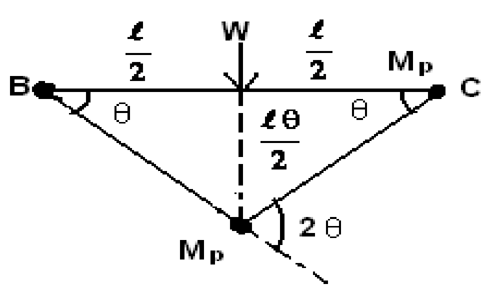
Degree of indeterminacy = (No. of closed loops x 3) – No. of releases

= (1 x 3) – 0 = 3

No. of possible plastic hinges = 5

No. of independent mechanisms = 5 – 3 = 2

Step 2: Beam Mechanism:



$$EWD = \frac{W\ \ell\theta}{2}$$

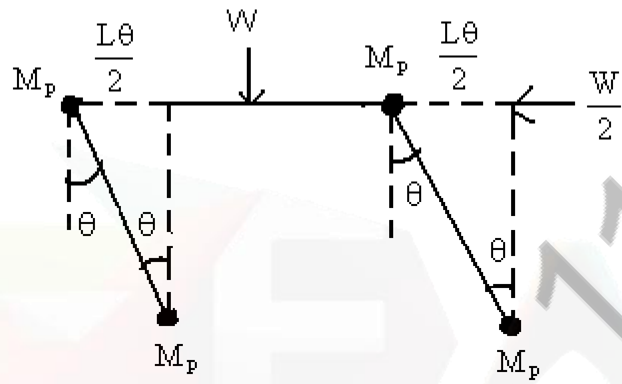
$$IWD = M_p\theta + M_p(2\theta) + M_p\theta = 4\,M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{2} = 4\,M_p\theta$$

$$W_C = \frac{8\,M_p}{\ell}$$

Step 3: Sway Mechanism:



$$EWD = \frac{W\ \ell\theta}{4}$$

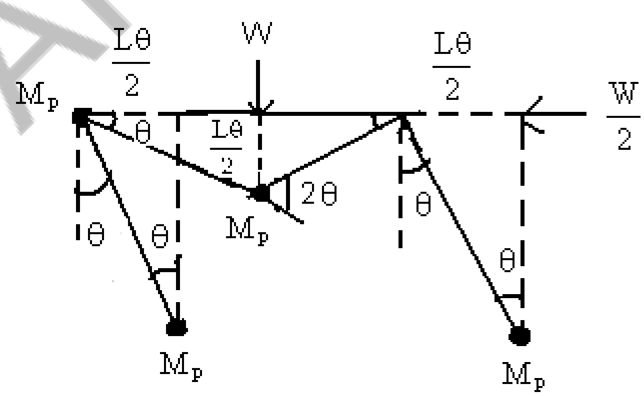
$$IWD = M_p\theta + M_p\theta + M_p\theta + M_p\theta = 4\,M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{4} = 4\,M_p\theta$$

$$W_C = \frac{16\,M_p}{\ell}$$

Step 4: Combined Mechanism:



$$EWD = \left(\frac{W\ell\theta}{2}\right) + \left(\frac{W\ell\theta}{4}\right) = \frac{3W\ell\theta}{4}$$

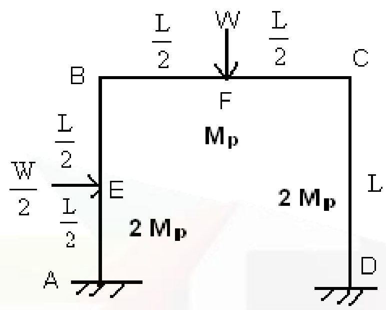
$$IWD = M_p\theta + M_p(2\theta) + M_p(2\theta) + M_p\theta = 6M_p\theta$$

$$EWD = IWD$$

$$\frac{3W\ell\theta}{4} = 6M_p\theta$$

$$W_C = \frac{8M_p}{\ell}$$

9. Find the collapse load for the frame shown in figure.(Nov/dec-2017,2016)



Solution:

Step 1: Degree of indeterminacy:

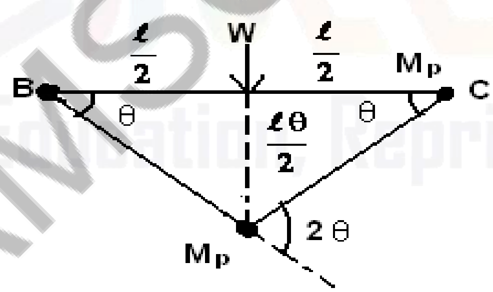
Degree of indeterminacy = (No. of closed loops x 3) – No. of releases

= (1 x 3) – 1 = 2

No. of possible plastic hinges = 5

No. of independent mechanisms = 5 – 2 = 3

Step 2: Beam Mechanism:



$$EWD = \frac{W\ell\theta}{2}$$

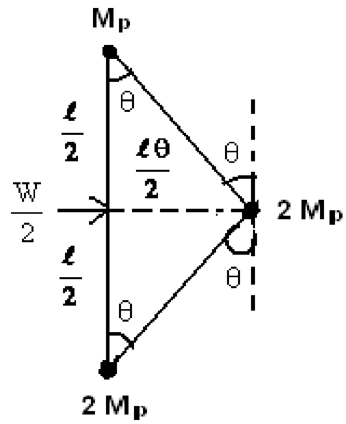
$$IWD = M_p\theta + M_p(2\theta) + M_p\theta = 4M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{2} = 4M_p\theta$$

$$W_C = \frac{8M_p}{\ell}$$

Step 3: Column Mechanism:



$$EWD = \frac{W\ell\theta}{4}$$

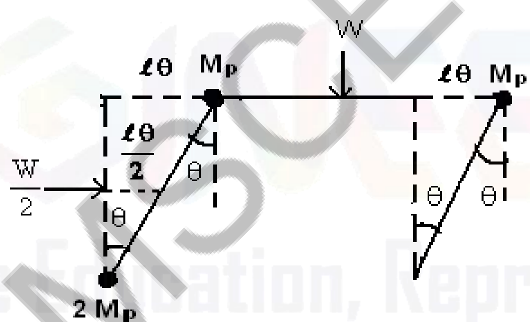
$$IWD = 2M_p\theta + 2M_p(2\theta) + M_p\theta = 7M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{2} = 7M_p\theta$$

$$W_C = \frac{28M_p}{\ell}$$

Step 3: Sway Mechanism:



$$EWD = \frac{W\ell\theta}{4}$$

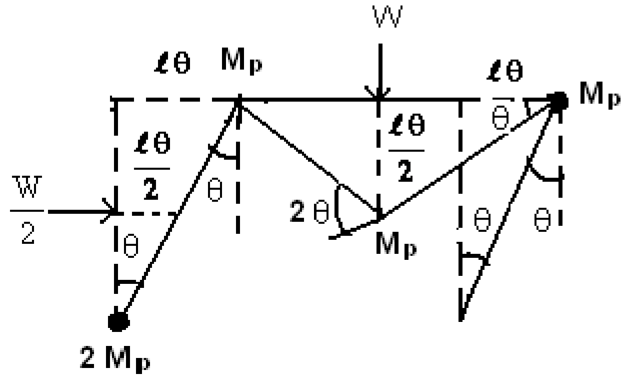
$$IWD = 2M_p\theta + M_p\theta + M_p\theta = 4M_p\theta$$

$$EWD = IWD$$

$$\frac{W\ell\theta}{4} = 4M_p\theta$$

$$W_C = \frac{16M_p}{\ell}$$

Step 4: Combined Mechanism:



$$EWD = \left(\frac{W\ell\theta}{4}\right) + \left(\frac{W\ell\theta}{2}\right) = \frac{3W\ell\theta}{4}$$

$$IWD = 2M_p\,\theta + M_p\,(2\theta) + M_p\,(2\theta) = 6\,M_p\,\theta$$

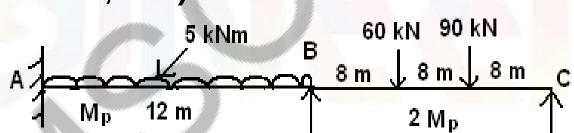
$$EWD = IWD$$

$$\frac{3W\ell\theta}{4} = 6\,M_p\theta$$

$$W_C = \frac{8\,M_p}{\ell}$$

Hence the collapse load, $W_C = \frac{8\,M_p}{\ell}$

10. A continuous beam ABC is loaded as shown in figure. Determine the required M_P if the load factor is 3.2.(May/june-2016,2018)



Solution:

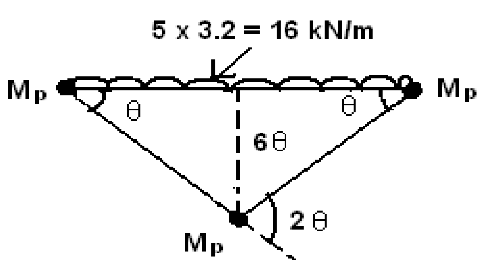
Step 1: Degree of indeterminacy:

Degree of indeterminacy = 5 – 3 = 2

No. of possible plastic hinges = 5

No. of independent mechanisms = 5 – 2 = 3

Step 2: Mechanism (1):



$$EWD = 16 \times \frac{1}{2} \times 12 \times 6 \ \theta$$

$$= 576\,\theta$$

$$IWD = M_p\theta + M_p(2\theta) + M_p\theta$$

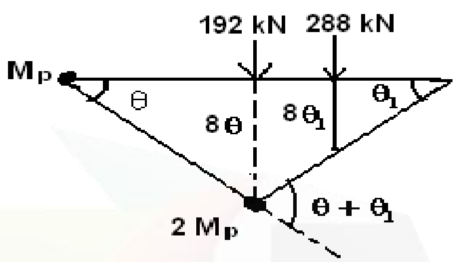
$$= 4\,M_p\theta$$

$$IWD = EWD$$

$$4M_p\theta = 576\,\theta$$

$$M_p = 144 \text{ kNm}$$

Step 3: Mechanism (2):



$$8\,\theta = 16\,\theta_1$$

$$\theta_1 = \frac{\theta}{2}$$

$$\theta + \theta_1 = \theta + \frac{\theta}{2} = \frac{3\theta}{2}$$

$$EWD = (192 \times 8\,\theta) + (288 \times 4\theta) = 2688\,\theta$$

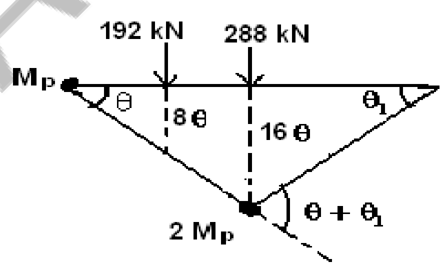
$$IWD = M_p\theta + 2M_p(\theta + \theta_1) = 4M_p\theta$$

$$IWD = EWD$$

$$4\,M_p\theta = 2688\,\theta$$

$$M_p = 672 \text{ kNm}$$

Step 4: Mechanism (3):



$$16\,\theta = 8\,\theta_1$$

$$\theta_1 = 2\theta$$

$$EWD = (192 \text{ x } 8\theta) + (288 \text{ x } 16\theta) = 6144\ \theta$$

$$IWD = M_P\theta + 2M_P(3\theta) = 7M_P\theta$$

$$IWD = EWD$$

$$7\ M_P\theta = 6144\ \theta$$

$$M_P = 877.71 \text{ kNm}$$

The required plastic moment of the beam section shall be $M_P = 877.71$ kNm.

UNIT 4 - CABLE AND SUSPENSION BRIDGES

PART - A (2 marks)

1. **What is the nature of forces in the cables? (AUC Apr/May 2011)**

   Cables of cable structures have only tension and no compression or bending.

2. **What are the types of stiffening girders? (Nov/Dec 2019)**
   - Suspension bridges with three hinged stiffening girders
   - Suspension bridges with two hinged stiffening girders

3. **What is the need for cable structures? (AUC May/June 2013)**
   - The main load bearing member.
   - Flexible throughout.
   - It can take only direct tension and cannot take any bending moment.

4. **What are cable structures?**

   Long span structures subjected to tension and uses suspension cables for supports. Examples of cable structures are suspension bridges, cable stayed roof.

5. **What is the true shape of cable structures?**

   Cable structures especially the cable of a suspension bridge is in the form of a catenary. Catenary is the shape assumed by a string / cable freely suspended between two points.

6. **Mention the different types of cable structures.**

   Cable structures are mainly of two types:

   (a) Cable over a guide pulley

   (b) Cable over a saddle

7. **Briefly explain cable over a guide pulley.**

   Cable over a guide pulley has the following properties:
   - Tension in the suspension cable = Tension in the anchor cable
   - The supporting tower will be subjected to vertical pressure and bending due to net horizontal cable tension.

8. **Briefly explain cable over saddle.**

   Cable over saddle has the following properties:

- Horizontal component of tension in the suspension cable = Horizontal component of tension in the anchor cable
- The supporting tower will be subjected to only vertical pressure due to cable tension.

9. What are the main functions of stiffening girders in suspension bridges? (Nov/dec-2018)

Stiffening girders have the following functions.

- They help in keeping the cables in shape
- They resist part of shear force and bending moment due to live loads.

10. Give the expression for calculating Tension T on a girder.(Nov/dec-2018)

The tension developed in the cable is given by

$$T = \sqrt{H^2 + V^2}$$

Where, H = horizontal component and V = vertical component.

.

11. What are cables made of?

Cables can be of mild steel, high strength steel, stainless steel, or polyester fibres. Structural cables are made of a series of small strands twisted or bound together to form a much larger cable.

Steel cables are either spiral strand, where circular rods are twisted together or locked coil strand, where individual interlocking steel strands form the cable (often with a spiral strand core).

Spiral strand is slightly weaker than locked coil strand. Steel spiral strand cables have a Young's modulus, E of 150 ± 10 kN/mm² and come in sizes from 3 to 90 mm diameter. Spiral strand suffers from construction stretch, where the strands compact when the cable is loaded.

12. Give the types of significant cable structures

Linear structures:

- Suspension bridges
- Draped cables
- Cable-stayed beams or trusses
- Cable trusses
- Straight tensioned cables

Three-dimensional structures:

- Bi-cycle roof
- 3D cable trusses
- Tensegrity structures

- Tensairity structures

13. What are the main functions of stiffening girders in suspension bridges?(Nov/dec-2018)

Stiffening girders will cause uniform load distribution in the cable no matter where the live load is present on the girder. They also maintain the parabolic shape of the cable. They also carry a part of bending moments and shear forces due to live loads.

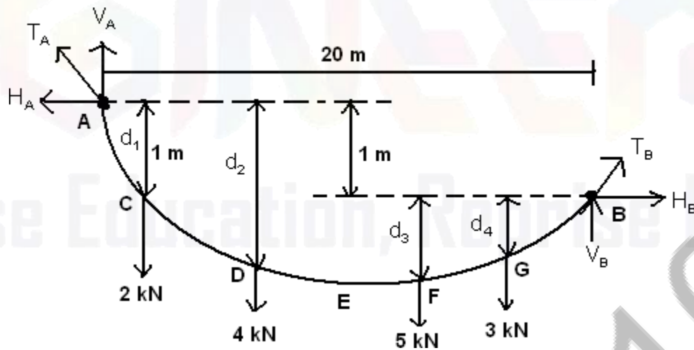
14. Write about the temperature effects on cable.(May/Hune-2016)

Tension forces in the cables are very sensitive to temperature changes. Temperature increase causes loss of tension in the cables. The latter is susceptible of causing loss of stability.
Low temperatures and the loss of ductility of certain materials could result in catastrophic failure.

PART - B (16 marks)

1. **A suspension cable is supported at two point "A" and "B", "A" being one metre above "B". the distance AB being 20 m. the cable is subjected to 4 loads of 2 kN, 4 kN, 5 kN and 3 kN at distances of 4 m, 8 m, 12 m and 16 m respectively from "A". Find the maximum tension in the cable, if the dip of the cable at point of application of first loads is 1 m with respect to level at A. find also the length of the cable. (Apr/May 2018,2016)**

Solution:



Step 1: Re actions :

$\sum V = 0$

$V_A + V_B = 14$

$\sum M@ B = 0$

$(V_A \times 20) - (H \times 1) - (2 \times 16) - (4 \times 12) - (5 \times 8) - (3 \times 4) = 0$

$20 V_A - H - 132 = 0$

$V_A = 0.05 H + 6.6$....................(1)

$\sum H = 0$

$H_A = H_B$

$\sum M \text{ @ } C = 0$

$(V_A \text{ x } 4) - (H \text{ x } 1) = 0$

$V_A = 0.25H$ .... (2)

sub. (2) in (1),

$0.25\,H = 0.05H + 6.6$

$H = 33\,kN$

$(1) \Rightarrow \quad V_A = 8.25 \text{ kN}$

$V_B = 5.75 \text{ kN}$

Step 2 : Maximum Tension in the cable :

$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{8.25^2 + 33^2} = 34.02 \text{ kN}$$

$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{5.75^2 + 33^2} = 33.49 \text{ kN}$$

Maximum Tension in the cable, $T_{max} = 34.09\,kN.$

Step 3: Length of the cable :

Here, $d_1 = 1m$

Equating moments about D to zero,

$(8.25 \text{ x } 8) - (33 \text{ x } d_2) = 0$

$d_2 = 2 \text{ m}$

Equating moments about D to zero,

$(-5.75 \text{ x } 8) + (33 \text{ x } d_3) = 0$

$d_3 = 1.39 \text{ m}$

Equating moments about D to zero,

$(-5.75 \text{ x } 4) + (33 \text{ x } d_4) = 0$

$d_4 = 0.69\,m$

$AC = \sqrt{4^2 + 1^2} = 4.12 \text{ m}$

$CD = \sqrt{4^2 + 2^2} = 4.47 \text{ m}$

$FG = \sqrt{4^2 + 1.39^2} = 4.23 \text{ m}$

$GB = \sqrt{4^2 + 0.69^2} = 4.06 \text{ m}$

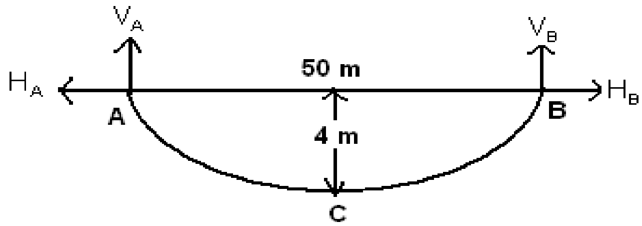
Length of the cable, $L = AC + CD + FG + BG + DF$

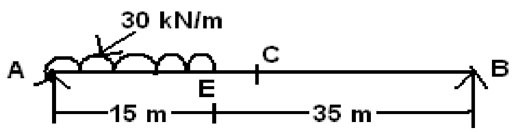
$= 4.12 + 4.47 + 4.23 + 4.06 + 4$

$L = 20.88\,m$

2. **A suspension bridge has a span 50 m with a 15 m wide runway. It is subjected to a load of 30 kN/m including self weight. The bridge is supported by a pair of cables having a central dip of 4 m. find the cross sectional area of the cable necessary if the maximum permissible stress in the cable materials is not to exceed 600 MPa.(Nov/Dec 2019,2017,2016)**

Solution:





Step 1: Re actions :

$$\sum V = 0$$

$$V_A + V_B = 450$$

$$\sum M @ A = 0$$

$$-(V_B x\ 50) + \left(\frac{30 \text{ x } 15^2}{2}\right) = 0$$

$$V_B = 67.5 \text{ kN}$$

$$V_A = 382.5 \text{ kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$(V_A \text{ x } 25) - (H \text{ x } 4) - (30 \text{ x } 15 \text{ x } (7.5+10)) = 0$$

$$H = 421.87 \text{ kN}$$

Step 2 : Maximum Tension in the cable :

$$T_A$$

$$T_B = \sqrt{V_B^2 + H^2} = \sqrt{67.5^2 + 421.87^2} = 427.24 \text{ kN}$$

Maximum Tension in the cable, $T_{max} = 569.46$ kN.

Step 3: Area :

$$T_{max} = \sigma \,.\, A$$

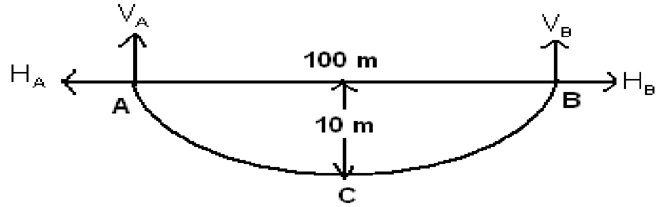
$$A = \frac{T_{max}}{\sigma} = \frac{569.46 \text{ x } 10^3}{600}$$

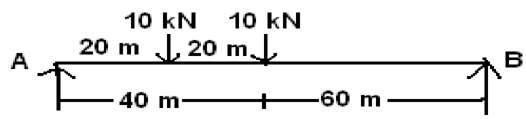
Area, $A = 949.1 \text{ mm}^2$.

3. A three hinged stiffening girder of a suspension bridge of 100 m span subjected to two point loads 10 kN each placed at 20 m and 40 m respectively from the left hand hinge. Determine the bending moment and shear force in the girder at section 30 m from each end. Also determine the maximum tension in the cable which has a central dip of 10 m.

(Nov/dec- 2018,2019)

Solution:





Step 1: Re actions :

$\sum V = 0$

$V_A + V_B = 20$

$\sum M @ B = 0$

$(V_A \text{ x } 100) - (10 \text{ x } 80) - (10 \text{ x } 60) = 0$

$V_A = 14 \text{ kN}$

$V_B = 6 \text{ kN}$

$\sum H = 0$

$H_A = H_B$

$\sum M @ C = 0$

$(V_A \text{ x } 50) - (H \text{ x } 10) - (10 \text{ x } 30) - (10 \text{ x } 10) = 0$

$H = 30 \text{ kN}$

Step 2 : Shear force :

SF at 30 m from left hand hinge.

$V_{30} = V_A - 10 - H \tan\theta$

here,

$$\tan\theta = \frac{4\,d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x} 10}{100^2}(100 - (2 \text{ x } 30))$$

$\tan\theta = 0.16$

$V_{30} = 14 - 10 - (30 \text{ x } 0.16)$

$V_{30} = -0.8 \text{ kN}$

SF at 30 m from right hand hinge.

$$V_{30} = V_B - H \tan\theta$$
$$= 6 - (30 \text{ x } 0.16)$$
$$V_{30} = 1.2 \text{ kN}$$

Step 3: Bending Moment:

BM at 30 m from left hand hinge.

$$BM_{30} = V_A \text{x} 30 - H \text{ x } y - 10 \text{ x } 10$$

here, y at 30 m from each end,

$$y = \frac{4d}{\ell^2} \text{ x } X(\ell - X^2) = \frac{4 \text{ x} 10}{100^2} \text{ x } 30(100 - 30)$$

$$y = 8.4 \text{ m}$$

$$BM_{30} = (14 \text{x} 30) - (30 \text{x} 8.4) - 100 = 68 \text{ kNm.}$$

BM at 30 m from right hand hinge.

$$BM_{30} = -V_B \text{x} 30 + H \text{ x } y$$
$$= -(6 \text{ x } 30) + (30 \text{ x } 8.4)$$
$$BM_{30} = 72 \text{ kNm.}$$

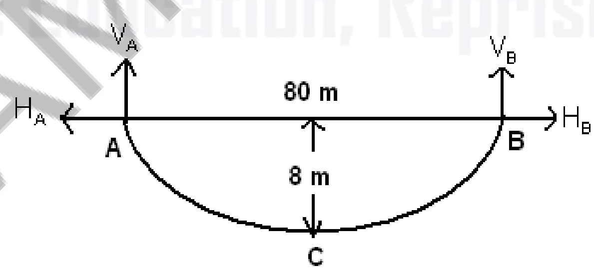
Step 4 : Maximum Tension in the cable :

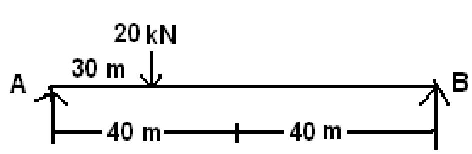
$$T_A = \sqrt{V_A^2 + H^2} = \sqrt{14^2 + 30^2} = 33.11 \text{kN}$$
$$T^B = \sqrt{V_B^2 + H^2} = \sqrt{6^2 + 30^2} = 30.59 \text{kN}$$

Maximum Tension in the cable, $T_{max} = 33.11$ k N.

4. A suspension bridge cable of span 80 m and central dip 8 m is suspended from the same level at two towers. The bridge cable is stiffened by a three hinged stiffening girder which carries a single concentrated load of 20 kN at a point of 30 m from one end. Sketch the SFD for the girder. (May/June 2013)

Solution:





Step 1: Re actions :

$$\sum V = 0$$

$$V_A + V_B = 20$$

$$\sum M@ B = 0$$

$$(V_A \text{ x } 80) - (20 \text{ x } 5\ 0) = 0$$

$$V_A = 12.5 \text{ kN}$$

$$V_B = 7.5 \text{ kN}$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$(V_A \text{ x } 40) - (20 \text{ x } 10) - (H \text{ x } 8) = 0$$

$$H = 37.5 \text{ kN}$$

Step 2 : Shear force :

SF at 40 m from left hand hinge.

$$V_{40} = V_A - 20 - H \tan\theta$$

here,

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 8}{80^2}(80 - (2 \text{ x } 40))$$

$$\tan\theta = 0$$

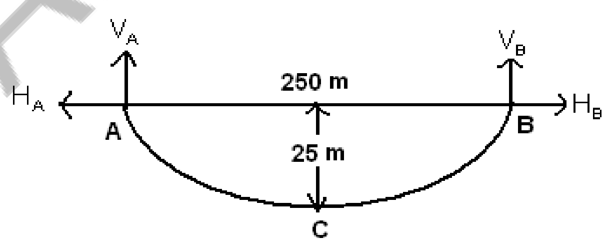
$$V_{40} = 12.5 - 20 - (37.5 \text{ x } 0)$$

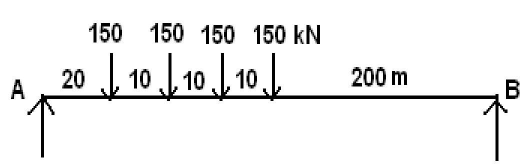
$$V_{40} = -7.5 \text{ kN}$$

5. A suspension bridge of 250 m span has two nos. of three hinged stiffening girders supported by cables with a central dip of 25 m. if 4 point loads of 300 kN each are placed at the centre line of the roadway at 20, 30, 40 and 50 m from left hand hinge. Find the shear force and bending moment in each girder at 62.5 m from each end. Calculate also the maximum tension in the cable. (Nov/dec-2018)

Solution:

The load system is shared equally by the two girders and cables. Take the loads as 150 kN each.





Step 1: Reactions :

$\sum V = 0$

$V_A + V_B = 600$

$\sum M @ B = 0$

$(V_A \text{ x } 250) - (150 \text{ x } 230) - (150 \text{ x } 220) - (150 \text{ x } 210) - (150 \text{ x } 200) = 0$

$V_A = 516 \text{ kN}$

$V_B = 84 \text{ kN}$

$\sum H = 0$

$H_A = H_B$

$\sum M @ C = 0$

$(V_A \text{x} 125) - (H \text{ x } 25) - (150 \text{ x} 105) - (150 \text{ x } 95) - (150 \text{ x } 85) - (150 \text{ x } 75) = 0$

$H = 420 \text{ kN}$

Step 2 : Shear force :

SF at 62.5 m from left hand hinge.

$V_{62.5} = V_A - 150 - 150 - 150 - 150 - H \tan\theta$

here,

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 25}{250^2}(250 - (2 \text{ x } 62.5))$$

$\tan\theta = 0.2$

$V_{62.5} = 516 - 150 - 150 - 150 - 150 - (420 \text{ x } 0.2)$

$V_{62.5} = -168 \text{ kN}$

SF at 62.5 m from right hand hinge.

$V_{62.5} = V_B - H \tan\theta$

$= 84 - (420 \text{ x } 0.2)$

$V_{62.5} = 0$

Step 3: Bending Moment :

BM at 62.5 m from left hand hinge.

$BM_{62.5} = V_A \text{ x } 62.5 - (150 \text{ x } 42.5) - (150 \text{ x } 32.5) - (150 \text{ x } 22.5) - (150 \text{ x} 12.5) - H \text{ x } y$

here, y at 62.5 m from each end,

$$y = \frac{4d}{\ell^2} \text{ x } X(\ell - X^2) = \frac{4 \text{ x } 25}{250^2} \text{ x } 62.5(250 - 62.5)$$

$y = 18.75 \text{ m}$

$BM_{62.5} = (516 \text{ x } 62.5) - (150 \text{ x } 42.5) - (150 \text{ x } 32.5) - (150 \text{ x } 22.5) - (150 \text{ x} 12.5) - (420 \text{ x} 18.75)$

$BM_{62.5} = 7875 \text{ kNm.}$

BM at 62.5 m from right hand hinge.

$$BM_{62.5} = -V_B x\, 62.5 + H \text{ x } y$$

$$= -(84 \text{ x } 62.5) + (420 \text{ x } 18.75)$$

$$BM_{62.5} = 2625 \text{ kNm.}$$

Step 4 : Maximum Tension in the cable :

Bending moment for the cable,

$$Hd = \frac{w\ell^2}{8}$$

$$w = \frac{H \text{ x } d \text{ x } 8}{\ell^2} = \frac{420 \text{ x } 25 \text{ x } 8}{250^2} = 1.344 \text{ kN / m}$$

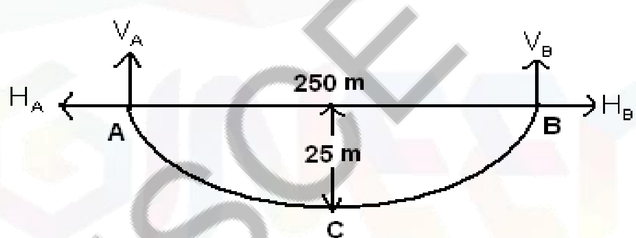
$$V_A = V_B = \frac{w\ell}{2} = \frac{1.344 \text{ x } 250}{2} = 168 \text{ kN}$$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{168^2 + 420^2} = 452.35 \text{ kN}$$

Maximum Tension in the cable, $T_{max} = 452.35$ kN.

6. A suspension bridge is of 160 m span. The cable of the bridge has a dip of 12 m. the cable is stiffened by a three hinged girder with hinges at either end and at centre. The dead load of the girder is 15 kN/m. find the greatest positive and negative bending moments in the girder when a single concentrated load of 340 kN passes through it. Also find the maximum tension in the cable.(Nov/dec-2018,2016)

Solution:



Step1: Bending Moment :

The uniformly distributed dead load will not cause any bending moment in the stiffening girder. The live load is a sin gle concentrated moving load.

$$\text{Max. + ve BM} = 0.096\ W\ell = 0.096 \text{ x } 340 \text{ x } 160$$

$$= 5222.4 \text{ kNm.}$$

This will occur at $0.211\ \ell = 0.211 \text{ x } 160$

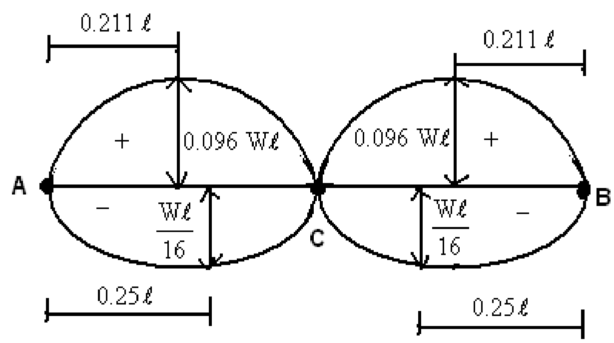
$= 33.76$ m from either end.

$$\text{Max. – ve BM} = -\frac{W\ell}{16} = -\frac{340 \text{ x } 160}{16}$$

$$= -3400 \text{ kNm.}$$

This will occur at $0.25\ \ell = 0.25 \text{ x } 160$

$= 40$ m from either end.



Step 2 : Maximum tension in the cable :

Dead load of the girder (transmitted to the cable directly)

$$p_d = 15 \text{ kN / m}$$

Equivalent udl transmitted to the cable due to the moving concentrated load,

$$p_\ell = \frac{2 \text{ x } 340}{160} = 4.25 \text{ kN / m}$$

Total load transmitted to the cable, $p = p_d + p_\ell = 15 + 4.25 = 19.25 \text{ kN / m}$

$$\text{Vertical reaction, } V = \frac{p\ell}{2} = \frac{19.25 \text{ x } 160}{2} = 1540 \text{ kN}$$

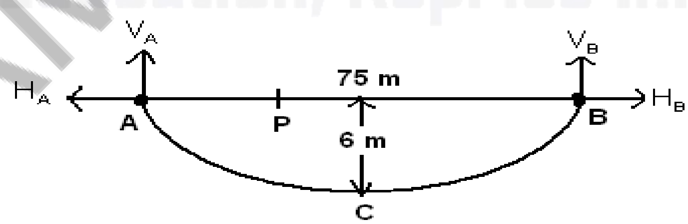
$$\text{Horizontal pull, } H = \frac{p\ell^2}{8d} = \frac{19.25 \text{ x } 160^2}{8 \text{ x } 12} = 5133.2 \text{ kN}$$

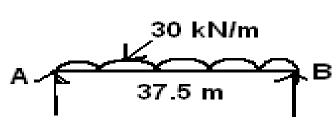
$$\text{Maximum tension, } T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{1540^2 + 5133.3^2}$$

$$T_{max} = 5359.3 \text{ kN.}$$

7. A suspension cable of 75 m horizontal span and central dip 6 m has a stiffening girder hinged at both ends. The dead load transmitted to the cable including its own weight is 1500 kN. The girder carries a live load of 30 kN/m uniformly distributed over the left half of the span. Assuming the girder to be rigid, calculate the shear force and bending moment in the girder at 20 m from left support. Also calculate the maximum tension in the cable.

Solution:





$$\ell = 75\text{m; } d = 6\text{m; } DL = 1500 \text{ kN; } LL = 30 \text{ kN / m}$$

Since the girder is rigid, the live load is transmitted to the cable as an udl whatever the position of the load.

Horizontal force due to live load, $H_\ell = \frac{P\ell}{8d} = \frac{(30 \text{ x } 37.5) \text{ x } 75}{8 \text{ x } 6} = 1757.8 \text{ kN}$

Horizontal force due to dead load, $H_d = \frac{P\ell}{8d} = \frac{1500 \text{ x } 75}{8 \text{ x } 6} = 2343.8 \text{ kN}$

Total horizontal force, $H = H_\ell + H_d = 1757.8 + 2343.8 = 4101.6 \text{ kN}$

$$V_A = V_B = \frac{\text{Total load}}{2} = \frac{W_\ell + W_d}{2}$$

$$= \frac{(30 \text{ x } 37.5) + 1500}{2} = 1312.5 \text{ kN}$$

Maximum tension in the cable :

$$T_{max} = \sqrt{H^2 + V^2} = \sqrt{4101.6^2 + 1312.5^2}$$

$$T_{max} = 4306.5 \text{ kN}$$

Dip at x = 20 m :

$$y = \frac{4d}{\ell^2} \text{ x } X(\ell - X^2) = \frac{4 \text{ x } 6}{75^2} \text{ x } 20(75 - 20) = 4.69 \text{ m}$$

$$\tan\theta = \frac{4d}{\ell^2}(\ell - 2x) = \frac{4 \text{ x } 6}{75^2} \text{ x } (75 - 2 \text{ x } 20) = 0.149$$

To find V_A and V_B :

$$V_A + V_B = 1125$$

Equating moments about A to zero

$$(V_B \text{ x } 75) - (30 \text{ x } 37.5 \text{ x } 18.75) = 0$$

$$V_B = 281.25 \text{ kN}$$

$$V_A = 843.75 \text{ kN}$$

Bending Moment at P :

$$BM_{20} = V_A \text{ x } 20 - H_\ell \text{ x } y - \frac{w\ell^2}{2}$$

$$= (843.75 \text{ x } 20) - (1757.8 \text{ x } 4.69) - \frac{30 \text{ x } 20^2}{2}$$

$$BM_{20} = 2630.92 \text{ kNm.}$$

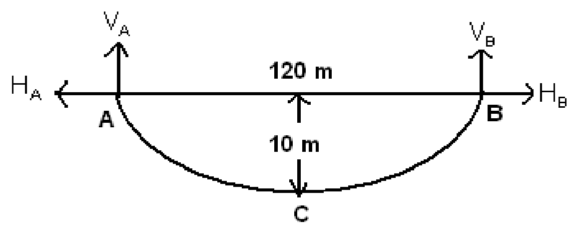
Shear force at P :

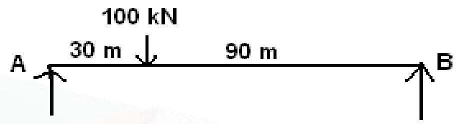
$$SF_{20} = V_A - H_\ell \text{ x } \tan\theta - w\ell = 843.75 - (1757.8 \text{ x } 0.149) - (30 \text{ x } 20)$$

$$SF_{20} = -18.16 \text{ kN.}$$

8. A suspension cable has a span of 120 m and a central dip of 10 m and is suspended from the same level at both towers. The bridge is stiffened by a stiffening girder hinged at the end supports. The girder carries a single concentrated load of 100 kN at a point 30 m from left end. Assuming equal tension in the suspension hangers. Calculate the horizontal tension in the cable and the maximum positive bending moment.

Solution:





Step 1: Re actions :

$$\sum V = 0$$

$$V_A + V_B = 100$$

$$\sum M @ A = 0$$

$$(100 \times 30) - (V_B \times 120) = 0$$

$$V_B = 25\,kN$$

$$V_A = 75\,kN$$

$$\sum H = 0$$

$$H_A = H_B$$

$$\sum M @ C = 0$$

$$-(V_B \times 60) + (H \times 10) = 0$$

$$H = 150\,kN$$

Step 2 : Maximum Tension in the cable :

Bending moment for the cable,

$$w = \frac{100}{\ell} = \frac{100}{120} = 0.83\ kN/m$$

$$V_A = V_B = \frac{w\ell}{2} = \frac{0.83 \times 120}{2} = 50\,kN$$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{50^2 + 150^2} = 158.1kN$$

Maximum Tension in the cable, $T_{max} = 158.1kN$.

Step 3 : Maximum positive Bending Moment :

Maximum positive Bending moment will occur at under the point load.

$$BM_{30} = V_A x\, 30 - H\, x\, y$$

here, y at 30 m from left end,

$$y = \frac{4d}{\ell^2}\ x\ X(\ell - X^2) = \frac{4\ x 10}{120^2}\ x\ 30(120 - 30)$$

$$y = 7.5\,m$$

$$BM_{30} = (75\ x\ 30) - (150\ x\ 7.5)$$

$$BM_{30} = 1125\ kNm.$$