UNIT – 3

FREQUENCY RESPONSE AND SYSTEM ANALYSIS

PART-A

1. What is meant by frequency response ? April/May 2017

A frequency response is the steady state response of s system when the input to the system is a sinusoidal signal.

2. List out the different frequency domain specification? (NOV/DEC 2015, MAY/JUNE 2016, Nov/Dec 2017)

The frequency domain specifications are

- Resonant peak
- Resonant frequency
- Bandwidth
- Cut- off rate
- Gain margin
- Phase margin

3. Define – Resonant Peak

The maximum value of the magnitude of closed loop transfer function called resonant peak.

4. What is bandwidth?

The bandwidth is the range of frequencies for which the system gain is more than - 3 dB. The bandwidth is a measure of the ability of a feedback system to reproduce the input signal, noise rejection characteristics and rise time.

5. Define Cut – off rate ?

The slope of the log – magnitude curve near the cut – off is called cut – off rate. The cut –off rate indicates the ability to distinguish the signal from noise.

6. Define – Gain margin? (MAY/ JUNE 2013)

The gain margin, K_g is defined as the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency. Gain margin $kg = \frac{1}{|G(j\omega)|_{\alpha=\omega pc}}$

7. Define Phase Cross over frequency? April/May 2019

The frequency at which, the phase of open loop transfer function is -180° is called phase cross over frequency ω_{pc}.

8. What is Phase margin? (MAY/JUNE 2013 & NOV/DEC 2011)

It is the amount of phase lag at the gain cross over frequency required to bring system to the verge of instability. The phase margin, $\gamma=180^o+\phi_{gc}$.

9. Define Gain cross over frequency? (APRIL/MAY 2011, May 2016, April/May 2019 & Nov/Dec 2019)

The gain cross over frequency ω_{gc}, is the frequency at which the magnitude of the open loop transfer function is unity.

10. What is Bode plot?

The Bode plot is the frequency response plot of the transfer function of a system. A bode plot consist of two graphs. One is the plot of magnitude of sinusoidal transfer versus log ω. The other is a plot of the phase angle of a sinusoidal function versus log ω.

11. What are the main advantages of Bode plot?

The main advantages are:

(i) Multiplication of magnitude can be to addition.
(ii) A simple method for sketching an approximate log curve is available
(iii) It is based on asymptotic approximation. Such approximation is sufficient if rough information on the frequency response characteristics is needed.
(iv) The phase angle curves can be easily draw if a template for the phase angle curve of 1+jω is available.

12. Define Corner frequency? April/May 2018

The frequency at which the two asymptotes meet in a magnitude plot is called corner frequency.

13. Define Phase lag and phase lead?

A negative phase angle is called phase lag. A positive phase angle is called phase lead.

14. What are M circles? (NOV/DEC 2015, MAY/JUNE 2016)

The magnitude M of closed loop transfer function with unity feedback will be in the form of circle on complex plane for each constant value of M. The family these circles are called M circles.

15. What is Nichols chart?

The chart consisting of M & N loci in the log magnitude versus phase diagram is called Nichol's chart.

16. What are two contours Nichol's chart?

Nichols chart of M and N contours superimposed on ordinary graph. The M contours are the magnitude of closed loop system in decibel and the N contours are the phase angle locus of closed loop s system.

17. What is non – minimum phase transfer function?

A transfer function which has one or more zeros in the right half S – plane is known as non – minimum phase transfer function.

18. What are the advantages of Nichols chart? (APRIL/MAY 2015)

The advantage are:

(i) It is used to find the close loop frequency response from open loop frequency response.
(ii) Frequency domain specification can be determined from Nichols chart.
(iii) The gain of the system can be adjust to satisfy the given specification.

19. What are N circles? **(NOV/DEC 2015, MAY/JUNE 2016)**

If the phase of closed loop transfer function with unity feedback is $\propto$, then N = tan $\propto$. For each constant value of N, a circle can be drawn in the complex plane. The family of these circles are called N circles.

20. What are the two types of compensation?

The two types of compensation are

(i) Cascade or series compensation
(ii) Feedback compensation or parallel compensation

21. What are the three types of compensator? **(MAY/JUNE 2013)**

The three types of compensators are

1. Lag compensator
2. Lead compensator
3. Lag – lead compensator

22. What are the uses of lead compensator? **(NOV/DEC 2011)**

The uses of lead compensator are

- Speeds up the transient response
- Increases the margin of stability of a system
- Increases the system error constant to a limited extent.

23. What is the use of lag compensator? **(APRIL/MAY 2011)**

The lag compensator improves the steady state behaviour of a system, while nearly preserving its transient response.

24. When lag – lead compensator is required?

The lag – lead compensator is required when both the transient and steady state response of a system has to be improved.

25. What is a compensator? **(APRIL/MAY 2011)**

A device inserted into the system for the purpose of satisfying the specification is called as a compensator.

26. When lag/ lead/ lag – lead compensation is employed? **(APRIL/MAY 2011, May/June 2016, April/May 2017, Nov/Dec 2017)**

Lag compensation is employed for stable system for improvement in steady state performance. Lead compensation is employed for stable/ unstable system for improvement in transient state performance. Lag – lead compensation is employed for stable/unstable system for improvement in both steady state and transient state performance.

27. What are the effects of adding a zero to a system?

Adding a zero to a system results in pronounced early peak to system response thereby the peak overshoot increase appreciably.

28. What are the characteristics of phase lead network? (APRIL/MAY 2015)

- In lead compensation, if the bandwidth increases, the speed off response will also get increased
- The lead compensator having the phase lead frequency response characteristics which will improve the transient response and will also extent to steady state response.

29. What is the significant of Nichol's plot? (NOV/DEC 2016)

The complete closed loop frequency response can be obtained by using Nichol's chart. All the frequency domain specification can be obtained by sketching open loop magnitude – phase plot on the Nichol's chart.

30. What is series compensation? (NOV/DEC 2016)

If the compensator is placed in the forward path of the plant then, the compensation is termed as series compensation.

PART – B
BODE PLOT

1. For the following transfer function, sketch the Bode plot. Also determine gain margin & phase margin.

$$G(s)H(s)=\frac{5}{s(10+s)(20+s)}$$

Solution:-
The sinusoidal transfer function G(jω) is obtained by replacing s by jω in the s – domain transfer function.

$$G(s)H(s)=\frac{5}{s\times 10(1+0.1s)20(1+0.05s)}$$

$$=\frac{5}{200s(1+0.1s)(1+0.05s)}$$

$$=\frac{0.025}{s(1+0.1s)(1+0.05s)}$$

Put s = jω

$$G(j\omega)H(j\omega)=\frac{0.025}{j\omega(1+j0.1\omega)(1+j0.05\omega)}$$

Magnitude plot

The corner frequencies are $\omega_{c1}=\frac{1}{0.1}=10\,\text{rad/sec}$

$$\omega_{c2}=\frac{1}{0.05}=20\ \text{rad/sec}$$

Team	Corner frequency rad /sec	Slope db/dec	Change in slope db/dec

$\dfrac{0.025}{j\omega}$	-	-20	-
$\dfrac{1}{1+j0.1\omega}$	$\omega_{c1}=\dfrac{1}{0.1}=10$	-20	-20 – 20 = -40
$\dfrac{1}{1+j0.05\omega}$	$\omega_{c2}=\dfrac{1}{0.05}=20$	-20	-40-20 = -60

Choose a low frequency ω_ℓ such that $\omega_\ell < \omega_{c1}$ & choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let ω_ℓ = 0.1 rad/ sec, ω_h = 50 rad/sec

Let $A = |G(j\omega)|$ in dB

Calculating of gain A at $\omega = \omega_\ell, \omega_{c1}, \omega_{c2} + \omega_h$

At $\omega = \omega_\ell$, $\quad A = 20\log\left|\dfrac{0.025}{j\omega}\right| \omega = \omega_\ell = 0.1$

$$= 20\log\left|\frac{0.02}{0.1}\right|$$

$$= -12.04\,\text{dB}$$

At $\omega = \omega_{c1}$ $\quad A = 20\log\left|\dfrac{0.025}{j\omega}\right| \omega = \omega_{c1} = 10$

$$= 20\log\left|\frac{0.025}{10}\right|$$

$$= -52.04 \text{ db}$$

At $\omega = \omega_{c2}$ $\quad A = \left[\text{slope from } \omega_{c1} \text{to } \omega_{c2} \times \log\dfrac{\omega_{c2}}{\omega_{c1}}\right] + \text{A at } \omega = \omega_{c1}$

$$= 40 \times \log\left|\frac{20}{10}\right| + (-52.04)$$

$$-12.04 + (-52.04) = -64.08\,\text{dB}$$

At $\omega = \omega_h$ $\quad A = \left[\text{slope from } \omega_{c2} \text{ to} \omega_h \times \log\dfrac{\omega_h}{\omega_{c2}}\right] + \text{A at} \, \omega = \omega_{c2}$

$$= -60 \times \log\left(\frac{50}{20}\right) + (-64.08) = -88 \text{ db}$$

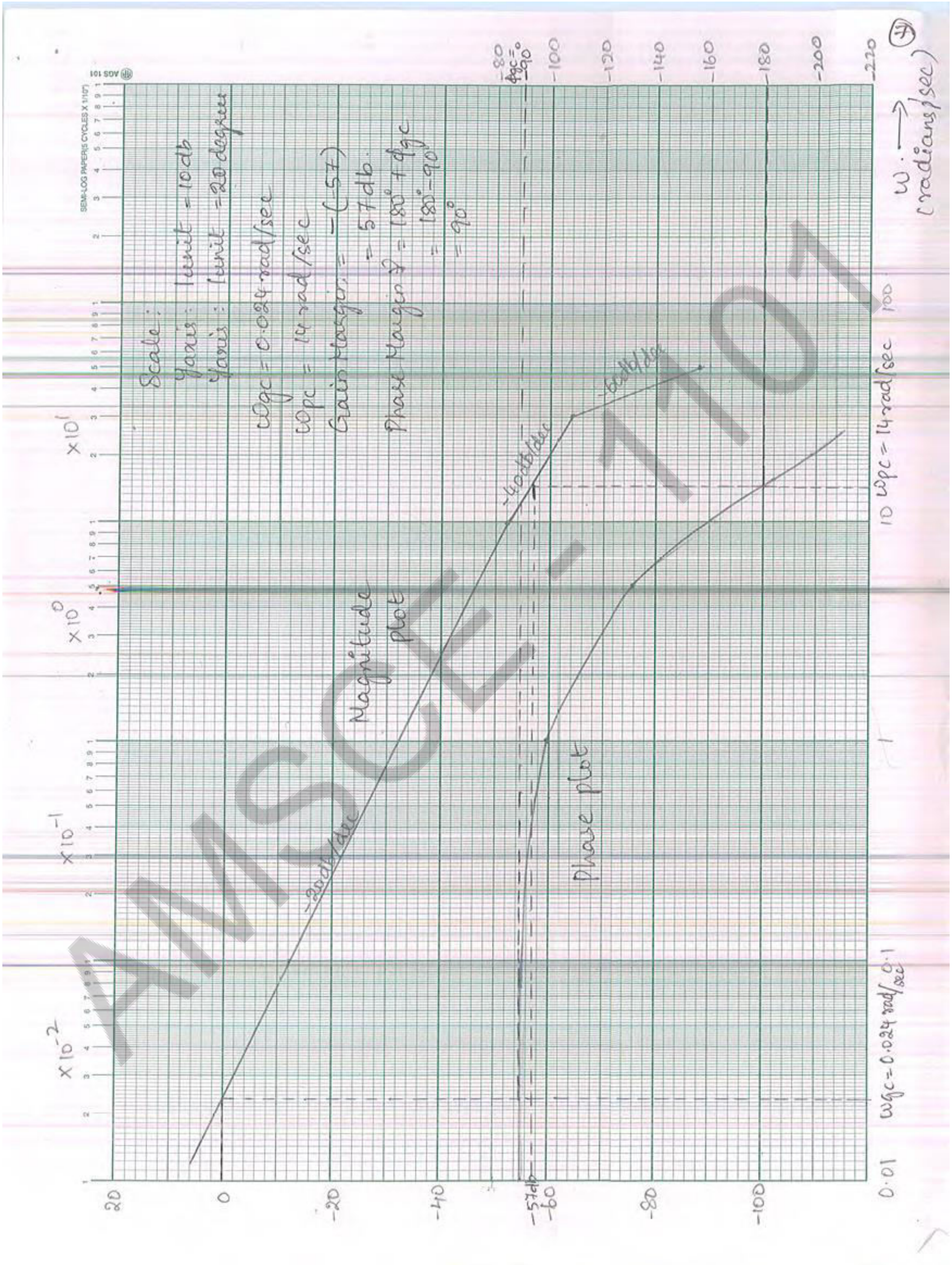
ω rad/sec	A dB
0.1	-12.04
10	-52.04
20	-64.08
50	-88

Phase angle plot

$\phi = \underline{|G}(j\omega) = -90 - \tan^{-1} 0.1\omega - \tan^{-1} 0.05\omega$

ω rad/sec	$\phi = \underline{|G}(j\omega)$ deg
0.01	-90.08
0.1	-90.85
1	-98.57
5	-130.6
10	-161.6

14	-179.45
15	-183.17
20	-198.43



From graph, gain crossover frequency $\omega_{gc} = 0.024$ rad/ sec

Phase crossover frequency ω_{pc} = 14rad/ sec
Gain margin = 57 db
Phase margin $\gamma = 90°$

2. Sketch the Bode plot for the following transfer function and determine the phase margin and gain margin $G(s) = \dfrac{20}{s(1+3s)(1+4s)}$

Solution:-

The sinusoidal transfer function of G(jω) is obtained by replacing s by jω in the given transfer function.

$$G(j\omega) = \frac{20}{j\omega(1+j3\omega)(1+j4\omega)}$$

Magnitude plot

The corner frequencies, $\omega_{c1} = \dfrac{1}{4} = 0.25\ \text{rad/sec}$

$$\omega_{c2} = \frac{1}{3} = 0.33\ \text{rad/sec}$$

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/sec
$\dfrac{20}{j\omega}$	-	-20	-
$\dfrac{1}{1+j3\omega}$	$\omega_{c1} = \dfrac{1}{4} = 0.25$	-20	-20 – 20 = - 40
$\dfrac{1}{1+j4\omega}$	$\omega_{c2} = \dfrac{1}{3} = 0.33$	-20	-40 - 20 = -60

Choose a frequency ω_ℓ such that $\omega_\ell < \omega_{c1}$ and choose a frequency ω_h such that $\omega_h > \omega_{c2}$

Let $\omega_\ell = 0.15\text{rad/sec}$ and $\omega_h = 2\text{rad/sec}$

Calculation of gain A at $\omega = \omega_\ell, \omega_{c1}, \omega_{c2} + \omega_h$

At $\omega = \omega_\ell$, $A = |G(j\omega)| = 20\log\left|\dfrac{20}{j\omega}\right| \omega = \omega_\ell = 0.15$

$$= 20\log\left|\frac{20}{0.15}\right| = 42.5\ \text{dB}$$

At $\omega = \omega_{c1}$, $A = |G(j\omega)| = 20\log\left|\dfrac{20}{0.15}\right| \omega = \omega_{c1} = 0.25$

$$= 20\log\left|\frac{20}{0.25}\right| = 38\text{dB}$$

At $\omega = \omega_{c2}$ $A = \left[\text{Slope from } \omega_{c1} \text{to} \omega_{c2} \times \log\dfrac{\omega_{c2}}{\omega_{c1}}\right] + \text{A at } \omega = \omega_{c1}$

$$= -40 \times \log\frac{0.33}{0.25} + 38 = 33\text{dB}$$

At $\omega = \omega_h$ $A = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_h \times \log\dfrac{\omega_h}{\omega_{c2}}\right] + \text{A at } \omega = \omega_{c2}$

$$= -60 \times \log\frac{2}{0.33} + 33 = -13.95 \approx 14\text{dB}$$

ω rad/sec	A dB

0.15	42.5
0.25	38
0.33	33
2	-14

Phase angle plot

$\phi = \underline{|G}(j\omega) = -90° - \tan^{-1}3\omega - \tan^{-1}4\omega$

ω rad/sec	$\phi = \underline{|G}(j\omega)$ deg
0.15	-146
0.2	-160
0.25	-172
0.33	-188
0.6	-218
1	-238
2	-253

From graph,

Gain cross over frequency, ω_{gc} = 1.12 rad/sec

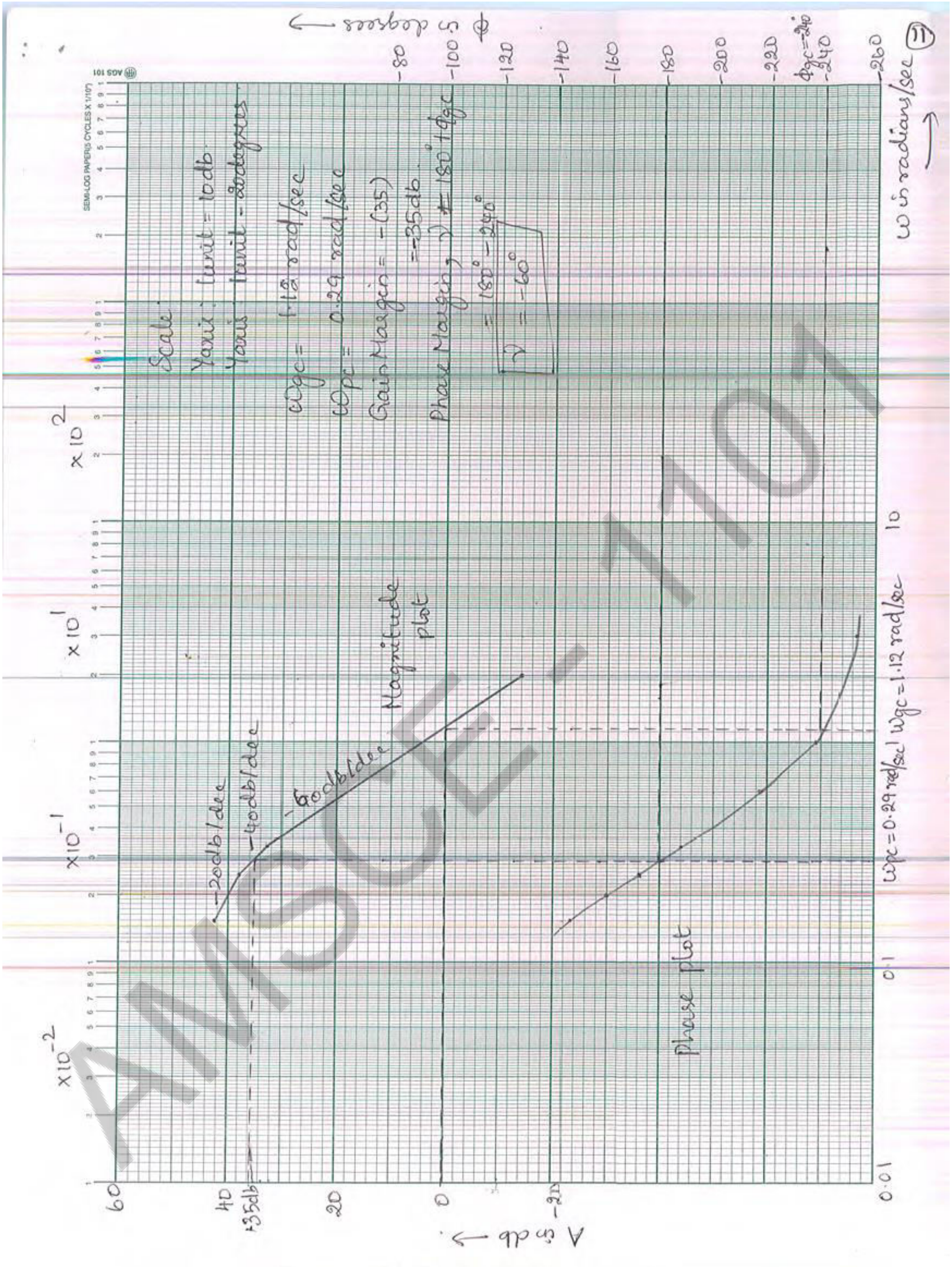
Phase cross over frequency, ω_{pc} = 0.29 rad/sec

Gain margin = -35db

Phase margin $\gamma = 180° + \phi gc$

$= 180° - 240°$

$= -60°$



3. Sketch the Bode plot for the following transfer function and determine the system gain K for the gain cross over frequency to be 5rad/ sec. $G(s)=\dfrac{ks^2}{(1+0.2s)(1+0.02s)}$ **APRIL/MAY 2017**

Solution:-

The sinusoidal transfer function G(jω) is obtained by replacing s by (jω) in the s-domain transfer function

Put s = jω

$$\therefore G(j\omega) = \frac{k(j\omega)^2}{(1+j0.2\omega)(1+j0.02\omega)}$$

Let k = 1

$$\therefore G(j\omega) = \frac{(j\omega)^2}{(1+j0.2\omega)(1+0.02\omega)}$$

Magnitude plot

The corner frequency are $\omega_{c1} = \frac{1}{0.2} = 5 \text{rad/sec}$

$$\omega_{c2} = \frac{1}{0.02} = 50 \text{rad/sec}$$

Term	Corner frequency rad/sec	Slope db/ dec	Change in slope db/deg
$(j\omega)^2$	-	+40	-
$\frac{1}{1+j0.2\omega}$	$\omega_{c1} = \frac{1}{0.2} = 5$	-20	40 – 20 = 20
$\frac{1}{1+j0.02\omega}$	$\omega_{c2} = \frac{1}{0.02} = 50$	-20	20 – 20 = 0

Choose a low frequency ω_ℓ such that $\omega_\ell < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let $\omega_\ell = 0.5 \text{rad/sec}$ and $\omega_h = 100 \text{rad/sec}$

Let $A = |G(j\omega)| \text{in db}$

Calculating of gain A at $\omega = \omega_\ell, \omega_{c1}, \omega_{c2} + \omega_h$

At $\omega = \omega_\ell$, $A = 20\log|(j\omega)^2|$

$$= 20\log(\omega)^2$$

$$= 20\log(0.5)^2 = -12 \text{db}$$

At $\omega = \omega_{c1}$, $A = 20\log|(j\omega)^2|$

$$= 20\log(\omega)^2$$

$$= 20\log(0.5)^2 = 28 \text{db}$$

At $\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A \text{ at } \omega = \omega_{c1}$

$$= 20 \times \log\frac{50}{5} + 28 = 48 \text{db}$$

At $\omega = \omega_h$ $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A \text{ at } \omega = \omega_{c2}$

$$= 0 \times \log\frac{100}{50} + 48 = 48 \text{db}$$

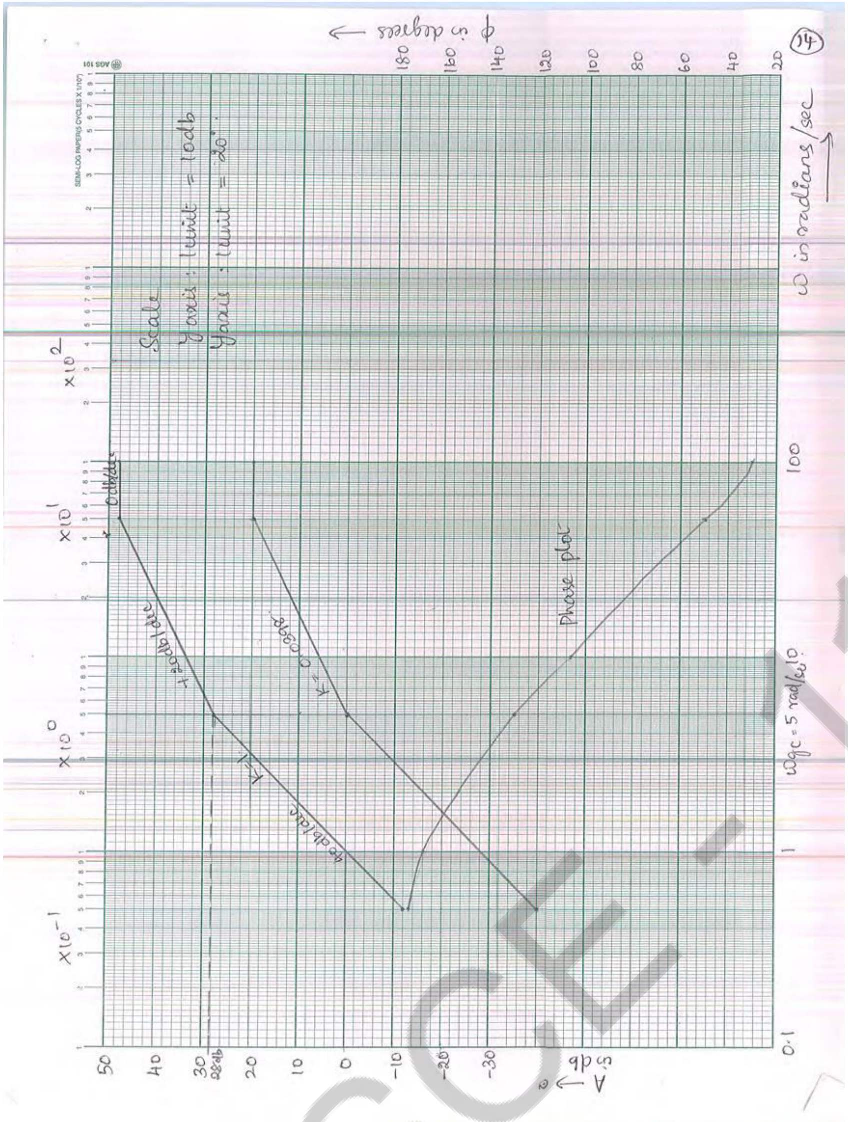


Table-1

ω rad / sec	A dB
0.5	-12
5	28
50	48
100	48

Phase plot

$\phi = \underline{\lfloor G}(j\omega) = 180° - \tan^{-1} 0.2\omega - \tan^{-1} 0.02\omega$

Table 2

ω = rad/sec	$\phi = \underline{\lfloor G}(j\omega)$ deg
0.5	174
1	168
5	130
10	106
50	50

100	30

Bode plot for the above table 1 & 2 is shown in fig.

To find K

Gain cross over frequency ω_{gc} = 5 rad/sec (given) At $\omega = \omega_{gc}$ = 5 rad/sec, the gain is 28 dB.
If gain cross over frequency is 5rad/sec, then at that frequency, the dB gain should be zero.

Hence to every point of magnitude plot a dB gain of -28dB should be added.
The value of k is calculated by equating
20 log k to -28 dB

20 log k = -28 dB

$$20\log k = -28\text{dB}; k = 10^{-28/20}; k = 0.0398$$

4. Given $G(s) = \frac{ke^{-0.2s}}{s(s+2)(s+8)}$. Find K so that the system is stable with

(a) gain margin equal to 6db (b) Phase margin equal to 45°

Solution:-

Put k =1 and convert the given transfer function to time constant form (or) bode form

$$\therefore G(s) = \frac{e^{-0.2s}}{s(s+2)(s+8)} = \frac{e^{-0.2s}}{s \times 2(1+0.5s) \times 8(1+0.125s)}$$

$$= \frac{0.0625e^{-0.2s}}{s(1+0.5s)(1+0.125s)}$$

The sinusoidal transfer function G(jω) is obtained by replacing s by jω.

$$\therefore G(j\omega) = \frac{0.0625e^{-j0.2\omega}}{j\omega(1+j0.5\omega)(1+j0.125\omega)}$$

Magnitude plot

The corner frequency are, $\omega_{c1} = \frac{1}{0.5} = 2\text{rad/sec}$

$\omega_{c2} = \frac{1}{0.125} = 8\text{rad/sec}$

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.0625}{j\omega}$	-	-20	-
$\frac{1}{1+j0.5\omega}$	$\omega_{c1} = \frac{1}{0.5} = 2$	-20	-20 – 20 = -40
$\frac{1}{1+j0.125\omega}$	$\omega_{c2} = \frac{1}{0.125} = 8$	-20	-40 – 20 = -60

Choose a low frequency ω_ℓ such that $\omega_\ell < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let $\omega_\ell = 0.5\text{rad/sec}$ and $\omega_h = 50\text{rad/sec}$

Calculation of gain A at $\omega = \omega_\ell, \omega_{c1}, \omega_{c2} + \omega_h$

At $\omega = \omega_\ell$, $A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\left|\frac{0.0625}{0.5}\right| = -18\,db$

At $\omega = \omega_{c1}$, $A = 20\log\left|\frac{0.0625}{j\omega}\right| = 20\log\left|\frac{0.0625}{2}\right| = -30\,db$

At $\omega = \omega_{c2}$, $A = \left[\text{slope form } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A \text{ at } \omega = \omega_{c1}$

$$= -40 \times \log\frac{8}{2} + (-30) = -54\,db$$

At $\omega = \omega_h$, $A = \left[\text{slope form } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A \text{ at } \omega = \omega_{c2}$

$$= -60 \times \log\frac{50}{8} + (-54) = -102\,db$$

ω rad/sec	A db
0.5	-18
2	-30
8	-54
50	-102

Phase angle plot

$$\phi = \underline{|G}(j\omega) = -0.2 \times \omega \times \frac{180}{\pi} - 90° - \tan^{-1} 0.5\omega - \tan^{-1} 0.125\omega$$

ω rad/sec	$\phi = \underline{|G}(j\omega)\,deg$
0.01	-90
0.1	-94
0.5	-114
1	-134
2	-172
3	-202
4	-226

The above Bode plot for the above transfer function is shown in fig.

To find K

With k = 1, gain margin = 32 db

But required gain margin is 6db. Hence to every point of magnitude plot, a db gain of 26 db is added.

$$20\log k = 26$$

$$k = 10^{26/20} = 19.95$$

Phase margin $\gamma = 180° + \phi_{gc}$

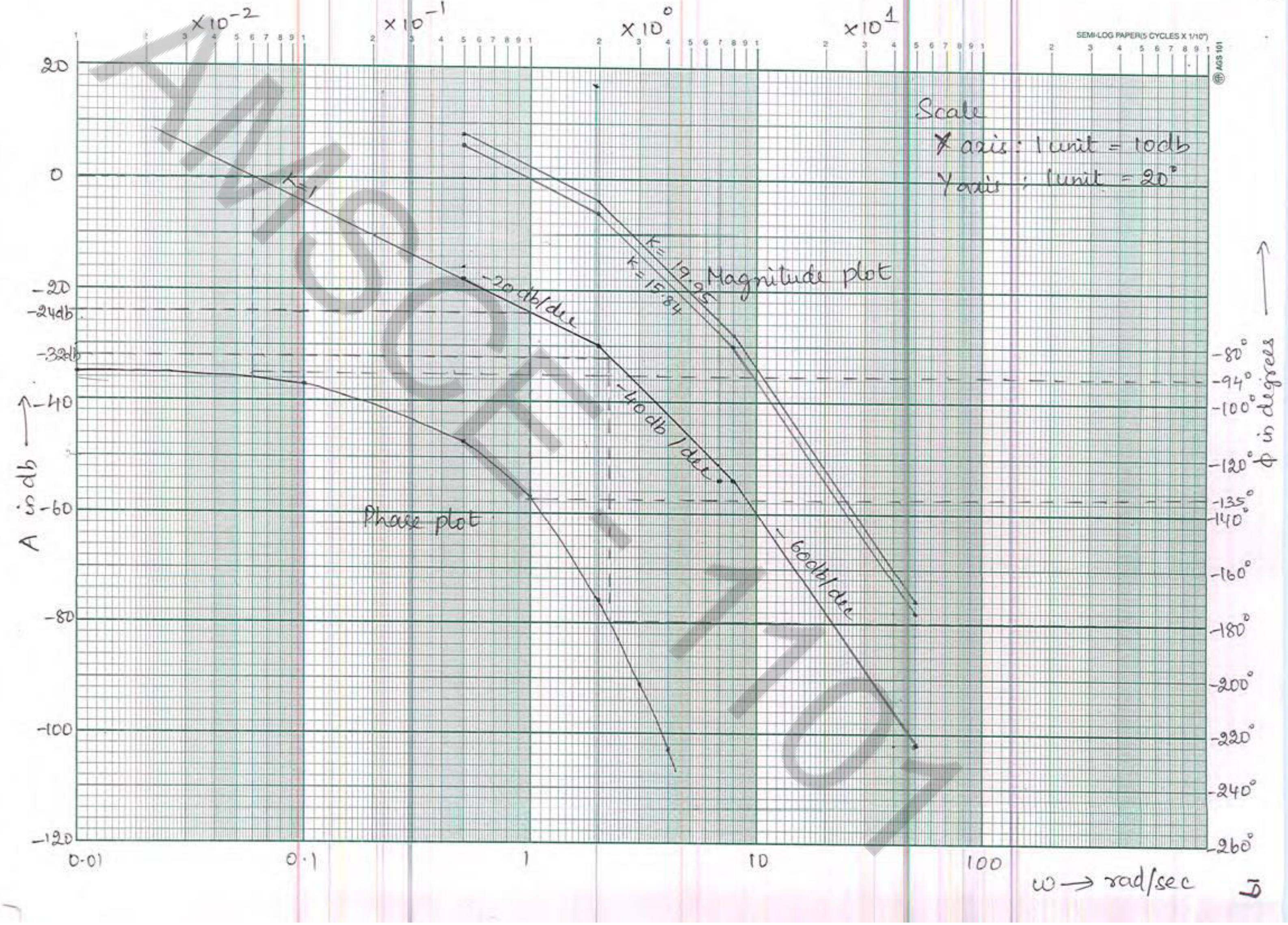
When $\gamma_{new} = 45°, \phi_{gc\,new} = \gamma_{new} - 180° = 45° - 180° = -135°$

When K = 1, the db gain at $\phi_{gc} = -135°$ is -24 db.

The gain must be made zero, to have PM = 45°. Hence to every point of magnitude plot a db gain of 24 db should be added.

The value of k is calculated by

$$20\log k = 24; k = 10^{24/20}; k = 15.84$$


Scale
X axis : 1 unit = 10db
Y axis : 1unit = 20°
Magnitude plot
Phase plot
K=1
K= 19.95
K= 15.84
-20db/dec
-40db/dec
-60db/dec
A in db
φ in degrees
ω → rad/sec
-24db
-32db
-94°
-135°
0.01
0.1
1
10
100
20
0
-20
-40
-60
-80
-100
-120
-80°
-100°
-120°
-140°
-160°
-180°
-200°
-220°
-240°
-260°

5. Sketch the bode plot for the following transfer function & determine phase margin

$$G(s)=\frac{75(1+0.2s)}{s(s^2+16s+100)}$$

$$G(s)=\frac{75(1+0.2s)}{s(s^2+16s+100)}=\frac{75(1+0.2s)}{s\times100\left(\frac{s^2}{100}+\frac{16s}{100}+1\right)}$$

$$=\frac{0.75(1+0.2s)}{s(1+0.01s^2+0.16s)}$$

Put s = jω

$$G(j\omega)=\frac{0.75(1+j0.2\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}$$

$$=\frac{0.75(1+j0.2\omega)}{j\omega(1-0.01\omega^2+j0.16\omega)}$$

Magnitude plot

The corner frequencies are, $\omega_{c1}=\frac{1}{0.2}=5\,rad/sec$

$\omega_{c2}=\omega_n=10\,rad/sec$

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{0.75}{j\omega}$	-	-20	-
$1+j0.2\omega$	$\omega_{c1}=\frac{1}{0.2}=5$	+20	-20 +20 = 0
$\frac{1}{1+0.01\omega^2+j0.16\omega}$	$\omega_{c2}=\omega_n=10$	-40	0 – 40 = -40

Choose a low frequency ω_ℓ such that $\omega_\ell<\omega_{c1}$ and choose a high frequency ω_h such that $\omega_h>\omega_{c2}$

Let $\omega_\ell=0.5\,rad/sec$ and $\omega_h=20\,rad/sec$

Calculation of gain A at $\omega=\omega_\ell,\omega_{c1},\omega_{c2}+\omega_h$

At $\omega=\omega_\ell$, $A=20\log\left|\frac{0.75}{j\omega}\right|=20\log\frac{0.75}{0.5}=3.5\,db$

At $\omega=\omega_{c1}$, $A=20\log\left|\frac{0.75}{j\omega}\right|=20\log\frac{0.75}{5}=-16.5\,db$

At $\omega=\omega_{c2}$, $A=\left[\text{slope from }\omega_{c1}\text{ to }\omega_{c2}\times\log\frac{\omega_{c2}}{\omega_{c1}}\right]+A\text{ at }\omega=\omega_{c1}$

$$=0\times\log\frac{10}{5}+(-16.5)=-16.5\,db$$

At $\omega=\omega_h$, $A=\left[\text{slope from }\omega_{c2}\text{ to }\omega_h\times\log\frac{\omega_h}{\omega_{c1}}\right]+A\text{ at }\omega=\omega_{c2}$

$$=-40+\log\frac{20}{10}+(-16.5)=-28.5\,db$$

ω rad/sec	A dB
0.5	3.5
5	-16.5
10	-16.5
20	-28.5

Phase angle plot.

$$\phi = \underline{|G}(j\omega) = \tan^{-1}0.2\omega - 90° - \tan^{-1}\frac{0.16\omega}{1-0.01\omega^2}, \text{for}\,\omega \le \omega_n$$

$$\phi = \underline{|G}(j\omega) = \tan^{-1}0.2\omega - 90°\left(\tan^{-1}\frac{0.16}{1-0.01\omega^2} + 180°\right)\text{for}\,\omega > \omega_n$$

ω rad/sec	$\phi = \angle G(j\omega)\,\text{deg}$
0.5	-88
1	-88
5	-92
10	-116
20	-148
50	-168
100	-174

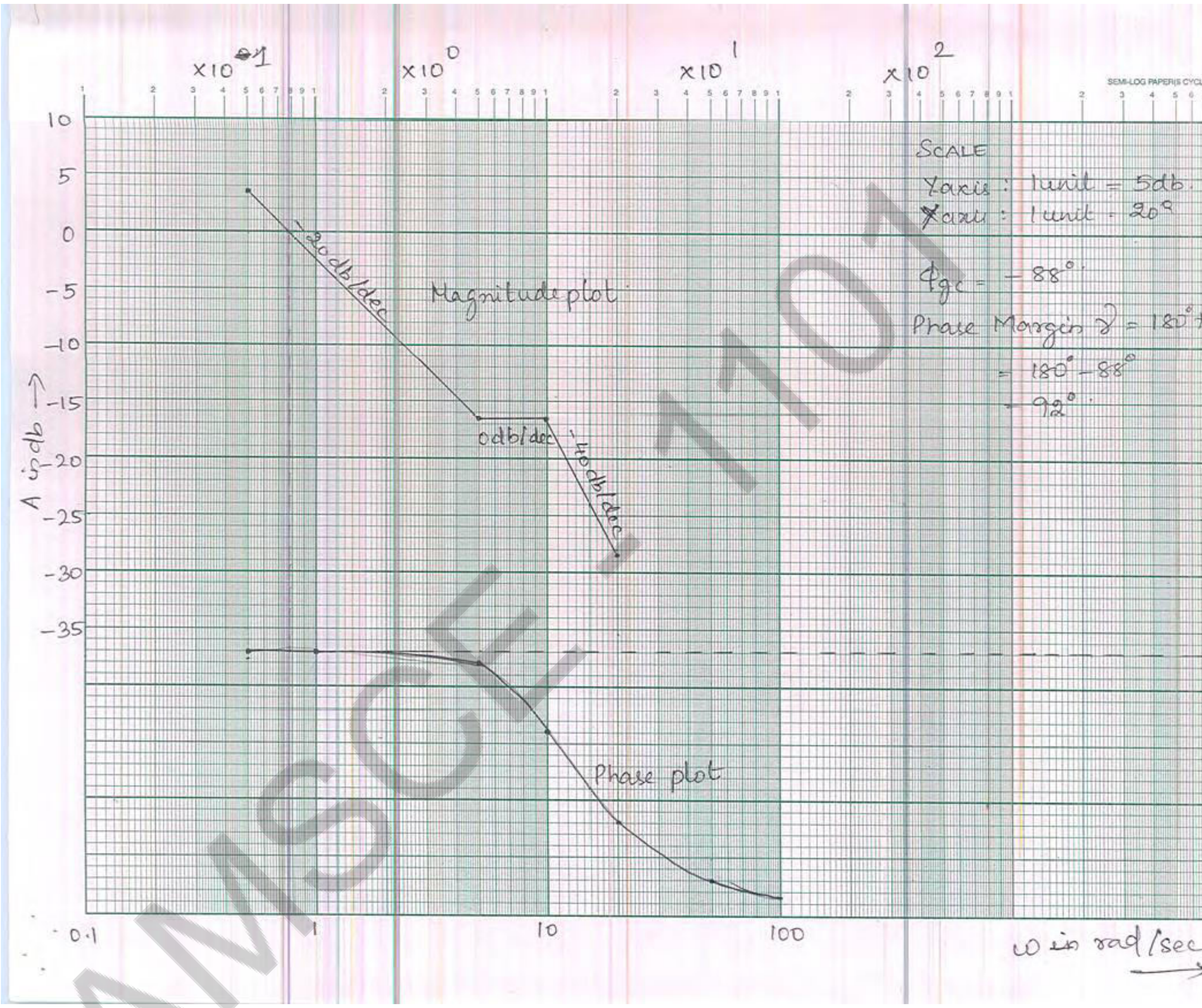
From graph

ϕ_{gc} = -88°

Phase Margin γ = 180° + ϕ_{gc}

= 180 ° - 88° = 92°

Gain Margin = + ∞ [As phase plot crosses the -180° at infinity. $|G(j\omega)|$ at infinity = -∞ db]



POLAR PLOT

1. The open loop transfer function of a unity feedback system is given by $G(s)=\frac{1}{s^2(1+s)(1+2s)}$**. Sketch the polar plot and determine the gain margin and phase margin**

Solution

Given that $G(s) = \dfrac{1}{s^2(1+s)(1+2s)}$

Put s = jω, $G(j\omega) = \dfrac{1}{(j\omega)^2(1+j\omega)(1+j2\omega)}$

$$= \frac{1\angle 0°}{\omega^2\angle 180°\sqrt{1+\omega^2}\angle \tan^{-1}\omega\sqrt{1+4\omega^2}\angle \tan^{-1}2\omega}$$

$$G(j\omega) = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}\angle\left(-180° - \tan^{-1}\omega - \tan^{-1}2\omega\right)$$

$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = \frac{1}{\omega^2\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$= \frac{1}{\omega^2\sqrt{1+5\omega^2+4\omega^4}}$$

$\angle G(j\omega) = -180° - \tan^{-1}\omega - \tan^{-1}2\omega$

Corner frequencies

$\omega_{c1} = \dfrac{1}{1} = 1$ rad / sec

$\omega_{c2} = \dfrac{1}{2} = 0.5$ rad / sec

Magnitude and phase plot of G(jω)

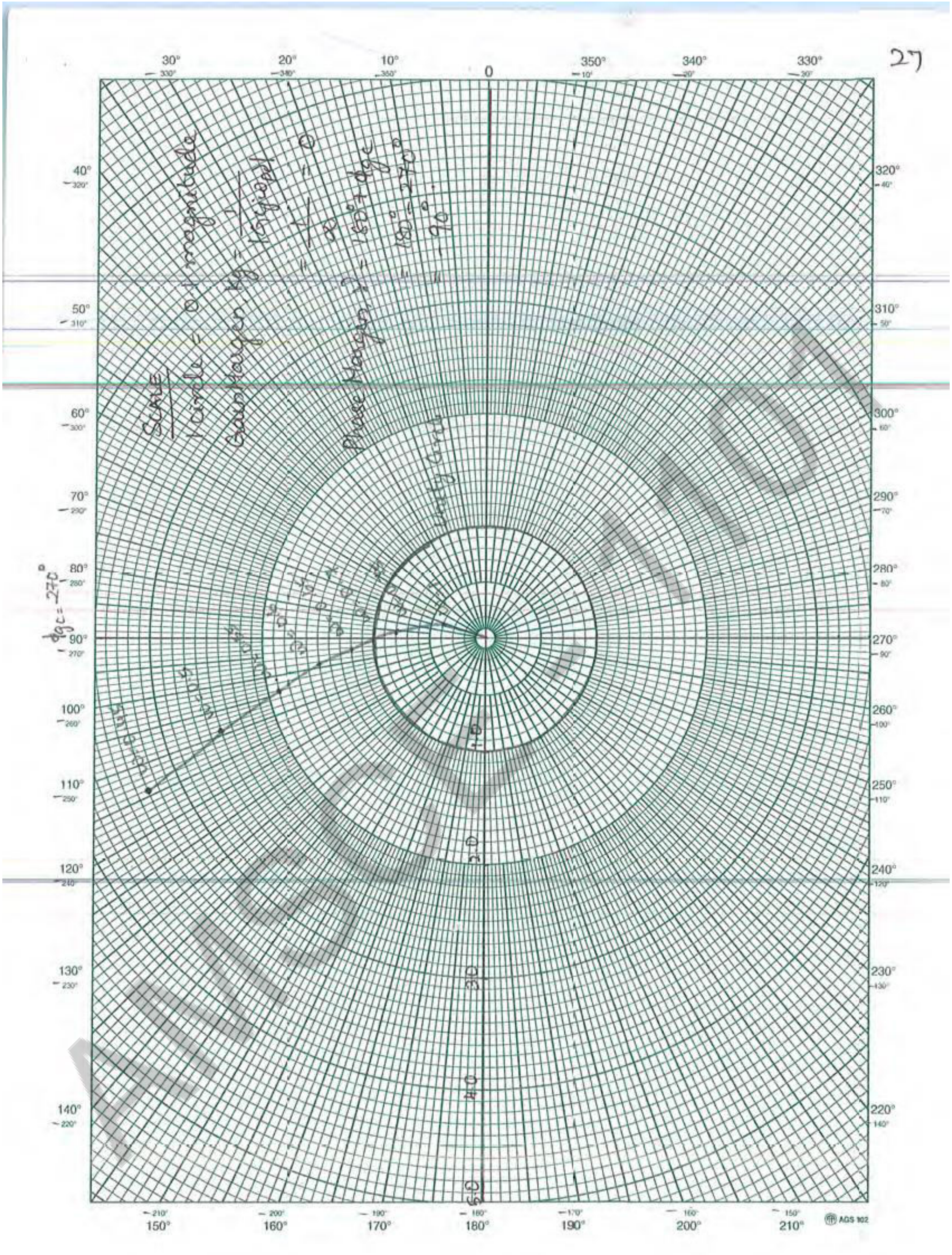
ω rad/sec	|G(jω)|	∠G(jω) Deg
0.45	3.33	-246
0.5	2.5	-251
0.55	1.9	-256
0.6	1.5	-261
0.65	1.2	-265
0.7	0.97 ≅ 1	-269
0.75	0.8	-273
1.0	0.3	-288

From Polar graph

Gain Margin, $K_g = \dfrac{1}{\infty} = 0$

Phase Margin, γ = 180° - 270°

= - 90°



2. The open loop transfer function of a unity feedback system is given by $G(s)=\dfrac{1}{s(1+s)^2}$. Sketch the polar plot and determine the gain margin and phase margin.

Solution

Given that $G(s) = \frac{1}{s(1+s)^2}$

Put s = jω

$\therefore$ G(jω) = $\frac{1}{j\omega(1+j\omega)^2} = \frac{1}{j\omega(1+j\omega)(1+j\omega)}$

$= \frac{1}{\omega\angle 90°\sqrt{1+\omega^2}\ \tan^{-1}\omega\sqrt{1+\omega^2}\ \tan^{-1}\omega}$

$= \frac{1}{\omega\left(\sqrt{1+\omega^2}\right)^2}\angle\left(-90° - 2\tan^{-1}\omega\right)$

$|G(j\omega)| = \frac{1}{\omega(1+\omega^2)} = \frac{1}{\omega+\omega^3}$

$\angle G(j\omega) = -90° - 2\tan^{-1}\omega$

Corner frequencies.

$\omega_{c1} = \frac{1}{1} = 1$ rad / sec

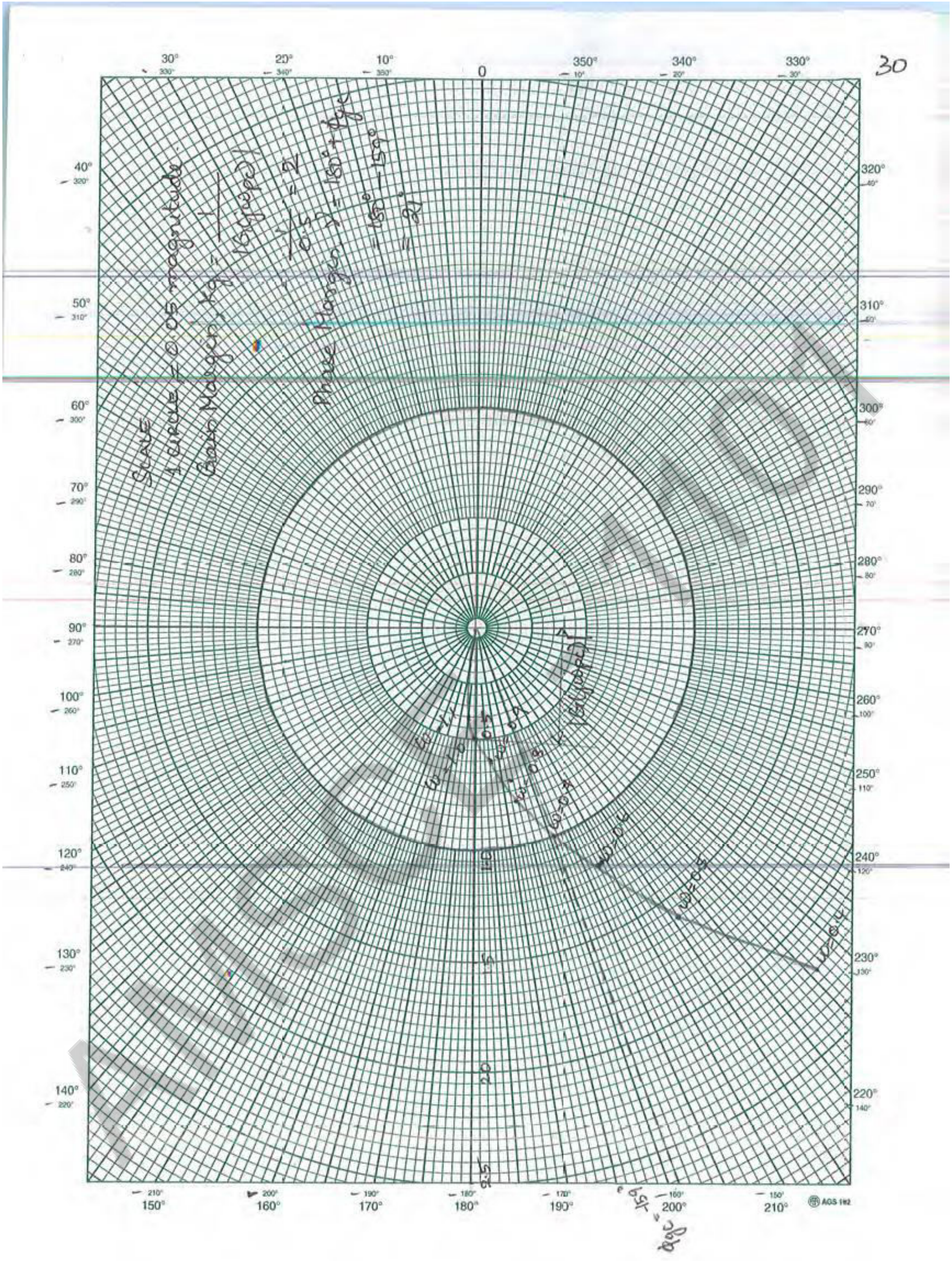
Table Magnitude and phase plot of G(jω)

ω rad/sec	|G(jω)|	∠G(jω) Deg
0.4	2.2	-134
0.5	1.6	-143
0.6	1.2	-151
0.7	1	-159
0.8	0.8	-167
0.9	0.6	-174
1.0	0.5	-180
1.1	0.4	-185

From Polar graph,

Gain Margin kg = 2

Phase Margin γ = 21°



3. Consider a unity feedback system having an open loop transfer function, $G(s)=\dfrac{k}{s(1+0.5s)(1+4s)}$

Sketch the polar plot and determine the value of k so that (i) Gain Margin is 20db
(ii) Phase Margin is 30°
Solution

Given that $G(s) = \dfrac{k}{s(1+0.5s)(1+4s)}$

Put k = 1 and s = jω in G(s)

$$\therefore\ G(j\omega) = \frac{1}{j\omega(1+j0.5\omega)(1+j4\omega)}$$

$$= \frac{1}{\omega\angle 90^\circ\sqrt{1+(0.5\omega)^2}\ \tan^{-1} 0.5\omega\sqrt{1+(4\omega)^2}\angle\tan^{-1} 4\omega}$$

$$= \frac{1}{\omega\sqrt{1+0.25\omega^2}\ \sqrt{1+16\omega^2}\ \left(+90^\circ+\tan^{-1} 0.5\omega+\tan^{-1} 4\omega\right)}$$

$$= \frac{1}{\omega\sqrt{1+0.25\omega^2}\ \sqrt{1+16\omega^2}}\angle\left(-90^\circ-\tan^{-1} 0.5\omega-\tan^{-1} 4\omega\right)$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+0.25\omega^2}\ \sqrt{1+16\omega^2}}$$

$\angle G(j\omega) = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 4\omega$

Corner frequencies

$\omega_{c1} = \dfrac{1}{4} = 0.25$ rad / sec

$\omega_{c2} = \dfrac{1}{0.5} = 2$ rad / sec

32 Table Magnitude and Phase plot of G(jω)

ω rad/sec	|G(jω)|	∠G(jω) Deg
0.3	2.11	-149
0.4	1.3	-159
0.5	0.87	-167
0.6	0.61	-174
0.8	0.35	-184
1.0	0.22	-193
1.2	0.15	-199

From polar plot, k = 1

Gain Margin kg = $\dfrac{1}{0.44} = 2.27$

Gain Margin in db = 20 log 2.27 = 7.12 db

Phase Margin, $\gamma = 180^\circ + \phi_{gc}$ - 180° - 165° = 15°

To find k

Case (i)

Let G_B be the magnitude of open loop transfer fn G(jω) at -180° with k = 1

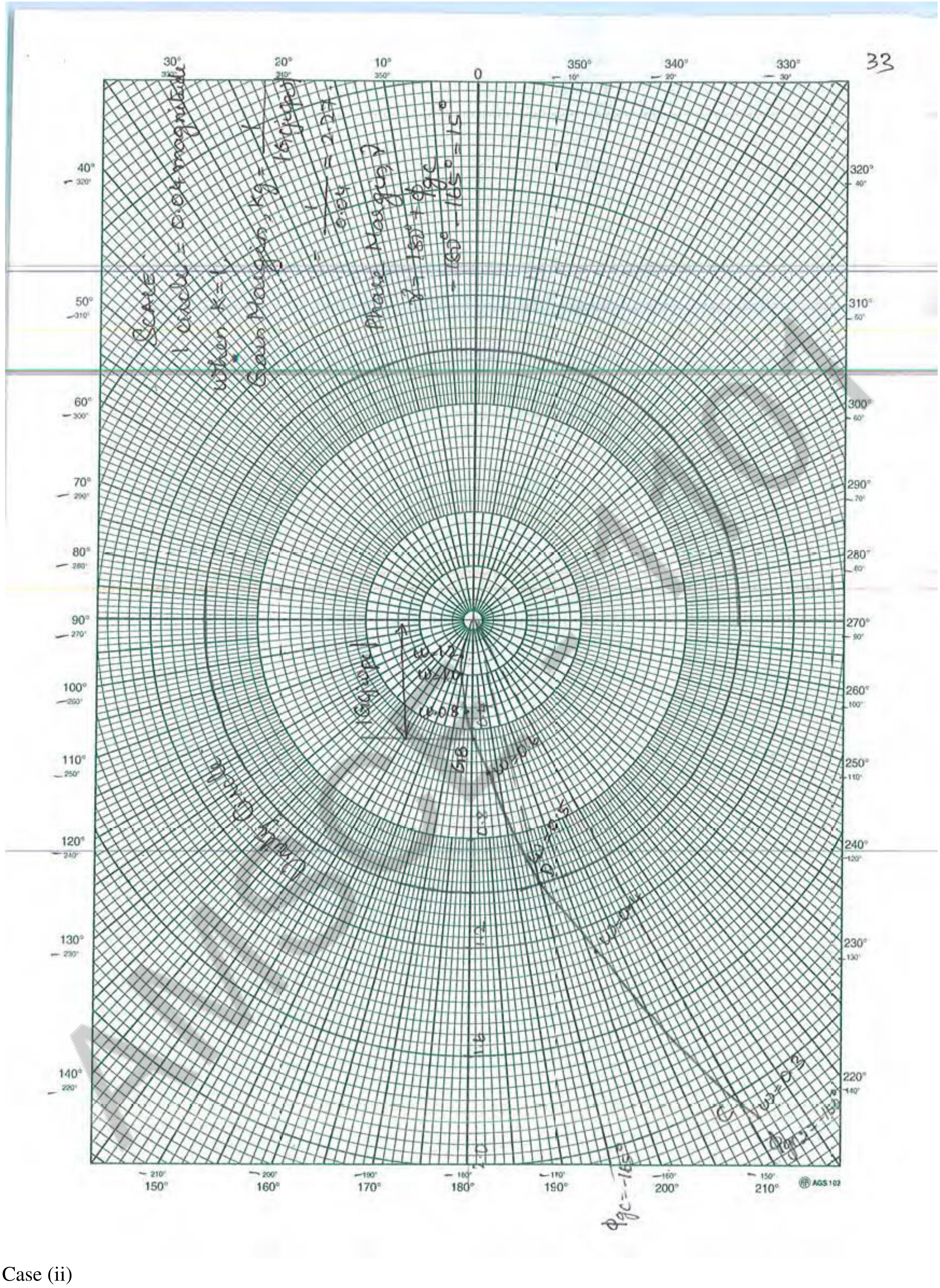
Let G_A be the magnitude of open loop transfer function G(jω) at -180° with k = ? & gain margin of 20 db.

Now $20 \log \dfrac{1}{G_A} = 20 \Rightarrow \log \dfrac{1}{G_A} = \dfrac{20}{20} = 1$

$\Rightarrow G_A = 0.1$

The value of $k = \dfrac{G_A}{G_B} = \dfrac{0.1}{0.44} = 0.227$

k = 0.227



Case (ii)

With k = 1, the phase margin is 15°. This has to be increased to 30°. Hence the gain has to be decreased.

Let ϕ_{gc2} be the phase of $G(j\omega)$ for a phase margin of 30°.

$\therefore\ 30° = 180° + \phi_{gc2}$

$\phi_{gc2} = 30° - 180° = -150°$

In the polar plot the -150° line cuts the locus of $G(j\omega)$ at point c and cut the unity circle at point D.

Let G_C be magnitude of G(jω) at point C

G_D be magnitude of G(jω) at point D

From polar plot, $G_C = 2.04$

$G_D = 1$

$$\text{Now } k = \frac{G_D}{G_C} = \frac{1}{2.04} = 0.49$$

k = 0.49

*** INCLUDE THIS ***

Compensator design using Bode plots

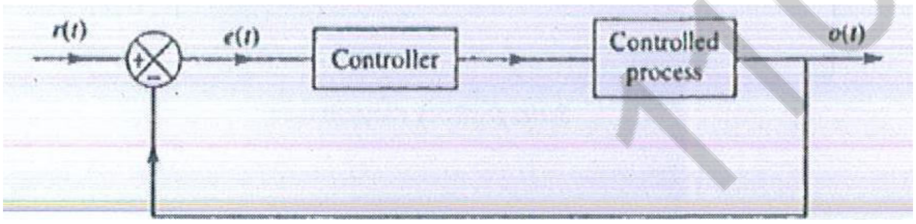
1. Write short notes on different types of compensation

Types of compensation

Series Compensation or Cascade Compensation

This is the most commonly used system where the controller is placed in series with the controlled process.

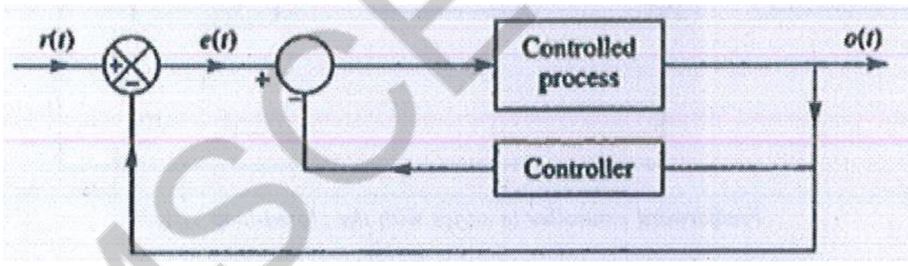
Figure shows the series compensation.



Series compensation

Feedback compensation or Parallel compensation

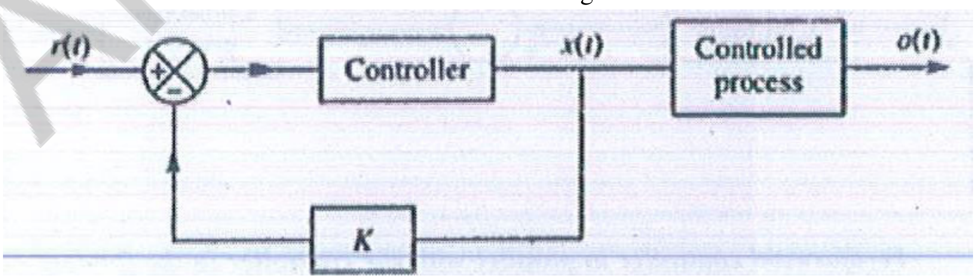
This is the system where the controller is placed in the sensor feedback path as shown in fig.



Feedback compensation or parallel compensation

State Feedback Compensation

This is a system which generates the control signal by feeding back the state variables through constant real gains. The scheme is termed state feedback. It is shown in Fig.



State feedback compensation

The compensation schemes shown in Figs above have one degree of freedom, since there is only one controller in each system. The demerit with one degree of freedom controllers is that the performance criteria that can be realized are limited.

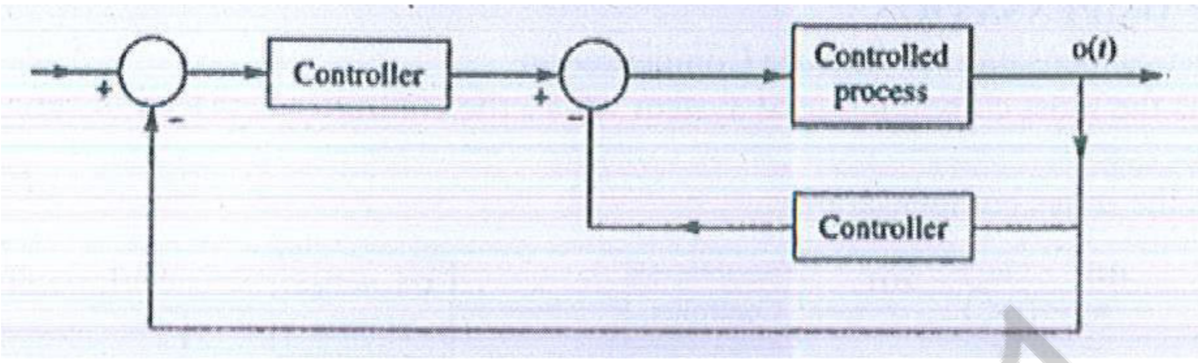
That is why there are compensation schemes which have two degree freedoms, such as:

a) Series – feedback compensation

b) Feed forward compensation

Series- Feedback Compensation

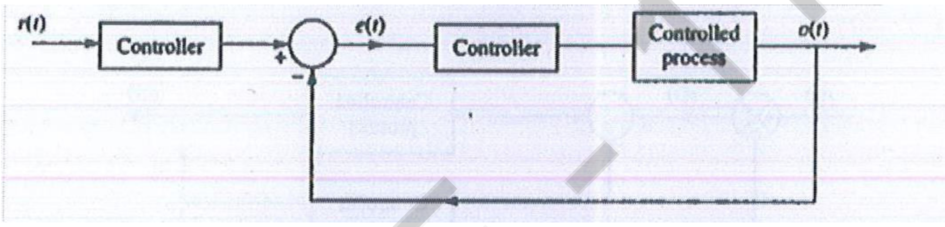
Series-feedback compensation is the scheme for which is series controller and a feedback controller are used. Figure shows the series-feedback compensation scheme.



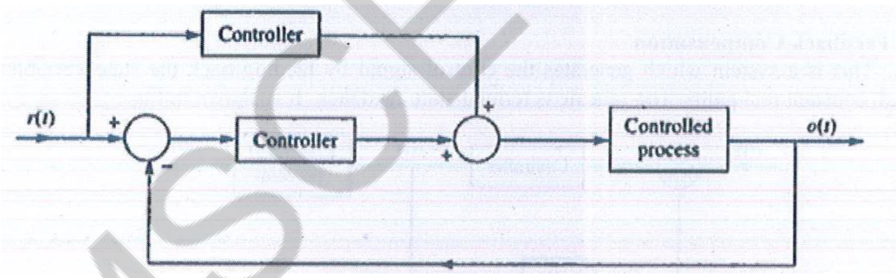
Series-feedback compensation.

Feed forward Compensation

The feed forward controller is placed in series with the closed-loop system which has a controller in the forward path. In Fig. Feed forward the is placed in parallel with the controller in the forward path. The commonly used controller in the above-mentioned compensation schemes are now described in the section below.



Feed forward controller in series with the closed-loop system.



Feed forward controller in parallel with the controller in the forward path.

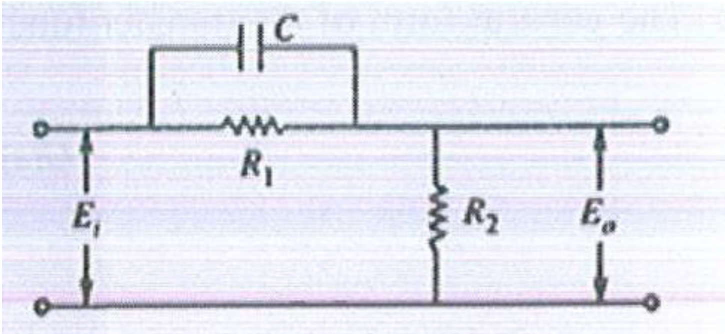
2. Realize the lead compensator using electrical network and obtain the transfer function

Lead Compensator

It has a zero and a pole with zero closer to the origin. The general form of the transfer function of the load compensator is

$$G(s)=\frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}}$$

$$G(j\omega)=\beta\frac{(\tau j\omega+1)}{\beta\tau j\omega+1}$$



$$\frac{E_0(s)}{E_i(s)} = \frac{R_2}{\dfrac{R_1 \times \dfrac{1}{Cs} + R_2\left(R_1 + \dfrac{1}{Cs}\right)}{R_1 + \dfrac{1}{Cs}}} = \frac{R_2R_1 + \dfrac{R_2}{Cs}}{R_1R_2 + \dfrac{1}{Cs}(R_1 + R_2)}$$

$$= \frac{CsR_1R_2 + R_2}{CsR_1R_2 + R_1 + R_2}$$

$$= \frac{R_2(CsR_1 + 1)}{(R_1 + R_2)\left(\dfrac{CsR_1R_2}{R_1 + R_2} + 1\right)}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right)\frac{CR_1s + 1}{\left(\dfrac{CR_1R_2s}{R_1 + R_2} + 1\right)}$$

Subsisting

$\tau = CR_1; \beta\tau = \dfrac{CR_1R_2}{R_1 + R_2}$ ($\because\ \tau = CR_1$)

Transfer function

$G(s) = \beta\ \dfrac{\tau s + 1}{\beta\tau s + 1}$

3. Realize the lag compensator using electrical network and obtain the transfer function

Lag Compensator

It has a zero and a pole with the zero situated on the left of the pole on the negative real axis. The general form of the transfer function of the lag compensator is

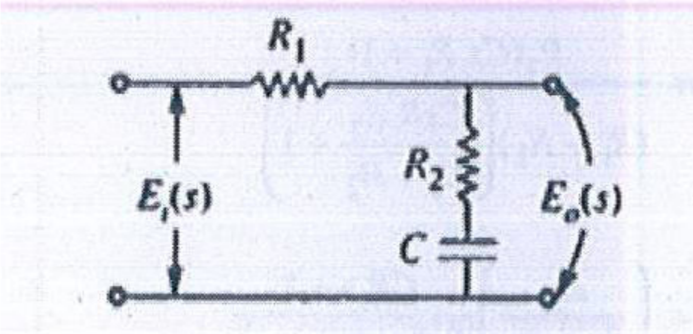
$$G(s) = \frac{s + \dfrac{1}{\tau}}{s + \dfrac{1}{\alpha\tau}} = \frac{\alpha(\tau s + 1)}{\alpha\tau s + 1}$$

Where $\alpha > 1$, $\tau > 0$.

Therefore, the frequency response of the above transfer function will be

$$G(j\omega) = \frac{\alpha(\tau j\omega + 1)}{\alpha\tau j\omega + 1}$$

$$E_0(s) = \frac{E_i(s)}{R_1 + R_2 + \dfrac{1}{Cs}}\left(R_2 + \frac{1}{Cs}\right)$$



Lag compensator

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}$$

$$= \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1}$$

$$= \frac{R_2C\left(s + \frac{1}{R_2C}\right)}{(R_1 + R_2)C\left(s + \frac{1}{(R_1 + R_2)C}\right)}$$

$$= \frac{R_2}{(R_1 + R_2)} \frac{s + \frac{1}{R_2C}}{\left(s + \frac{1}{(R_1 + R_2)C}\right)} = \frac{R_2}{(R_1 + R_2)} \frac{\left(s + \frac{1}{R_2C}\right)}{\left(s + \frac{R_2}{(R_1 + R_2)R_2C}\right)}$$

Now comparing with

$$G(s) = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}}$$

$$\frac{1}{\tau} = \frac{1}{R_2C}; \qquad \frac{1}{\alpha\tau} = \frac{R_2}{(R_1 + R_2)R_2C};$$

$$\frac{1}{a\tau} = \frac{R_2}{(R_1 + R_2)} \frac{1}{\tau} \qquad \left(\because \frac{1}{\tau} = \frac{1}{R_2C}\right)$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

Therefore

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{\alpha} \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}}$$

4. Realize the lag-lead compensator using electrical network and obtain the transfer function

Lag-Lead Compensator

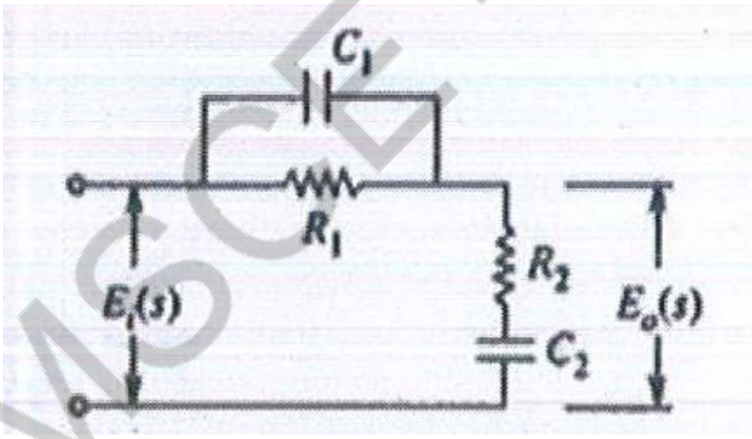
The lag-lead compensator is the combination of a lag compensator and a lead compensator. The lag-section is provided with one real pole and one real zero, the pole being to the right of zero, whereas the lead section has one real pole and one real came with the zero being to the right of the pole.

The transfer function of the lag-lead compensator will be

$$G(s)=\left(\frac{s+\frac{1}{\tau_1}}{s+\frac{1}{\alpha\tau_1}}\right)\left(\frac{s+\frac{1}{\tau_2}}{s+\frac{1}{\beta\tau_2}}\right)$$

The figure shows lag lead compensator

$$E_0(s)=\frac{E_i(s)}{\dfrac{R_1\times\frac{1}{sC_1}}{R_1+\frac{1}{sC_1}}+R_2+\frac{1}{sC_2}}\left(R_2+\frac{1}{sC_2}\right)$$



Where $\alpha > 1$, $\beta < 1$.

$$\frac{E_0(s)}{E_i(s)}=\frac{\left(R_1\times\frac{1}{sC_1}\right)\left(R_2+\frac{1}{sC_2}\right)}{R_1\times\frac{1}{sC_1}+\left(R_2+\frac{1}{sC_2}\right)\left(R_1+\frac{1}{sC_1}\right)}$$

$$=\frac{\frac{(sC_1R_1+1)}{sC_1}\,\frac{(sC_2R_2+1)}{sC_2}}{\frac{R_1}{sC_1}+\frac{(R_2sC_2+1)}{sC_2}\,\frac{(R_1sC_1+1)}{sC_1}}$$

$$=\frac{\dfrac{(1+sC_1R_1)(1+sC_2R_2)}{s^2C_1C_2}}{\dfrac{R_1sC_2+R_2sC_2+1+R_1R_2s^2C_1C_2+R_1sC_1}{s^2C_1C_2}}$$

$$=\frac{(1+sC_1R_1)(1+sC_2R_2)}{s^2R_1R_2C_1C_2+s(R_1C_1+R_2C_2)+1+R_1sC_2}$$

$$=\frac{(1+sC_1R_1)(1+sC_2R_2)}{s^2R_1R_2C_1C_2+s(R_1C_1+R_2C_2)+1+R_1sC_2}$$

$$=\frac{C_1R_1C_2R_2\left(s+\dfrac{1}{C_1R_1}\right)\left(s+\dfrac{1}{C_2R_2}\right)}{R_1R_2C_1C_2\left[s^2+\left\{\dfrac{1}{R_2C_2}+\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}\right\}s+\dfrac{1}{R_1R_2C_1C_2}\right]}$$

$$=\frac{\left(s+\dfrac{1}{C_1R_1}\right)\left(s+\dfrac{1}{C_2R_2}\right)}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$$

The above transfer functions are comparing with

$$G(s)=\frac{\left(s+\dfrac{1}{\tau_1}\right)\left(s+\dfrac{1}{\tau_2}\right)}{\left(s+\dfrac{1}{\alpha\tau_1}\right)\left(s+\dfrac{1}{\beta\tau_2}\right)}$$

Then $\dfrac{1}{\tau_1}=\dfrac{1}{C_1R_1}$ $\quad\dfrac{1}{\tau_2}=\dfrac{1}{C_2R_2}$

$$\frac{1}{\alpha\tau_1}+\frac{1}{\beta\tau_2}=\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}$$

$$\frac{1}{\alpha\beta\tau_1\tau_2}=\frac{1}{R_1R_2C_1C_2}$$

$\tau_1 = C_1 R_1$

$\tau_2 = C_2 R_2$

$\alpha\beta\tau_1 \tau_2 = R_1 R_2 C_1 C_2$

$\alpha\beta = 1$ or $\beta = \dfrac{1}{\alpha}$

Therefore

$$G(s)=\frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\alpha\tau_1}\right)\left(s+\frac{\alpha}{\tau_2}\right)} \qquad \text{where } \alpha > 1$$

$$\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}=\frac{1}{\alpha\tau_1}+\frac{\alpha}{\tau_2}$$

M&N circles

1. Prove that the loci of the constant magnitude of closed loop transfer function is a circle

Constant M circles

Consider the polar plot of the open loop transfer function of a unity feedback system. A point on the polar plot is given by:

$G(j\omega) = x + jy$

The closed loop frequency response is given by

$$T(j\omega)=\frac{C(j\omega)}{R(j\omega)}=\frac{G(j\omega)}{1+G(j\omega)}=\frac{x+jy}{1+x+jy}$$

$$\therefore \qquad \left|T(j\omega)\right|^2=\frac{x^2+y^2}{(1+x)^2+y^2}$$

Let $\qquad \left|T(j\omega)\right| = M$

$$\therefore \qquad M^2=\frac{x^2+y^2}{(1+x)^2+y^2}$$

$M^2 (1+ x)^2 + M^2 y^2 = x^2 + y^2$

Rearranging, we have

$$x^2 (M^2 - 1) + 2xM^2 + y^2 (M^2 - 1) = - M^2 \qquad \text{--------------------(a)}$$

$$x^2+\frac{2M^2}{M^2-1}x+y^2=-\frac{M^2}{M^2-1}$$

Making a perfect square of the terms, we have,

$$\left(x^2+\frac{M^2}{M^2-1}\right)^2+y^2=-\frac{M^2}{M^2-1}+\frac{M^4}{\left(M^2-1\right)^2}$$

$$=\frac{-M^2\left(M^2-1\right)+M^4}{\left(M^2-1\right)^2}$$

$$= \left(\frac{M}{M^2 - 1} \right)^2$$

Represents a circle with a radius of $\frac{M^2}{M^2 - 1}$ and centre at $\left(-\frac{M^2}{M^2 - 1}, 0 \right)$.

For various assumed values of M, a family of circles can be drawn which represent the above equation.

These circles are called constant M-circles.

Properties of M-circles:

1. For M =1, the centre of the circle is at $\left(\underset{M \to 1}{Lt} \frac{-M^2}{M^2 - 1}, 0 \right)$. ie., (-∞, 0).

The radius is also infinity

Substituting M = 1 in equation (a), we have

2x = -1

Or x = $-\frac{1}{2}$

This M = 1 is a straight line parallel to y axis at $x = -\frac{1}{2}$.

2. For M > 1, centre of the circle is on the negative real axis and as M → ∞, the centre approaches

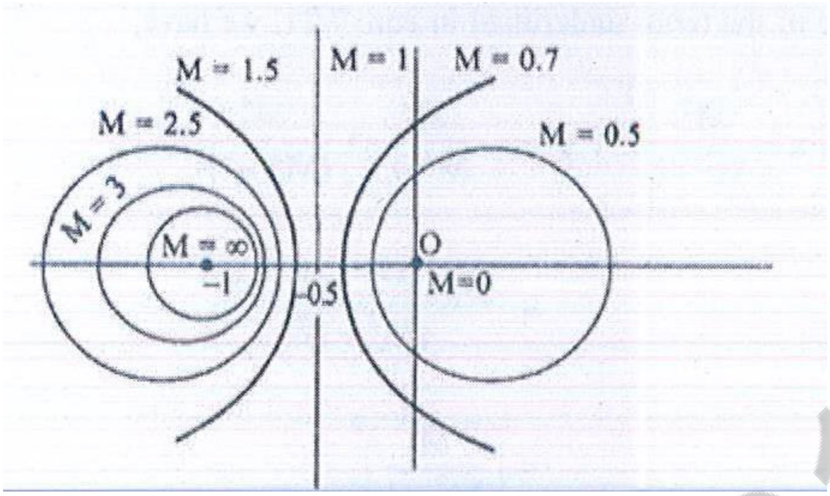
(-1, j0) point and the radius approaches zero; ie (-1, j0) point represents a circle of M = ∞.

3. For 0 < M < 1, $-\frac{M^2}{M^2 - 1}$ is positive and hence the centre is on the positive real axis.

4. For M = 0, the centre is at (0, 0) and radius is 0; ie., origin represents the circle for M = 0.

5. As M is made smaller and smaller than unity, the centre moves from +∞ towards the origin on the positive real axis.

The M circles are sketched in Fig. below



2. Prove that the loci of the constant phase angle of closed loop transfer function is a circle

Constant N circles

Constant N circles are obtained for the points on the open loop polar plot which result in constant phase angle for the closed loop system. Consider the phase angle of the closed loop transfer function

$$\angle T(j\omega) = \theta = \angle \frac{x + jy}{1 + x + jy}$$
$$= \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}$$

Taking tangent of the angles on both sides of equation. 7.23, we have

$$\tan\theta = \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y^2}{x(1+x)}} = \frac{y}{x^2 + y^2 + x}$$

Let $\tan\theta = N$

Then $\frac{y}{x^2 + y^2 + x} = N$

Rearranging, we get,

$$N(x^2 + x) + Ny^2 - y = 0$$

$$N\left(x + \frac{1}{2}\right)^2 + N\left(y - \frac{1}{2N}\right)^2 = \frac{N}{4} + \frac{1}{4N}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4}\left(\frac{N^2+1}{N^2}\right)$$

Represents the equation of a family of circles for different values of N with centre at

$$\left(-\frac{1}{2}, \frac{1}{2N}\right)$$

And Radius $$= \frac{\sqrt{N^2+1}}{2N}$$

These circles are known as constant N circles.

The constant N-circles are shown in Figure. Instead of marking the values of N on the various circles, value of $\alpha = \tan^{-1} N$ are marked so that the phase angle can be read from the curves.