IC8451 CONTROL SYSTEMS

SYLLABUS

COURSE OBJECTIVES

The student should be made to:

- To understand the use of transfer function models for analysis physical systems and introduce the control system components.
- To provide adequate knowledge in the time response of systems and steady state error analysis.
- To accord basic knowledge in obtaining the open loop and closed–loop frequency responses of systems.
- To introduce stability analysis and design of compensators
- To introduce state variable representation of physical systems

UNIT I SYSTEMS AND REPRESENTATION 9

Basic elements in control systems: – Open and closed loop systems – Electrical analogy of mechanical and thermal systems – Transfer function – AC and DC servomotors – Block diagram reduction techniques – Signal flow graphs.

UNIT II TIME RESPONSE 9

Time response: – Time domain specifications – Types of test input – I and II order system response – Error coefficients – Generalized error series – Steady state error – Root locus construction- Effects of P, PI, PID modes of feedback control –Time response analysis.

UNIT III FREQUENCY RESPONSE 9

Frequency response: – Bode plot – Polar plot – Determination of closed loop response from open loop response - Correlation between frequency domain and time domain specifications

UNIT IV STABILITY AND COMPENSATOR DESIGN 9

Characteristics equation – Routh Hurwitz criterion – Nyquist stability criterion- Performance criteria –Effect of Lag, lead and lag-lead compensation on frequency response-Design of Lag, lead and laglead compensator using bode plots.

UNIT V STATE VARIABLE ANALYSIS 9

Concept of state variables – State models for linear and time invariant Systems – Solution of state and output equation in controllable canonical form – Concepts of controllability and observability.

L	T	P	C
3	**2**	**0**	**4**

TOTAL (L: 45+T:30): 75 PERIODS

COURSE OUTCOMES

At the end of the course, the student should have the:

- Ability to develop various representations of system based on the knowledge of Mathematics, Science and Engineering fundamentals.
- Ability to do time domain and frequency domain analysis of various models of linear system.
- Ability to interpret characteristics of the system to develop mathematical model.
- Ability to design appropriate compensator for the given specifications.
- Ability to come out with solution for complex control problem.

- Ability to understand use of PID controller in closed loop system.

TEXT BOOKS

T1. Nagarath, I.J. and Gopal, M., “Control Systems Engineering”, New Age International Publishers, 2017.
T2. Benjamin C. Kuo, “Automatic Control Systems”, Wiley, 2014.

REFERENCES

R1. Katsuhiko Ogata, “Modern Control Engineering”, Pearson, 2015.
R2. Richard C.Dorf and Bishop, R.H., “Modern Control Systems”, Pearson Education,2009.
R3. John J.D., Azzo Constantine, H. and Houpis Sttuart, N Sheldon, “Linear Control System Analysis and Design with MATLAB”, CRC Taylor& Francis Reprint 2009.
R4. Rames C.Panda and T. Thyagarajan, “An Introduction to Process Modelling Identification and Control of Engineers”, Narosa Publishing House, 2017.
R5. M.Gopal, “Control System: Principle and design”, McGraw Hill Education, 2012.
R6. NPTEL Video Lecture Notes on “Control Engineering “by Prof. S. D. Agashe, IIT Bombay.

Unit – I

SYSTEMS COMPONENTS AND THEIR REPRESENTATION

Part – A

1. What is control system? Nov/Dec 2016
   A system consists of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity then the system is called control system

2. Define open loop control systems Nov/Dec 2017
   The control system in which the output quantity has no effect upon the input quantity is called open loop control system. This means that the output is not feedback to the point for correction

3. Define closed loop control systems Nov/Dec 2017
   The control system in which the output has an effect upon the input quantity so as to maintain the desired output values are called closed loop control systems

4. What are the components of feedback control system? Nov/Dec 2016
   The component of feedback control system are plant, feedback path elements, error detector actuator and controller

5. Distinguish between open loop and closed loop system May/June 2013, 2016, Nov/Dec 2019

S.No.	OPEN LOOP	CLOSED LOOP
1.	Inaccurate	Accurate
2.	Simple and economical	Complex and costlier
3.	The changes in output due to external disturbance are not corrected	The changes in output due to external disturbance are corrected automatically
4.	Always stable	Generally great efforts are needed to design a stable system

6. Define transfer function Nov/Dec 2011& April/May 2017
   The transfer function of a system is defined as the ratio of the Laplace transform of output to Laplace transform of input with zero initial conditions.

7. What are the basic elements used for modeling mechanical translational system May/June13/Nov/Dec16&17/April/May 19
   - Mass M,.Kg
   - Stiffness of spring K, N/m
   - Viscous friction coefficient dashpot B,N-sec/m

8. What are the basic elements used for modeling mechanical rotational system? April/May 2019
   - Moment of inertia J,Kg- m^2 /rad
   - Dashpot with rotational frictional coefficient B,N-m/(rad/sec)
   - Torsional spring with stiffness K,N-m/rad

9. Name two types of electrical analogous for mechanical system
   The two types of analogies for the mechanical system are
   - Force voltage analogy
   - Force current analogy

10. What is block diagram? Nov/Dec 2015, April/May 2017
    A Block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals.

11. What are the basic component s of block diagram? April/May 2017
The basic elements of block diagram are blocks, branch point and summing point

12. What is the basis for framing the rules of block diagram reduction technique?
The rules for block diagram reduction technique are framed such that any modification made on the diagram does not alter the input output relation

13. What is a signal flow graph?
A signal flow graph is a diagram that represents of set of simultaneous algebraic equations. By taking Laplace transform the time domain differential equations governing a control system can be transferred to a set of algebraic equations in a s-domain.

14. What is transmittance?
The transmittance is the gain acquired by the signal when it travels from one node to another node is signal flow graph.

15. What is sink and source?
Source is the input node in the signal flow graph and it has only outgoing branches. Sink is a output node in the signal flow graph and it has only incoming branches.

16. Write Masons Gain formula April/May 2015/2016, April/May 2018, April/May 2019
Masons gain formula states that the overall gain of the system as follows overall gain,

$$T(s) = \frac{\Sigma_K \Delta_K P_K}{\Delta}$$

T(s) = Transfer Function of the system
K = Number of forward path in the signal flow
P_K = Forward path gain of the Kth forward pain
Δ = 1- (Sum of individual loop gains) + (Sum of gain products of all possible combinations of two both touching loops) – (Sum of gain products of all possible combinations of three non touching loops) +….
Δ_K = (Δ for the part of the graph which is not touching Kth forward path)

17. Write the analogues electrical elements in force voltage analogy for the elements of mechanical translational system
Force – f – Voltage, e
Velocity, V – current , i
Displacement, x – charge, q
Fricitional coefficient , B – Resistance, R
Mass, M – inductance, L
Stiffness, K – Inverse of capacitance 1/C
Newton's second law – Kirchhoff's voltage law.

18. Write the analogous electrical elements in force current analogy for the elements of mechanical translational system
Force, f – current, i
Velocity, V – Voltage, e
Displacement, x – flnx Φ
Fricitional coefficient, B – Conductance, G = 1/R
Mass, M – capacitance C
Stiffness, K – Inverse of inductance, 1/L
Newton's second law – Kirchhoff's current law

19. Write the analogous electrical elements in torque voltage analogy for the elements of mechanical rotational system
Torque, T – Voltage, e
Angular Velocity, ω - current , i
Angular Displacement, θ - charge, q

Frictional coefficient, B – Resistance, R
Moment of Inertia, J– inductance, L
Stiffness of the spring, K-– Inverse of capacitance 1/C
Newton's second law – Kirchhoff's current law

20. Write the analogous electrical elements in torque current analogy for the elements of mechanical rotational system
Torque, t – current , i
Angular Velocity, ω - voltage , e
Angular Displacement, θ - flux, Φ
Frictional coefficient , B- Conductance , G = 1/R
Moment of Inertia, I- capactitance, C
Stiffness of the spring , K – Inverse of inducatance, 1/L
Newton's second law – Kirchhoff's current law

21. Write the force balance equation of an ideal mass, dashpot and spring element
Let a force f be applied to an ideal mass M. The mass will offer an opposing for f_m which is proportional to acceleration.

$$f = f_m = Md^2X / dt^2$$

Let a force f be applied to an ideal dashpot, with viscous frictional coefficient B. the dashpot will offer an opposing force f_b which is proportional to velocity.

$$f = f_b = B\frac{dX}{dt}$$

Let a force f be applied to an ideal spring, with spring constant K. The spring will offer an opposing force f_k which is proportional to displacement.

$$f = f_k = K\,X$$

22. Why negative feedback is invariably preferred in closed loop system?
The negative feedback results in better stability in steady state and rejects any disturbance signals.

23. State the principles of homogeneity (or) superposition
The principles of superposition and homogeneity states that if the system has responses
$y_1(t)$ and $y_2(t)$ for the inputs $x_1(t)$ and $x_2(t)$ respectively then the system response to the linear combination of the individual outputs $a_1x_1(t) + a_2x_2(t)$ its given by linear combination of the individual outputs $a_1y_1(t) + a_2y_2(t)$ where a_1 ,a_2 are constant.

$y_1(t)$ and $y_2(t)$ for the inputs $x_1(t)$ and $x_2(t)$ respectively then the system respsonse to the linear combination of the individual outputs $a_1x_1(t) + a_2x_2(t)$is given by linear combination of the individual outputs $a_1y_1(t) + a_2y_2(t)$ where $a_{1,}$ a_2 are constant

24. What are the basic properties of signal flow graph?
The basic properties of signal flow graph are
- Signal flow graph is applicable to linear systems
- It consists of nodes and branches
- A node adds the signal of all incoming branches and transmits this sum to all outgoing branches.
- Signals travel along branches only in the marked direction and is multiplied by the gain of the branch.
- The algebraic equations must be in form of cause and effect relationship

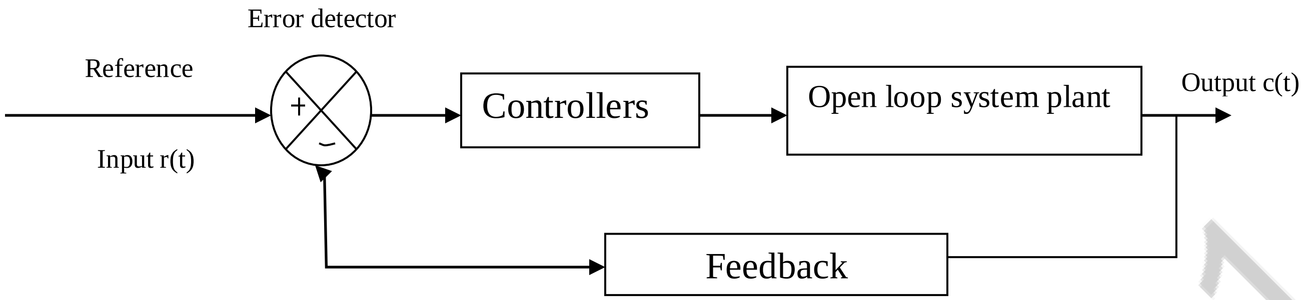
25. Define non touching loop
The loops are said to be non touching if they do not have common nodes

26. List the advantages of closed loop system? Nov/Dec 2015 & April/May 2017
It is accurate.

The change in output due to external disturbances are corrected automatically.

Part B and C question & Answers

1. Explain the features of closed loop feedback Control system May/ June 2015
   Control system in which the output has an effect upon the input quantity in order to maintain the desired output values are called closed loop systems.



The open loop system can be modified as closed loop system by providing a feedback. The provision of feedback automatically corrects the changes in output due to disturbances. Hence the closed loop system is also called automatic control systems

The general block diagram of an automatic control system is shown in fig. In consists of an error detector, a controller, plant and feedback path elements.

The reference signal (or input signal) corresponds to desired output. The feedback path elements samples the output and converts it to a signal of same type as that of reference signal. The feedback signal is proportional to output signal and it is fed to the error detector.

The error signal generated by the error detector is difference between reference signal and feedback signal. The controller modifier and amplifies the error signal to produce better control action. The modified error signal is fed to the plant to correct its output.

Advantages of closed loop system:

1. The closed loop systems are accurate.
2. The closed loop systems are accurate even in the presence of non linearties.
3. The sensitivity of the systems may be made small to make the system more stable
4. The closed loop systems are less affected by noise.

Disadvantages of closed loop systems.

1. The closed loop systems are complex and costly
2. The feedback in closed loop system may lead to oscillatory response.
3. The feedback reduces the overall gain of the system
4. Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system

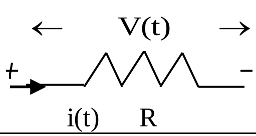
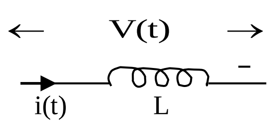
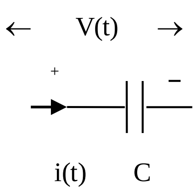
2.Compare open loop and closed loop control system **Nov/Dec 2016, Nov/Dec 2018**

S.No.	Open Loop	Closed loop
1.	Any change in output has no effect on the input (i.e.) feedback does not exists	Changes in output, affects the input which is possible by use of feedback
2.	Output measurement is not required for operation of system	Output measurement is necessary

3.	Feedback element is absent	Feedback element is present
4.	Error detector is absent	Error detector is necessary
5.	It is inaccurate and unreliable	Highly accurate and reliable
6.	Highly sensitive to the disturbances	Less sensitive to the disturbances
7.	Highly sensitive to the environmental changes	Less sensitive to the environmental changes
8.	Bandwidth is small	Bandwidth is large
9.	Simple to construct and cheap	Completed to design and hence costly
10.	Generally are stable in nature	Stability is the major consideration while designing
11.	Highly affected by non linearities	Reduced effect of non linearities

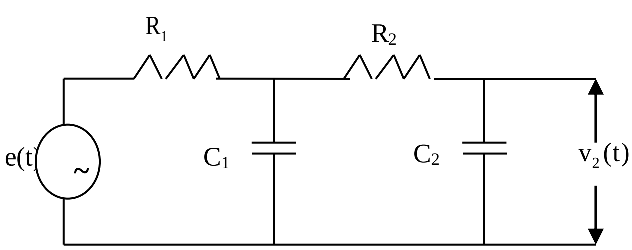
Mathematical Models of Electrical Systems

- → The basic elements of electrical system are resistor, inductor, capacitor
- → The differential equations of the electrical systems can be formed by applying Kirchoff's laws

COMPONENTS	VOLTAGE ACROSS THE ELEMENT	CURRENT THRO' THE ELEMENT
Resistors 	$V(t) = Ri(t)$	$i(t) = \dfrac{V(t)}{R}$
Inductor 	$v(t) = L\dfrac{di(t)}{dt}$	$i(t) = \dfrac{1}{L}\int v(t)\,dt$
Capacitor 	$v(t) = \dfrac{1}{C}\int i(t)dt$	$i(t) = C\dfrac{dv(t)}{dt}$

Problems

1. Obtain the transfer function of the electrical network shown in fig.

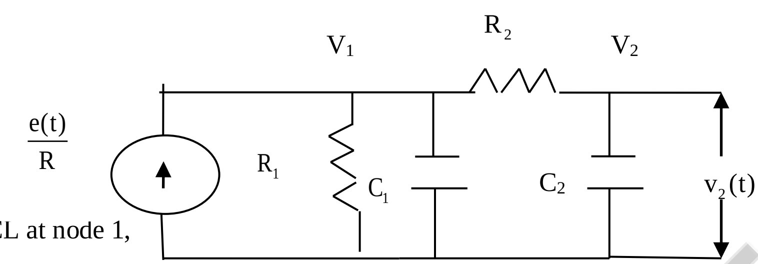


Input – e(t)

Output – $V_2(t)$

Transfer function $= \dfrac{V_2(s)}{E(s)}$

Using source transformation technique, voltage source is converted into current source.



Apply KCL at node 1,

$$\frac{V_1}{R_1} + C_1\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} = \frac{e(t)}{R} \qquad (1)$$

Apply KCL at node 2

$$\frac{V_2 - V_1}{R_2} + C_2\frac{dV_2}{dt} = 0 \qquad (2)$$

Equations (1) and (2) forms differential equations/mathematical form of the electrical network shown in fig,

To find transfer function $V_2(s)/E(s)$

Apply Laplace transform to eqn (1)

$$\frac{V_1(s)}{R_1} + C_1 sV(s) + \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$V_1(s)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1} \qquad (3)$$

Apply Laplace transform to eqn (2)

$$\frac{V_2(s)}{R_2} - \frac{V_1(s)}{R_2} + C_2 s V_2(s) = 0$$

$$\therefore V_2(s)\left[\frac{1}{R_2} + C_2 s\right] - V_1(s)\left[\frac{1}{R_2}\right] = 0$$

$$V_1(s) = \left[1 + sC_2R_2\right]V_2(s) \qquad \rightarrow (4)$$

$$L\left[x(t)\right] = X(s)$$

$$L\left[\frac{dx(t)}{dt}\right] = sX(s)$$

$$L\left[\frac{d^2x(t)}{dt^2}\right] = s^2X(s)$$

$$L\left[\int x(t)dt\right] = \frac{X(s)}{s}$$

Substituting for $V_1(s)$ from (4) in (3), we get

$$V_2(s)[1 + sC_2R_2]\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

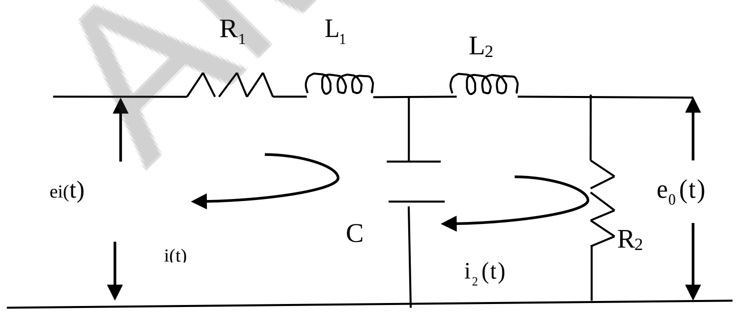
$$V_2(s)\left[(1 + sC_2R_2)\left(\frac{R_2 + sC_1R_1R_2 + R_1}{R_1R_2}\right) - \frac{R_1}{R_1R_2}\right] = \frac{E(s)}{R_1}$$

$$V_2(s)\left[\frac{(1 + sC_2R_2)(R_2 + R_1 + sC_1R_1R_2) - R_1}{R_1R_2}\right] = \frac{E(s)}{R_1}$$

$$\frac{V_2(s)}{E(s)} = \frac{R_2}{(1 + sC_2R_2)(R_1 + R_2 + sC_1R_1R_2) - R_1} \qquad (5)$$

Eqn (5) Is the required transfer function

2. Obtain the transfer function of the following n/w



Input → $e_i(t)$

Output $\rightarrow e_o(t)$

Transfer function = $\dfrac{E_0(s)}{E_i(s)}$

Applying KVL to mesh 1,

$$e_i(t) = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C}\int (i_i - i_2)\,dt \qquad \rightarrow (1)$$

Applying KVL to mesh 2,

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C}\int (i_2 - i_1)\,dt \qquad \rightarrow (2)$$

$$e_0(t) = i_2(t)\ R_2 \qquad \rightarrow (3)$$

Applying Laplace transform to eqns. (1), (2) and (3)

$$E_i(s) = R_1 I_1(s) + L_1 s I_1(s) + \frac{1}{Cs}\left[I_1(s) - I_2(s)\right]$$

$$E_i(s) = \left[R_1 + L_1 s + \frac{1}{Cs}\right] I_1(s) - \frac{1}{Cs} I_2(s) \qquad (4)$$

$$0 = L_2 s I_2(s) + R_2 I_2(s) + \frac{1}{Cs}\left[I_2(s) - I_1(s)\right]$$

$$0 = -\frac{1}{Cs} I_1(s) + \left[R_2 L_2 s \frac{1}{Cs}\right] I_2(s) \qquad \rightarrow (5)$$

$$E_0(s) = R_2 I_2(s) \qquad \rightarrow (6)$$

Expressing eqn (4) & (5) in matrix form

$$\begin{bmatrix} E_i(s) \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + L_1 s + \frac{1}{Cs} & -\frac{1}{Cs} \\ -\frac{1}{Cs} & R_2 + L_2 s + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} R_1 + L_1 s + \frac{1}{Cs} & -\frac{1}{Cs} \\ -\frac{1}{Cs} & R_2 + L_2 s + \frac{1}{Cs} \end{vmatrix}$$

$$= \left[\left(R_1 + L_1 s + \frac{1}{Cs}\right)\left(R_2 + L_2 s + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2\right]$$

$$\Delta I_2(s) = \begin{vmatrix} R_1 + L_1 s + \frac{1}{Cs} & E_i(s) \\ -\frac{1}{Cs} & 0 \end{vmatrix} = E_i(s)\frac{1}{Cs}$$

$$I_2(s) = \frac{\Delta I_2(s)}{\Delta} = \frac{E_i(s)\frac{1}{Cs}}{\left[\left(R_1 + L_1 s + \frac{1}{Cs}\right)\left(R_2 + L_2 s + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2\right]}$$

$$E_0(s) = I_2(s) R_2$$

$$= \frac{E_i(s)\frac{1}{Cs}R_2}{\left[\left(R_1 + L_1 s + \frac{1}{Cs}\right)\left(R_2 + L_2 s + \frac{1}{Cs}\right) - \left(\frac{1}{Cs}\right)^2\right]} = \frac{E_i(s)\frac{1}{\cancel{Cs}}R_2}{\frac{\left[\left(R_1Cs + L_1Cs^2 + 1\right)\left(R_2Cs + L_2\left(s^2+1\right) - 1\right)\right]}{C^{\cancel{2}}s^{\cancel{2}}}}$$

$$= \frac{E_i(s) R_2 Cs}{\left[\left(R_1Cs + L_1Cs^2 + 1\right)\left(R_2Cs + L_2Cs^2 + 1\right) - 1\right]}$$

$$\boxed{\frac{E_0(s)}{E_i(s)} = \frac{R_2 C_s}{\left[L_1Cs^2 + R_1Cs + 1\right]\left[L_2Cs^2 + R_2Cs + 1\right] - 1}}$$

3. An electrical circuit is shown in fig. obtain the transfer function relating the output voltage $e_o(t)$ to the input voltage $e_i(t)$ in the form

$$\frac{E_0(s)}{E_i(s)} = K_g \frac{(1+sT_1)}{(1+sT_2)}$$

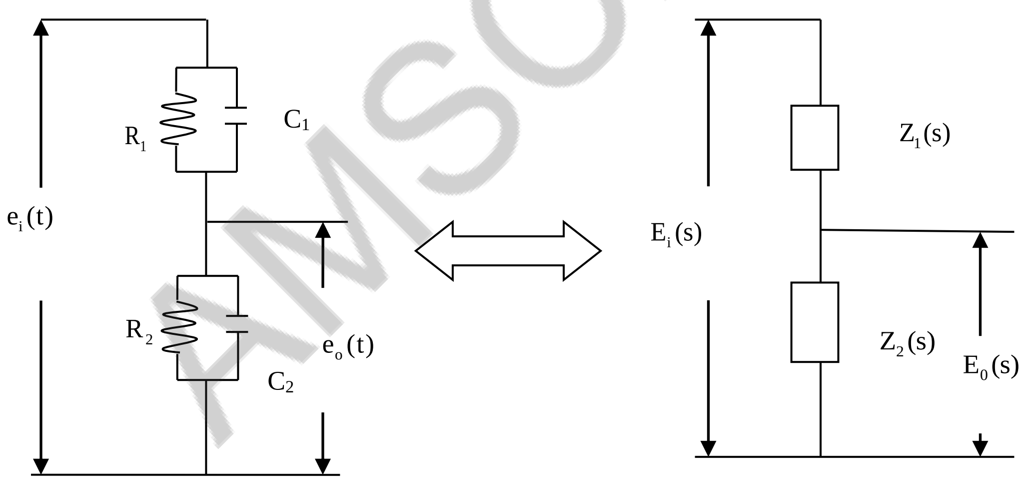


Fig (a) fig(b)

Sol: The components R_1 & C_1 form one parallel combination R_2 & C_2 form are one parallel combination and representation in fig (b)

$$\therefore Z_1(s) = \frac{R_1 \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{(1+R_1C_1s)}$$

similarly,

$$Z_2(s) = \frac{R_2}{(1+R_2C_2s)}$$

By voltage division d rule,

$$E_0(s) = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}, E_i(s)$$

$$\frac{E_0(s)}{E_i(s)} - \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{\frac{R_2}{1+R_2C_2s}}{\frac{R_1}{1+R_1C_1s} + \frac{R_2}{1+R_2C_2s}}$$

$$= \; = \frac{\frac{R_2}{\cancel{(1+R_2C_2s)}}}{\frac{R_1(1+R_2C_2s)+(1+R_1C_1s)R_2}{(1+R_1C_1s)\,\cancel{(1+R_2C_2s)}}}$$

$$= \frac{R_2(1+R_1C_1s)}{R_1(1+R_2C_2s)+R_2(1+R_1C_1s)} = \frac{R_2(1+R_1C_1s)}{R_1+R_2+R_1R_2C_1s+R_1R_2C_2s}$$

$$= \frac{R_2(1+R_1C_1s)}{(R_1+R_2)\left[1+\frac{R_1R_2C_1s+R_1R_2C_2s}{R_1+R_2}\right]}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2(1+R_1C_1s)}{(R_1+R_2)\left[1+\frac{R_1R_2C_1+R_1R_2C_2}{R_1+R_2}\right]^3}$$

$$\frac{E_0(s)}{E_i(s)} = K_g \frac{(1+sT_1)}{(1+sT_2)} \quad \text{where} \quad K_g = \frac{R_2}{R_1+R_2};$$

$$T_1 = R_1C_1; \quad T_2 = \left[1+\frac{R_1R_2C_1+R_1R_2C_2}{R_1+R_2}\right]$$

Mathematical Modeling of Mechanical Systems

1. What are the basic elements of Mechanical rotational systems? Write its torque balance equations Nov/Dec 2015, May/ June 2015

   The basic elements of Mechanical rotational system are

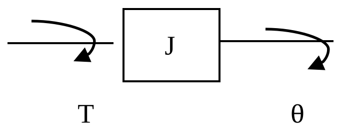
   (i) Moment of Inertia (J)

   (ii) Viscous fiction (B)

   (iii) Torsional stiffness (K)

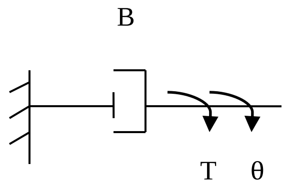
Torque Balance Equation

1. Moment of Inertia



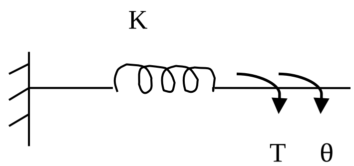
$$T_J(t) = J.\frac{d^2\theta}{dt^2}$$

2. Dashpot [one end is fixed]



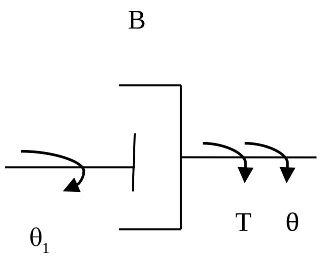
$$T_B(t) = B\frac{d\theta}{dt}$$

3. Torsional Spring [one end is fixed]



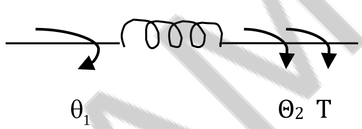
$$T_K(t) = K\theta$$

4. Dashpot [both ends are free]



$$T_B(t) = B\frac{d}{dt}(\theta_1 - \theta_2)$$

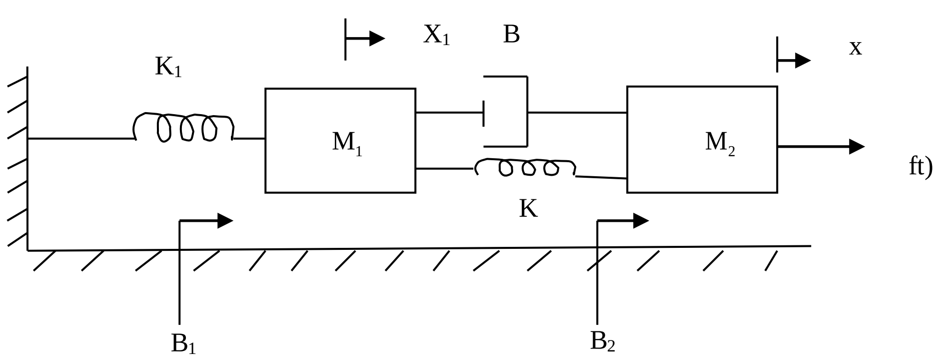
5. Spring [both ends are free]



$$T_K(t) = K(\theta_1 - \theta_2)$$

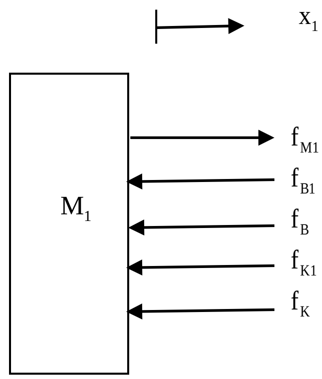
2. Write the differential equations governing the mechanical system and determine the transfer function

(May/June 2016) (April/May 2019)



Sol

Free body diagram for Mass 1



$$f_{M1} = M_1 \frac{d^2x_1}{dt^2} \qquad f_{K1} = K_1X_1$$

$$f_{B1} = B_1.\frac{dx_1}{dt} \qquad f_K = K(x_1 - x)$$

$$f_B = B.\frac{d}{dt}(x_1 - x)$$

By Newton's second law,

Σ applied force = Σ opposing force

$$f(t) = f_{M2} + f_{B2} + f_B + f_K$$

$$f(t) = M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B.\frac{d}{dt}(x - x_1) + K(x - x_1) \rightarrow (3)$$

On taking Laplace transform,

$$H_2s^2X(s) + B_2sX(s) + Bs[X(s) - X_1(s)]$$
$$+K[X(s) - X_1(s)] \;=\; F(s)$$
$$X(s) = [M_2s^2 + (B_2 + B)s + k] - X_1(s)[Bs + K] = F(s) \quad (4)$$

Substituting eqn (2) in eqn (4)

$$X(s) = \left[M_2s^2 + (B_2 + B)s + K\right] - X(s)\frac{(Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K + K_1)}$$

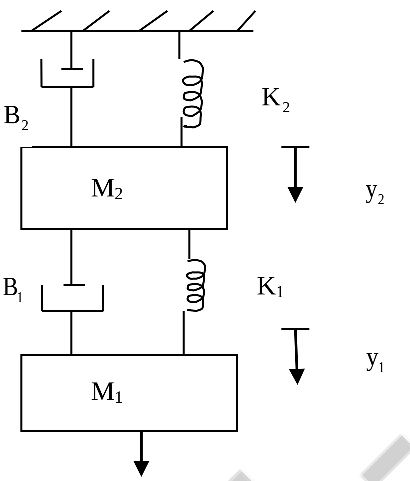
$$X(s)\left[\frac{\left[M_1s^2 + (B_1 + B)s + (K_1 + K)\right]\left[M_2s^2 + (B_2 + B)s + K\right] - (Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)}\right] = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (K_1 + K)}{[M_1 s^2 + (B_1 + B)s + (K_1 + K)][M_2 s^2 + (B_2 + B)s + K] - (Bs + K)^2} \rightarrow (5)$$

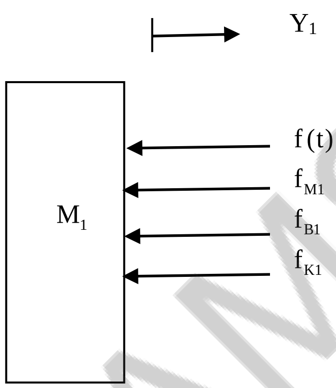
Eqn (1) and (3) forms the Mathematical model/differential equation of the given mechanical system equation (V) is the required transfer function

3.For the mechanical translational system shown in fig. determine the differential equation and obtain the transfer function $\frac{Y_2(s)}{F(s)}$

Nov/Dec 2019



Sol: Free body diagram for M_1



$$f_{M1} = M_1 \frac{d^2 y_1}{dt^2}$$

$$f_{K1} = K_1 (y_1 - y_2)$$

$$f_{B1} = B_1 \frac{d}{dt}(y_1 - y_2)$$

By Newton's second law,

$$f(t) = f_{M1} + f_{B1} + f_{K1}$$

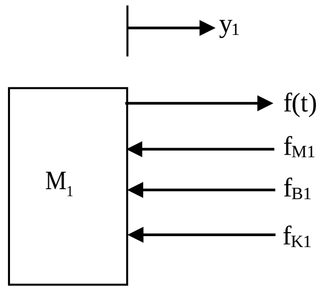
$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + B_1 \frac{d}{dt}(y_1 - y_2) + K_1 (y_1 - y_2) \rightarrow (1)$$

Taking Laplace transform

$$F(s) = M_1 s^2 Y_1(s) + B_1 s\left[Y_1(s) - Y_2(s)\right] + K_1\left[Y_1(s) - Y_2(s)\right]$$

$$F(s) = \left[M_1 s^2 + B_1 s + K_1\right] Y_1(s) - \left[B_1 s + K_1\right] Y_2(s) \qquad \rightarrow (2)$$

Free body diagram for M_2



$$f_{M2} = M_2 . \frac{d^2 y_2}{dt^2} \qquad f_{K1} = K_1 (y_2 - y_1)$$

$$f_{B1} = B_1 . \frac{d}{dt}(y_2 - y_1) \qquad f_K = K_2 (y_2)$$

$$f_B = B_2 . \frac{dy_2}{dt}$$

By Newton's second Law,

$$f_{M2} + f_{B1} + f_B + f_{K1} + f_K = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + B_1 \frac{d}{dt}(y_2 - y_1) + B\frac{dy_2}{dt} + K_1 (y_2 - y_1) + K_2 y_2 = 0 \qquad \rightarrow (3)$$

Taking Laplace transform

$$M_2 s^2 Y_2(s) + B_1 s\left[Y_2(s) - Y_1(s)\right] + BsY_2(s) + K_1\left[Y_2(s) - Y_1(s)\right] + K_2 Y_2(s) = 0$$

$$\left[M_2 s^2 + B_1 s + Bs + K_1 + K_2 Y_2(s)\right] - \left[B_1 s + K_1\right] Y_1(s) = 0$$

$$\left[M_2 s^2 + (B_1 + B)s + (K_1 + K_2)\right] Y_2(s) = \left[B_1 s + K_1\right] Y_1(s)$$

$$Y_1(s) = \frac{M_2 s^2 + (B_1 + B)s + (K_1 + K_2)}{(B_1 s + K_1)} Y_2(s) \qquad \rightarrow (4)$$

Substituting equation (4) in equation (2)

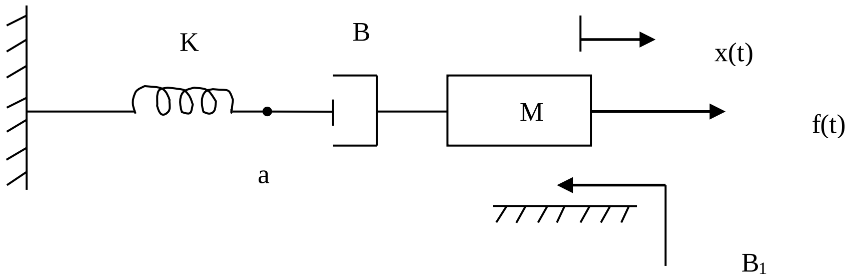
$$F(s) = \frac{(M_1 s^2 + B_1 s + K_1)\left[M_2 s^2 + (B_1 + B)s + (K_1 + K_2)\right] Y_2(s)}{(B_1 s + K_1)} - \left[B_1 s + K_1\right] Y_2(s)$$

$$F(s) = \frac{(M_1 s^2 B_1 s + K_1)\left[M_2 s^2 + (B_1 + B)s + (K_1 + K_2)\right] - \left[B_1 s + K_1\right]^2}{(B_1 s + K_1)} Y_2(s)$$

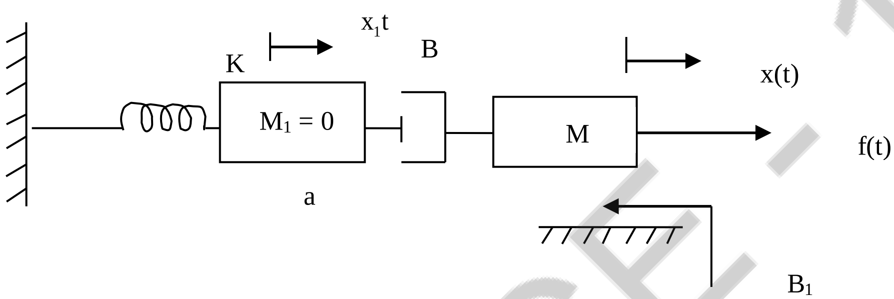
$$\frac{Y_2(s)}{F(s)} = \frac{B_1 s + K_1}{\left[M_1 s^2 + B_1 s + K_1\right]\left[M_2 s^2 + \left(B_1 + B\right)s + \left(K_1 + K_2\right)\right] - \left[B_1 s + K_1\right]^2} \rightarrow (5)$$

Eqn (1) and (3) forms the mathematical model/ differential equations of the given system eqn (5) gives the required transfer function of the given mechanical system

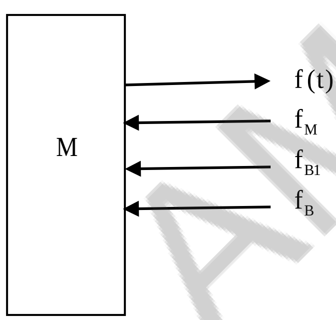
4. Determine the transfer function of the system shown in fiq.



Sol: Given system can be redrawn as follows



Free body diagram for Mass M



$$f_M = M\frac{d^2x}{dt^2}$$

$$f_{B1} = B_1\frac{dx}{dt}$$

$$f_B = B.\frac{d}{dt}\left(x - x_1\right)$$

By Newton's second law

$$f(t) = f_M + f_{B1} + f_B$$

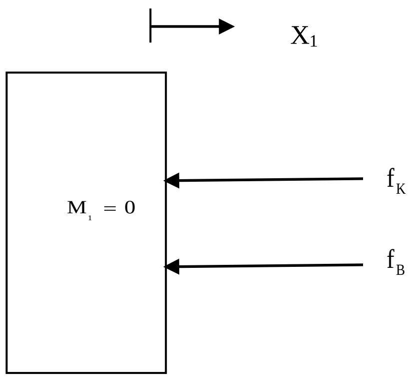
$$M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B.\frac{d}{dt}(x - x_1) = f(t) \qquad \rightarrow (1)$$

On taking Laplace transform

$$Ms^2X(s) + B_1sX(s) + Bs[X(s) - X_1(s)] = F(s)$$

$$[Ms^2 + (B_1 + B)s]X(s) - BsX_1(s) = F(s) \qquad \rightarrow (2)$$

Free body diagram for $M_1 = 0$



$$f_K = Kx_1$$

$$f_B = B.\frac{d}{dt}(x_1 - x)$$

By Newton's second law,

$$f_B + f_K = 0$$

$$\therefore B\frac{d}{dt}(x_1 - x) + Kx_1 = 0 \qquad \rightarrow (3)$$

On taking Laplace transform

$$Bs[X_1(s) - X(s)] + KX_1(s) = 0$$

$$[Bs + K]X_1(s) + BsX(s) = 0$$

$$X_1(s)[Bs + K] = Bs.X(s)$$

$$X_1(s) = \frac{Bs}{(Bs + K)}.X(s) \qquad \rightarrow (4)$$

On substituting eqn. (4) in eqn (2)

$$[Ms^2 + (B_1 + B)s]X(s) - \frac{(Bs)^2}{Bs + K}X(s) = F(s)$$

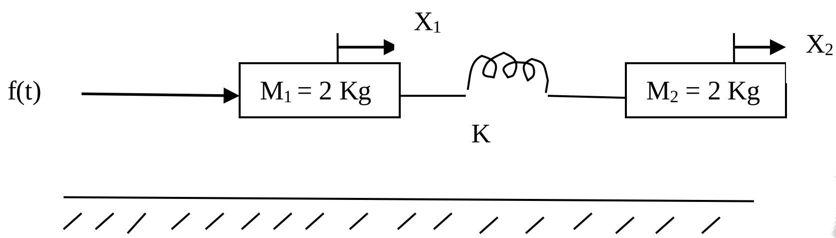
$$\left[\frac{\left[Ms^2+(B_1+B)s\right]\left[Bs+K\right]-(Bs)^2}{Bs+K}\right]X(s)=F(s)$$

$$\frac{X(s)}{F(s)}=\frac{Bs+K}{\left[Ms^2+(B_1+B)s\right]\left[Bs+K\right]-(Bs)^2} \qquad \rightarrow (5)$$

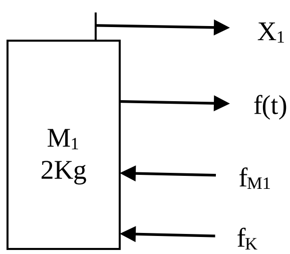
Equation (1) and (3) are differential equations governing the given system

Equation (5) is the required transfer function of the given mechanical translational system

5.Derive the transfer function of system shown in fig May/June 2015



Freebody diagram for M_1



$$f_{M1}=M_1\frac{d^2x_1}{dt^2}$$
$$f_K=K\left(x_1-x_2\right)$$

By Newton’s second law,

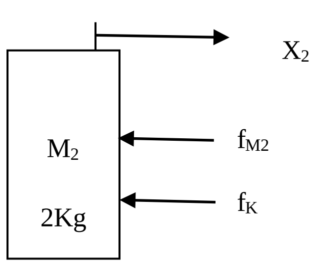
$$f(t)=f_{M1}+f_K$$
$$f(t)=M_1\frac{d^2x_1}{dt^2}+K\left(x_1-x_2\right) \qquad \rightarrow (1)$$

On taking Laplace transform,

$$F(s)=M_1s^2X_1(s)+K\left(X_1(s)-X_2(s)\right)$$

$$F(s)=\left(M_1s^2+K\right)X_1(s)-KX_2(s) \qquad \rightarrow (2)$$

Free body diagram for M_2



$$f_{M2} = M_2 . \frac{d^2x_2}{dt^2}$$
$$f_K = K\left(x_2 - x_1\right)$$

By Newton' second law

$$f_{M2} + f_K = 0$$
$$M_2 \frac{d^2x_2}{dt^2} + K\left(x_2 - x_1\right) = 0 \qquad \rightarrow (3)$$

On taking Laplace transform

$$M_2 s^2 X_2(s) + K\left[X_2(s) - X_1(s)\right] = 0$$
$$\left(M_2 s^2 + K\right)X_2(s) - KX_1(s) = 0$$
$$\left(M_2 s^2 + K\right)X_2(s) = KX_1(s)$$
$$X_2(s) = \frac{K}{M_2 s^2 + K} X_1(s) \qquad \rightarrow (4)$$
$$X_1(s) = \frac{M_2 s^2 + K}{K} X_2(s) \qquad \rightarrow (5)$$

On substituting eqn (4) in (2)

$$F(s) = \left(M_1 s^2 + K\right)X_1(s) - \frac{K^2}{M_2 s^2 + K} X_1(s)$$
$$F(s) = \frac{\left(M_1 s^2 K\right)\left(M_2 s^2 + K\right) - K^2}{M_2 s^2 + K} X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + K}{\left(M_1 s^2 + K\right)\left(M_2 s^2 + K\right) - K^2} \qquad \rightarrow (6)$$

Put $M_1 = 2$; $M_2 = 2$ in eqn (6)

$$\frac{X_1(s)}{F(s)} = \frac{2s^2}{\left(2s^2 + K\right)\left(2s^2 + k\right) - K^2}$$
$$\frac{X_1(s)}{F(s)} = \frac{2s^2}{\left(2s^2 + K\right)^2 - K^2}$$
$$= \frac{2s^2}{4s^4 + K^2 + 4Ks^2 - K^2}$$
$$= \frac{2s^2}{4s^4 + 4Ks^2} = \frac{s^2}{2\left(s^4 + Ks^2\right)}$$
$$= \frac{s^2}{2s^2\left(s^2 + K\right)} = \frac{1}{2\left(s^2 + K\right)}$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{2\left(s^2+K\right)} \qquad \rightarrow (7)$$

On substituting eqn (5) in eqnn (2)

$$F(s) = \frac{(M_1s^2+K)(M_2s^2+K)}{K}X_2(s) - KX_2(s)$$

$$F(s) = \left[\frac{(M_1s^2+K)(M_2s^2+K)-K^2}{K}\right]X_2(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{K}{\left[\left(M_1s^2+K\right)\left(M_2s^2+K\right)-K^2\right]} \qquad \rightarrow (8)$$

put $M_1 = 2,\ M_2 = 2$

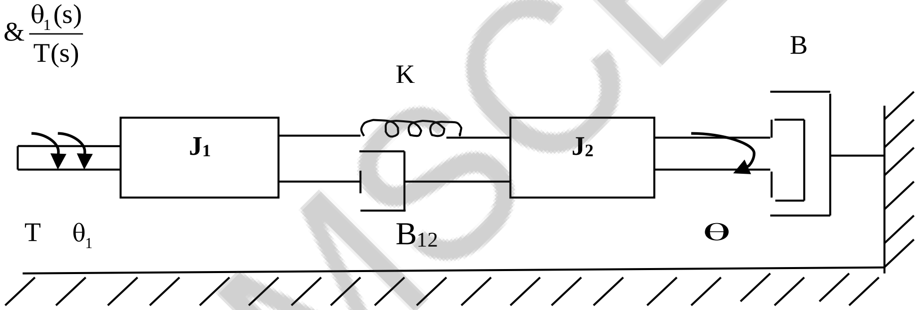
$$\therefore \frac{X_2(s)}{F(s)} = \frac{K}{\left[\left(2s^2+K\right)^2 - K^2\right]} = \frac{K}{2\left(s^4+Ks^2\right)}$$

$$\therefore \frac{X_2(s)}{F(s)} = \frac{K}{2s^2\left(s^2+K\right)} \qquad \rightarrow (9)$$

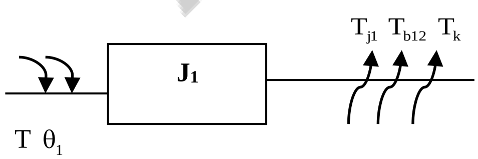
Eqn (7) & (9) are the required transfer function.

6.For the mechanical rotational system shown in fig determine the transfer function.

$$\frac{\theta(s)}{T(s)} \& \frac{\theta_1(s)}{T(s)}$$



Sol : Free body diagram for mass with moment of intertia J_1



$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2};\ T_{b12} = B_{12}\frac{d}{dt}(\theta_1 - \theta);\ T_K = K(\theta_1 - \theta)$$

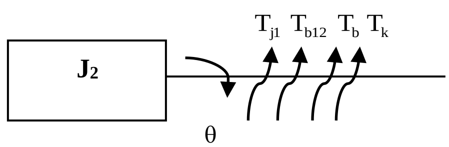
By Newton's second law, $T_{j1} + T_{b12} + T_k = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_{12}.\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta) = T \qquad (1)$$

On taking Laplace transform,

$$J_1 s^2 \theta_1(s) + sB_{12}\left[\theta_1(s) - \theta(s)\right] + K\left[\theta_1(s) - \theta(s)\right] = T(s)$$
$$\theta_1(s)\left[J_1 s^2 + sB_{12} + K\right] - \theta(s)\left[sB_{12} + K\right] = T(s) \qquad (2)$$

Free body diagram of mass with moment if intertia J_2



$$T_{J2} = J_2 \frac{d^2\theta}{dt^2};\ T_{b12} = B_{12}\frac{d}{dt}(\theta - \theta_1)$$
$$T_b = B\frac{d\theta}{dt};\ T_K = K(\theta - \theta_1)$$

By Newton's second law

$$T_{j2} + T_{b12} + T_b + T_k = 0$$

$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}\left(\theta - \theta_1\right) + B\frac{d\theta}{dt} + K\left(\theta - \theta_1\right) = 0 \qquad \rightarrow (3)$$

On taking Laplace transform,

$$J_2 s^2 \theta(s) + B_{12}s\left[\theta(s) - \theta_1(s)\right] + Bs\left[\theta(s)\right] + K\left[\theta(s) - \theta_1(s)\right] = 0$$
$$\theta(s)\left[J_2 s^2 + s\left(B_{12} + B\right) + K\right] - \theta_1(s)\left[B_{12}s + K\right] = 0$$
$$\theta_1(s) = \left[\frac{J_2 s^2 + s(B_{12} + B) + K}{B_{12}s + K}\right]\theta(s) \qquad (4)$$
$$\theta(s) = \left[\frac{B_{12}s + K}{J_2 s^2 + s(B_{12} + B) + K}\right]\theta_1(s) \qquad (5)$$

Substituting equation (4) in eqn (2)

$$\frac{\left[J_1 s^2 + B_{12}s + K\right]\left[J_2 s^2 + s(B_{12}+B) + K\right]}{(B_{12}s + K)}\theta(s) - \left[B_{12}s + K\right]\theta(s) = T(s)$$

$$\frac{\left[J_1 s^2 + B_{12}s + K\right]\left[J_2 s^2 + s(B_{12}+B) + K\right] - (B_{12}s+K)^2}{B_{12}s + K}\theta(s) = T(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{B_{12}s + K}{\left[J_1 s^2 + B_{12}s + K\right]\left[J_2 s^2 + s(B_{12}+B) + K\right] - \left(B_{12}s+K\right)^2}} \qquad (6)$$

Substituting eqn (5) in eqn (2)

$$\theta_1(s)\left[J_1 s^2 + sB_{12} + K\right] - \frac{(B_{12}s+K)^2}{J_2 s^2 + s(B_{12}+B) + K}\theta_1(s) = T(s)$$

$$\theta_1(s)\frac{\left[(J_1 s^2 + B_{12}s + K)(J_2 s^2 + s(B_{12}+B) + K) - (B_{12}s+K)^2\right]}{J_2 s^2 + s(B_{12}+B) + K} = T(s)$$

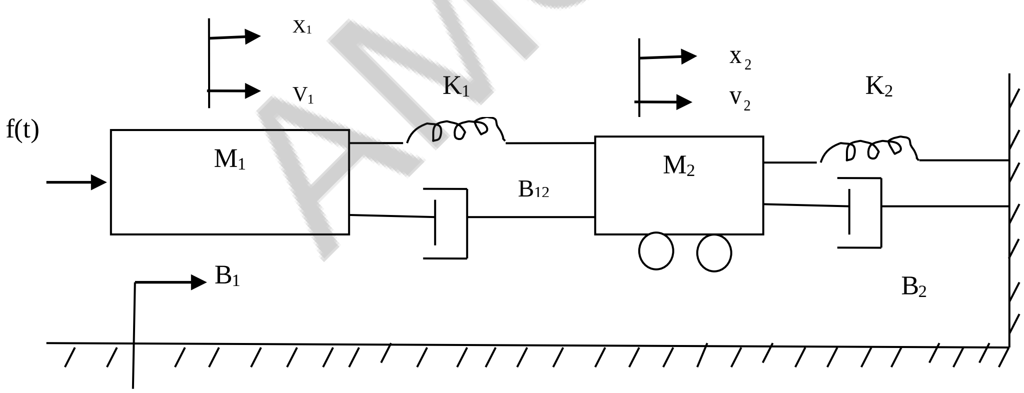
$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{J_2 s^2 + s(B_{12}+B) + K}{(J_1 s^2 + B_{12}s + K)(J_2 s^2 + s(B_{12}+B) + K) - (B_{12}s+K)^2}} \qquad (7)$$

The equation (1) and (3) are called differential equations of the given mechanical rotational systems.

The equation (6) and (7) are the required transfer functions of the given mechanical rotational systems.

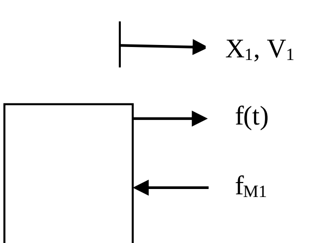
Analogous Systems

7.Write the differential equations governing the mechanical system shown in fig. Draw the force voltage and force current electrical analogous circuits and verify by writing mech and node equations.



Solution:

Freebody diagram for mass M_1



$$f_{M1} = M_1 \frac{d^2x_1}{dt^2}; \quad f_{K1} = K_1(x_1 - x_2)$$

$$f_{B1} = B_1 \frac{dx_1}{dt}$$

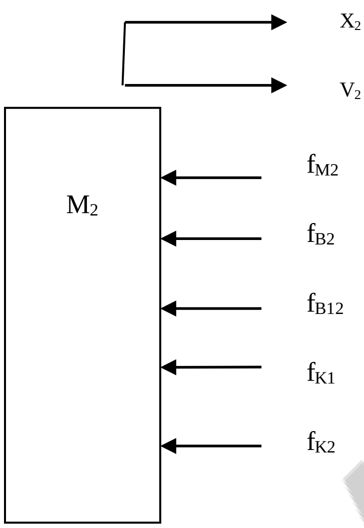
$$f_{B12} = B_{12} \frac{d}{dt}(x_1 - x_2)$$

By Newtons second law,

$$f_{M1} + f_{B1} + f_{B12} + f_{K1} = f(t)$$

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = f(t) \qquad \rightarrow (1)$$

Free body diagram for Mass M_2



$$f_{M2} = M_2 \frac{d^2x_2}{dt^2}$$

$$f_{B2} = B_2 \frac{dx_2}{dt}$$

$$f_{B12} = B_{12} \frac{d}{dt}(x_2 - x_1)$$

$$f_{K1} = K_1(x_2 - x_1)$$

$$f_{K2} = K_2 x_2$$

By Newtons second law,

$$f_{M2} + f_{B2} + f_{K2} + f_{B12} + f_{K1} = 0$$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_{12} \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) = 0 \qquad \rightarrow (2)$$

On replacing the displacements by velocity in differential equations (1) and (2) of the mechanical system

$$\left[(\text{i.e.,})v, \frac{d^2x}{dt^2} = \frac{dv}{dt}; \frac{dx}{dt} = v; x = \int v dt\right]$$

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + B_{12}(v_1 - v_2) + K_1 \int (v_1 - v_2) dt = f(t) \qquad \rightarrow (3)$$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2 dt + B_{12}(v_2 - v_1) + K_1 \int (v_2 - v_1) dt = 0 \qquad \rightarrow (4)$$

FORCE VOLTAGE ANALOGOUS CIRCUIT

The electrical analogous elements for mechanical system are given below

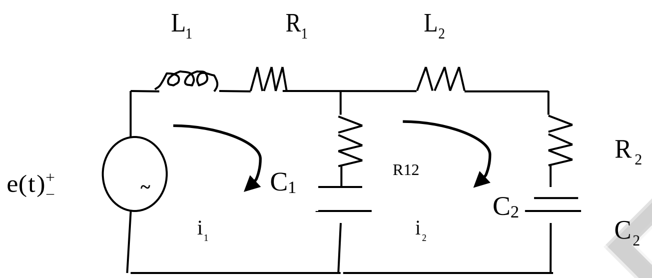
$f(t) \rightarrow e(t);\ v_1 = i;\ v_2 = i_2;$

$M_1 \rightarrow L_1 \quad B_1 \rightarrow R_1 \quad K_1 = 1/C_1$

$M_2 \rightarrow L_2 \quad B_2 \rightarrow R_2 \quad K_2 = 1/C_2$

$B_{12} \rightarrow R_{12}$

Force voltage electrical analogous circuits is shown below



Applying KVL to mesh 1

$$L_1 \frac{di_1}{dt} + R_1 i_1 + R_{12}(i_1 - i_2) + \frac{1}{c_1}\int (i_1 - i_2) dt = e(t) \qquad (5)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{c_2}\int i_2 dt + R_{12}(i_2 - i_1) + \frac{1}{c_1}\int (i_2 - i_1) dt = 0 \quad (6)$$

It is observed that the mesh basis equations

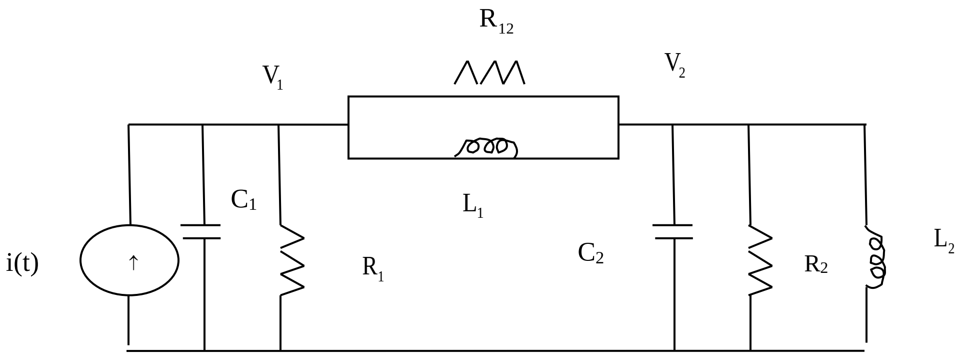
Eqn (5) and (6) ar similar to the differential equations

Eqn (3) and (4) governing the mechanical system

FORCE CURRENT ANALOGOUS CIRCUIT

The electrical analogous elements for the elements of mechanical system

$$f(t) = i(t) \qquad M_1 \rightarrow C_1 \qquad B_1 \rightarrow 1/R_1 \qquad K_1 \rightarrow 1/L_1$$
$$v_1 = V_1 \qquad M_2 \rightarrow C_2 \qquad B_2 \rightarrow 1/R_2 \qquad K_2 \rightarrow 1/L_2$$
$$v_2 = V_2 \qquad B_{12} \rightarrow 1/R_{12}$$



Force-current electrical analogous circuit

Applying KCL at node 1

$$C_1\frac{dV_1}{dt}+\frac{1}{R_1}V_1+\frac{1}{R_{12}}(V_1-V_2)+\frac{1}{L}\int (V_1-V_2)dt = i(t) \qquad \rightarrow (7)$$

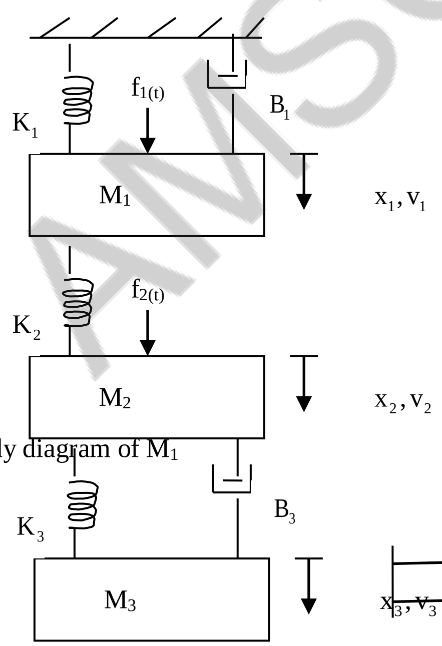
$$C_2\frac{dV_2}{dt}+\frac{1}{R_2}V_2+\frac{1}{L_2}\int V_2 dt+\frac{1}{R_{12}}(V_2-V_1)+\frac{1}{L}\int(V_2-V_1)dt = 0 \qquad \rightarrow (8)$$

It is observed that the node basis equations

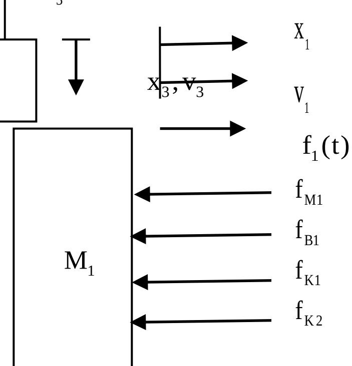
Eqn (7) and (8) are similar to the differential equations

Eqn (3) and (4) governing the mechanical system

8. Write the differential equations the mechanical system shown in fig.Draw the force – voltage and force current electrical analogous circuit and verify by writing mesh and node equations. NOV/DEC 2015



Sol: Freebody diagram of M_1

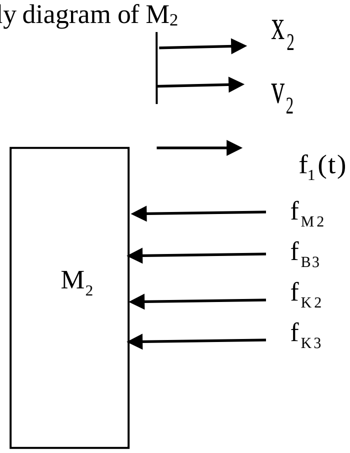


$$f_{M1} = M_1.\frac{d^2x_1}{dt^2} \qquad f_{B1} = B_1.\frac{dx_1}{dt}$$
$$f_{K2} = K_2\left(x_1 - x_2\right);\ f_{K1} = K_1x_1$$

By newton's second law,

$$f_{M1} + f_{B1} + f_{K1} + f_{K2} = f_1(t)$$
$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + K_2\left(x_1 - x_2\right) + K_1x_1 = f_1(t) \qquad \rightarrow (1)$$

Free body diagram of M_2

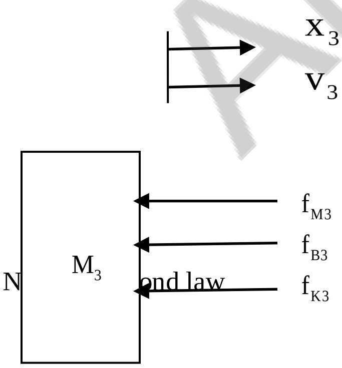


$$f_{M2} = M_2.\frac{d^2x_2}{dt^2} \qquad f_{B3} = B_3.\frac{d}{dt}\left(x_2 - x_3\right)$$
$$f_{K2} = K_2\left(x_2 - x_1\right);\ f_{K3} = K_3\left(x_2 - x_3\right)$$

By Newtons second law,

$$f_{M2} + f_{B3} + f_{K2} + f_{K3} = f_2(t)$$
$$M_2\frac{d^2x_2}{dt^2} + B_3\frac{d}{dt}\left(x_2 - x_3\right) + K_2\left(x_2 - x_1\right) + K_3\left(x_2 - x_3\right) = f_2(t) \qquad \rightarrow (2)$$

Free body diagram for M_3



$$f_{M3} = M_3\frac{d^2x_3}{dt^2}$$
$$f_{B3} = B_3\frac{d}{dt}\left(x_3 - x_2\right)$$
$$f_{K3} = K_3\left(x_3 - x_2\right)$$

By N ond law

$$f_{M3} + f_{B3} + f_{K3} = 0$$

$$M_3 \frac{d^2 x_3}{dt^2} + B_3 \frac{d}{dt}\left(x_3 - x_2\right) + K_3\left(x_3 - x_2\right) = 0 \rightarrow (3)$$

On replacing the displacements by velocity in the differential equation (1) and (2) and (3) governing the mechanical system

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + K_1 \int v_1 dt + K_2 \int \left(v_1 - v_2\right) dt = f_1(t) \qquad \rightarrow (4)$$

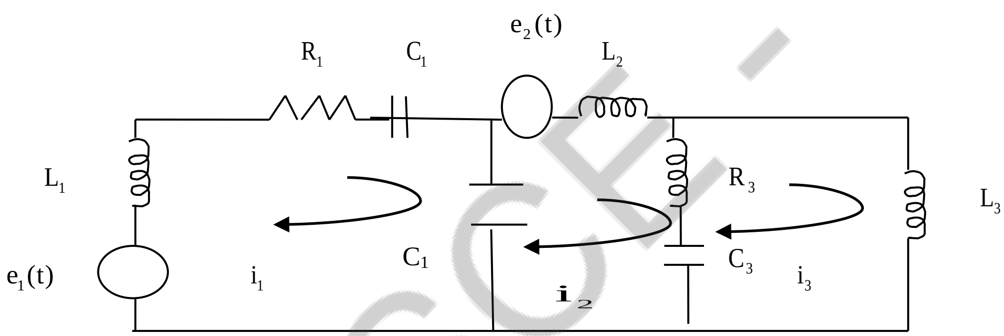
$$M_2 \frac{dv_2}{dt} + B_3\left(v_2 - v_3\right) + K_2 \int \left(v_2 - v_1\right) dt + K_3 \int \left(v_2 - v_3\right) dt = f_2(t) \qquad \rightarrow (5)$$

$$M_3 \frac{dv_3}{dt} + B_3\left(v_3 - v_2\right) + K_3 \int \left(v_3 - v_2\right) dt = 0 \qquad \rightarrow (6)$$

FORCE VOLTAGE ANALOGOUS CIRCUIT

The electrical analogous elements for the elements of mechanical system are given below.

$f_1(t) = e_1(t)$	$M_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$
$f_2(t) = e_2(t)$	$M_2 \rightarrow L_2$	$B_2 \rightarrow R_2$	$K_2 \rightarrow 1/C_2$
$v_1 = i_1$	$M_3 \rightarrow L_3$		$K_3 \rightarrow 1/C_3$
$v_2 = i_2$			
$v_3 = i_3$			



Applying KVL to mesh (1), (2) and (3)

$$L_1\frac{di_1}{dt}+R_1i_1+\frac{1}{C_1}\int i_1dt+\frac{1}{C_2}\int(i_1-i_2)dt=e_1(t) \qquad (7)$$

$$L_2\frac{di_2}{dt}+R_3(i_2-i_3)+\frac{1}{C_3}\int(i_2-i_3)dt+\frac{1}{C_2}\int(i_2-i_1)dt=e_2(t) \qquad (8)$$

$$L_3\frac{di_3}{dt}+R_3(i_3-i_2)+\frac{1}{C_3}\int(i_3-i_2)dt=0 \qquad (9)$$

It is observed that the mesh equations (7), (8) and (9) are similar to the differential equations (4), (5) and (6) governing the mechanical system

FORCE CURRENT ANALOGOUS CIRCUIT

The electrical analogous elements for the elements of mechanical system are given below

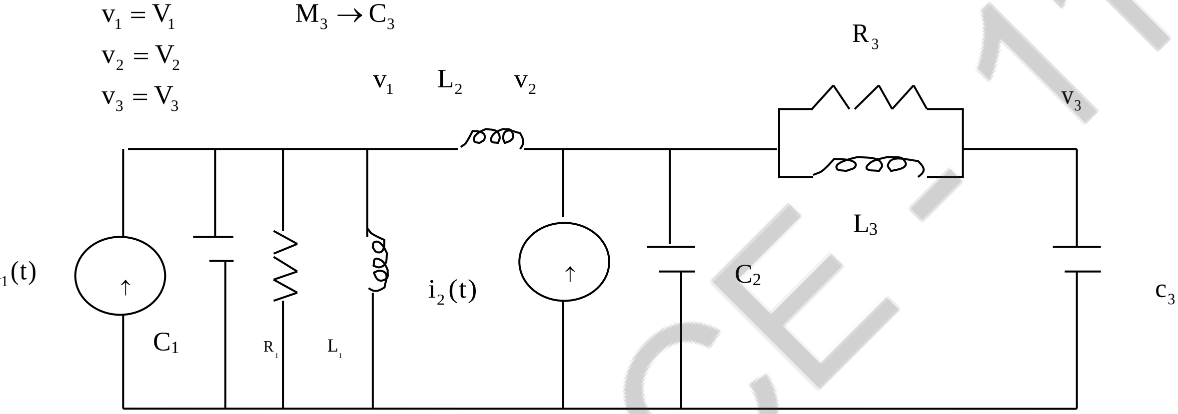
$f_1(t)=i_1(t)$ $\quad M_1\rightarrow C_1$ $\quad B_1\rightarrow 1/R_1$ $\quad K_1\rightarrow 1/L_1$

$f_2(t)=i_2(t)$ $\quad M_2\rightarrow C_2$ $\quad B_3\rightarrow 1/R_3$ $\quad K_2\rightarrow 1/L_2$

$v_1=V_1$ $\quad M_3\rightarrow C_3$

$v_2=V_2$

$v_3=V_3$



On applying KCL at node (1), (2) and (3)

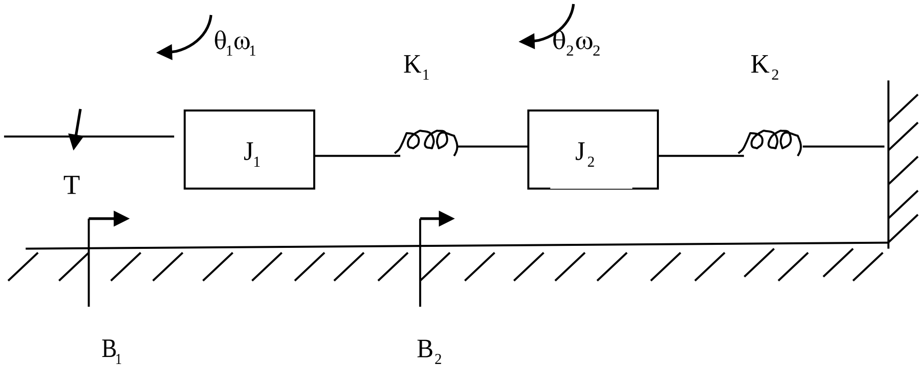
$$C_1\frac{dV_1}{dt}+\frac{1}{R_1}V_1+\frac{1}{L_1}\int V_1dt+\frac{1}{L_2}\int (V_1-V_2)dt=i_1(t) \qquad (10)$$

$$C_2\frac{dV_2}{dt}+\frac{1}{R_3}(V_2-V_3)+\frac{1}{L_3}\int(V_2-V_3)dt+\frac{1}{L_2}\int(V_2-V_1)dt=i_2(t) \qquad (11)$$

$$C_3\frac{dV_3}{dt}+\frac{1}{R_3}(V_3-V_1)+\frac{1}{L_3}\int(V_3-V_1)dt=0 \qquad (12)$$

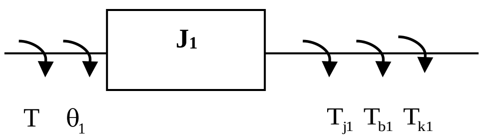
It is observed that node basis equations (10). (11) and (12) are similar to the differential equations (4), (5) and (6) governing the mechanical system

9. Write the differential equations governing the mechanical rotational system shown in fig. Draw the torque voltage and torque – current electrical analogous circuits and verifying by writing mesh and node equations.



Sol

Free body diagram of J_1



$$T_{ji} = J_1 \frac{d^2\theta_1}{dt^2}$$

$$T_{b1} = B_1 \frac{d\theta_1}{dt}$$

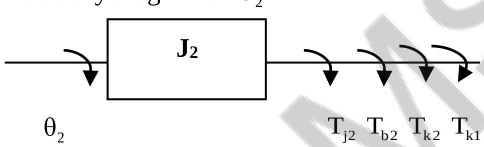
$$T_{k1} = K_1(\theta_1 - \theta_2)$$

By newtons second law

$$T_{j1} + T_{b1} + T_{k1} = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) = T \qquad (1)$$

Free body diagram of J_2



$$T_{j2} = J_2 \frac{d^2\theta_2}{dt^2}; \quad T_{b2} = B_2 \frac{d\theta_2}{dt}; \quad T_{k2} = K_2\theta_2; \; T_{k_1} = K_1(\theta_2 - \theta_1)$$

By Newtons second law,

$$T_{j2} + T_{b2} + T_{k2} + T_{k1} = 0$$

$$J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2\theta_2 + K_1(\theta_2 - \theta_1) \qquad (2)$$

On replacing the angular displacements by angular velocity in the differential equations (1) and (2) governing the mechanical rotational system, we get

$$\left(\text{i.e } \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt};\ \frac{d\theta}{dt} = \omega;\ \theta = \int \omega dt\right)$$

$$J_1 \frac{d\omega_1}{dt^2} + B_1\omega_1 + K_1\int (\omega_1 - \omega_2)\, dt = T \qquad \rightarrow (3)$$

$$J_2 \frac{d\omega_2}{dt^2} + B_2\omega_2 + K_2\int (\omega_2 dt) + K_1\int (\omega_2 - \omega_1)\, dt = 0 \quad \rightarrow (4)$$

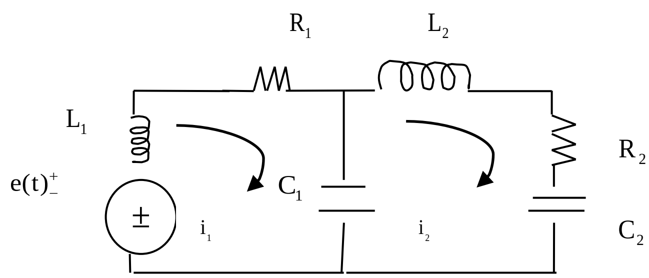
TORQUE – VOLTAGE ANALOGOUS CIRCUIT

The electrical analogous elements for the elements if mechanical rotational system are given below

$T \rightarrow e(t)$ $\quad J_1 \rightarrow L_1\ B_1 \rightarrow R_1$ $\quad K_1 \rightarrow 1/C_1$

$\omega_1 \rightarrow i_1$ $\quad J_2 \rightarrow L_2\ B_2 \rightarrow R_2$ $\quad K_2 \rightarrow 1/C_2$

$\omega_2 \rightarrow i_2$



Applying KVL to mesh (1) and (2)

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2) = e(t) \qquad (5)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_1}\int (i_2 - i_1) dt = 0 \qquad (6)$$

It is observed that mesh basis equations (5) and (6) are similar to differential equations (3) and (4) governing the mechanical system.

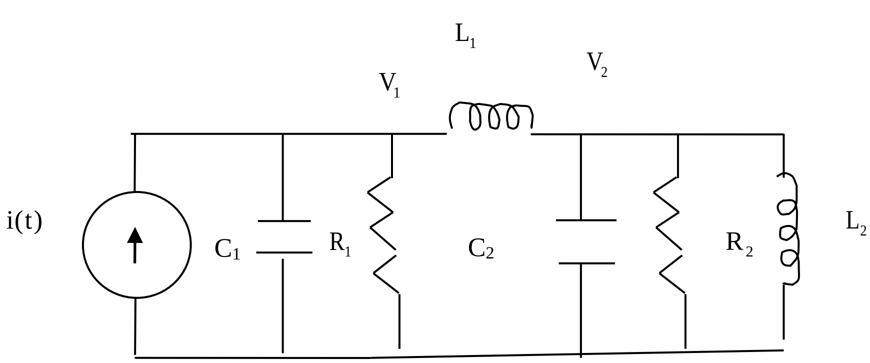
TORQUE CURRENT ANALOGOUS CIRCUIT

The electrical analogous elements for the elements of mechanical rotational system are given below.

$T \rightarrow i(t)\ J_1 \rightarrow C_1\ B_1 \rightarrow 1/R_1$ $\quad K_1 \rightarrow 1/L_1$

$\omega_1 \rightarrow V_1\ J_2 \rightarrow C_2\ B_2 \rightarrow 1/R_2$ $\quad K_2 \rightarrow 1/L_2$

$\omega_2 \rightarrow V_2$



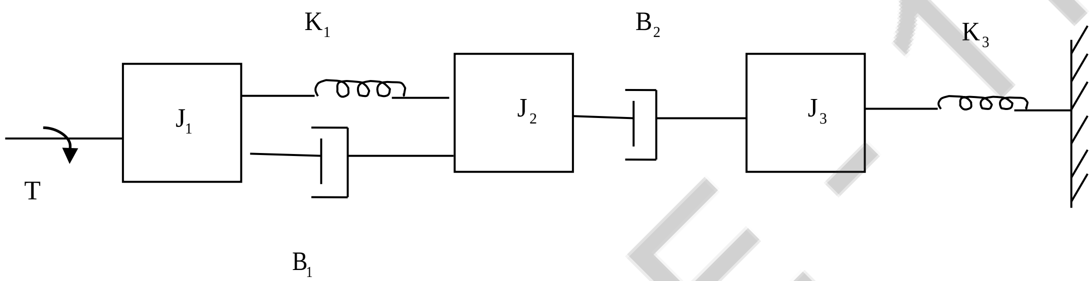
Applying KCL at node (1) and (2)

$$C_1 \frac{dV_1}{dt} + \frac{1}{R_1} V_1 + \frac{1}{L_1} \int (V_1 - V_2) dt = i(t)$$

$$C_2 \frac{dV_2}{dt} + \frac{1}{R_2} V_2 + \frac{1}{L_2} \int V_2 dt + \frac{1}{L_1} \int (V_2 - V_1) dt = 0 \quad (8)$$

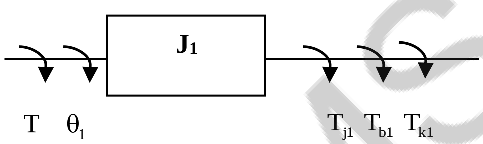
It is observed that the node basis equations (7) and (8) are similar to the differential equations (3) and (4) governing the mechanical system.

10.Write the differential equations governing the mechanical rotational system shown in fig. Draw the torque-voltage and torque-current electrical analogous circuits and verify by writing mesh and node equations.



Solution

Freebody diagram of J_1



$$T_{ji} = J_1 \frac{d^2\theta_1}{dt^2}$$

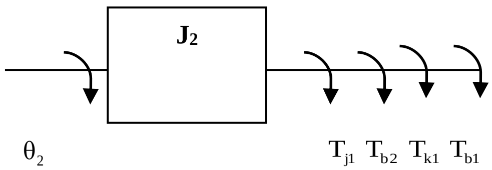
$$T_{b1} = B_1 \frac{d(\theta_1 - \theta_2)}{dt}$$

$$T_{k1} = K_1 (\theta_1 - \theta_2)$$

By newtons second law, $T_{j1} + T_{b1} + T_{k1} = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d(\theta_1 - \theta_2)}{dt} + K_1(\theta_1 - \theta_2) = T \qquad (1)$$

Freebody diagram of J_2

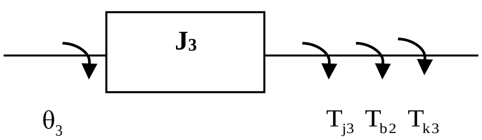


$$T_{j2} = J_2 \frac{d^2\theta}{dt^2}$$
$$T_{b2} = B_2 \frac{d(\theta_2 - \theta_3)}{dt}$$
$$T_{k1} = K_1(\theta_2 - \theta_1)$$
$$T_{b1} = B_1 \frac{d(\theta_2 - \theta_1)}{dt}$$

By newtons second law,

$$T_{j2} + T_{b2} + T_{b1} + T_{k1} = 0$$
$$J_2 \frac{d^2\theta}{dt^2} + B_2 \frac{d}{dt}(\theta_2 - \theta_3) + B_1 \frac{d(\theta_2 - \theta_1)}{dt} + K_1(\theta_2 - \theta_1) = 0 \qquad (2)$$

Free body diagram of J_3



$$T_{j3} = J_3 \frac{d^2\theta_3}{dt^2}$$
$$T_{b2} = B_2 \frac{d(\theta_3 - \theta_2)}{dt}$$
$$T_{k3} = K_3\theta_3$$

By Newton's second law,

$$T_{j3} + T_{b2} + T_{k3} = 0$$
$$J_3 \frac{d^2\theta_3}{dt^2} + B_2 \frac{d(\theta_3 - \theta_2)}{dt} + K_3\theta_3 = 0 \qquad (3)$$

On replacing the angular displacements by angular velocity in the differential equations (1) and (2) governing the mechanical rotational system, we get

$$(\text{i.e} \frac{d^2\theta}{dt^2}; \frac{d\omega}{dt} = \frac{d\theta}{dt} = \omega;\ \theta = \int \omega dt)$$

$$J_1 \frac{d\omega_1}{dt} + B_1(\omega_1 - \omega_2) + K_1 \int (\omega_1 - \omega_2) dt = T \qquad \rightarrow (4)$$
$$J_2 \frac{d\omega_2}{dt} + B_1(\omega_2 - \omega_1) + B_2(\omega_2 - \omega_3) + K_1 \int (\omega_2 - \omega_1) dt = 0 \qquad \rightarrow (5)$$
$$J_3 \frac{d\omega_3}{dt} + B_2(\omega_3 - \omega_2) + K_3 \int \omega_3 dt = 0 \qquad \rightarrow (6)$$

TORQUE – VOLTAGE ANALOGOUS CIRCUIT

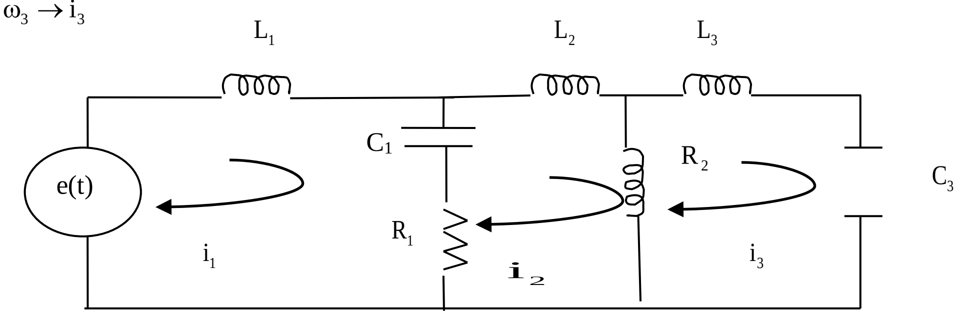
The electrical analogous elements for the elements of mechanical rotational system are given below.

$T \rightarrow e(t)$ $\quad J_1 \rightarrow L_1 \;\; B_1 \rightarrow R_1$ $\quad K_1 \rightarrow 1/C_1$

$\omega_1 \rightarrow i_1$ $\quad J_2 \rightarrow L_2 \;\; B_2 \rightarrow R_2$ $\quad K_3 \rightarrow 1/C_3$

$\omega_2 \rightarrow i_2$ $\quad J_3 \rightarrow L_3$

$\omega_3 \rightarrow i_3$



Applying KVL to mesh (1), (2) and (3)

$$L_1\frac{di_1}{dt}+R_1(i_1-i_2)+\frac{1}{C_1}\int(i_1-i_2)dt=e(t) \qquad (7)$$

$$L_2\frac{di_2}{dt}+R_1(i_2-i_1)+R_2\left(i_2-i_3\right)+\frac{1}{C_2}\int(i_2-i_1)dt=0 \qquad (8)$$

$$L_3\frac{di_3}{dt}+R_2(i_3-i_2)+\frac{1}{C_3}\int i_3dt=0 \qquad (9)$$

It is observed that mesh basis equation (7), (8) and (9) are similar to the differential equations (4), (5) and (6) governing the mechanical system.

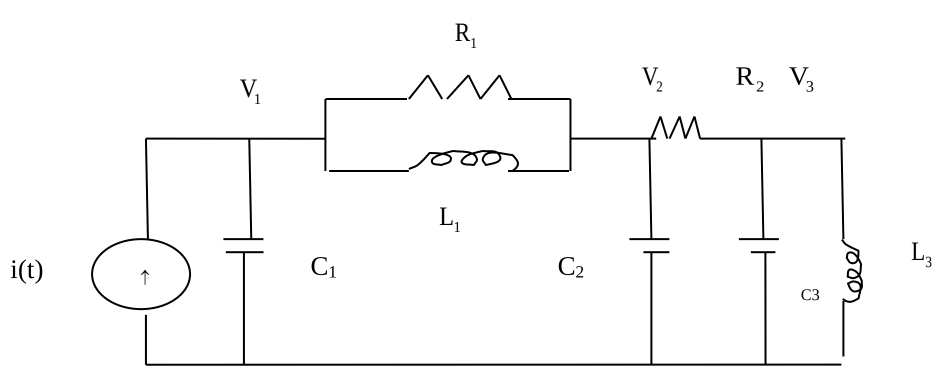
TORQUE – CURRENT ANALOGOUS CIRCUIT

The electrical analogous elements for the elements of mechanical rotational system are given below.

$T \rightarrow i(t)$ $\quad \omega_1 \rightarrow V_1$ $\quad J_1 \rightarrow C_1 \;\; B_1 \rightarrow 1/R_1$ $\quad K_1 \rightarrow 1/L_1$

$\omega_2 \rightarrow V_2$ $\quad J_2 \rightarrow C_2 \;\; B_2 \rightarrow 1/R_2$ $\quad K_2 \rightarrow 1/L_3$

$\omega_3 \rightarrow V_3$ $\quad J_3 \rightarrow C_3$



Applying KCL at node (1), (2) and (3)

$$C_1\frac{dV_1}{dt}+\frac{1}{R_1}\left(V_1-V_2\right)+\frac{1}{L_1}\int\left(V_1-V_2\right)dt=i(t) \quad \rightarrow (10)$$

$$C_2\frac{dV_2}{dt}+\frac{1}{R_1}\left(V_2-V_1\right)+\frac{1}{R_2}\left(V_2-V_3\right)+\frac{1}{L_1}\int\left(V_2-V_1\right)dt=0 \quad \rightarrow (11)$$

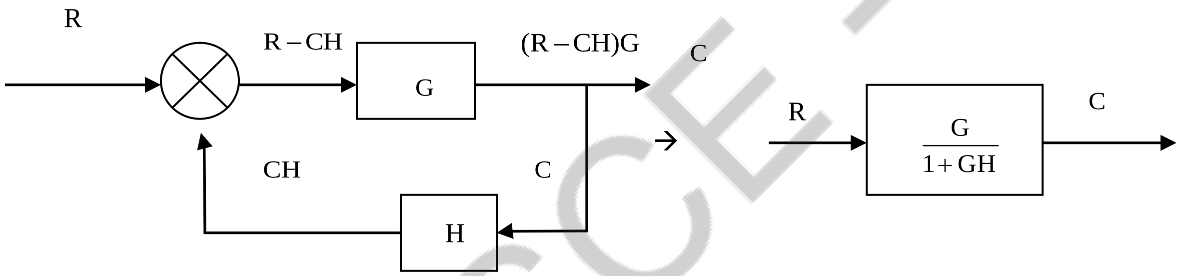
$$C_3\frac{dV_3}{dt}+\frac{1}{R_2}\left(V_3-V_2\right)+\frac{1}{L_3}\int V_3dt=0 \quad \rightarrow (12)$$

It is observed that the node basis equations (10), (11), and (12) are similar to the differential equations (4), (5) and (6) governing the mechanical system

BLOCK DIAGRAMS

1. Write the rule for eliminating negative and positive feedback in block diagram reduction NOV/DEC 2015

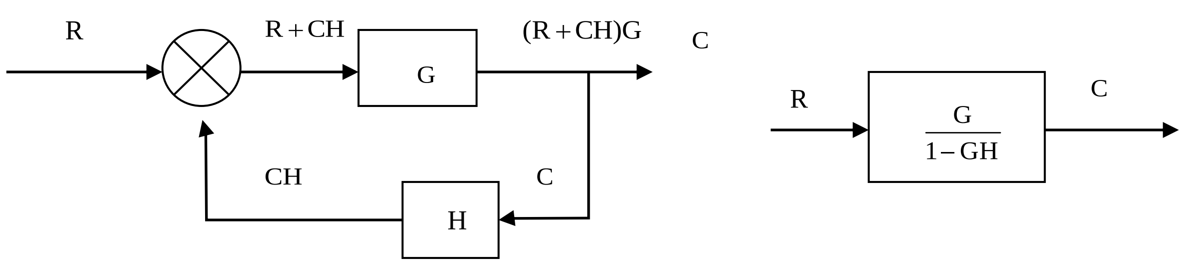
(A) Elimination of –ve feedback loop



Proof

$$C=(R-CH)G$$
$$C=RG-CHG$$
$$C+CHG=RG$$
$$C(1+HG)=RG$$
$$\frac{C}{R}=\frac{G}{(1+GH)}$$

(B) Elimination of positive feedback loop

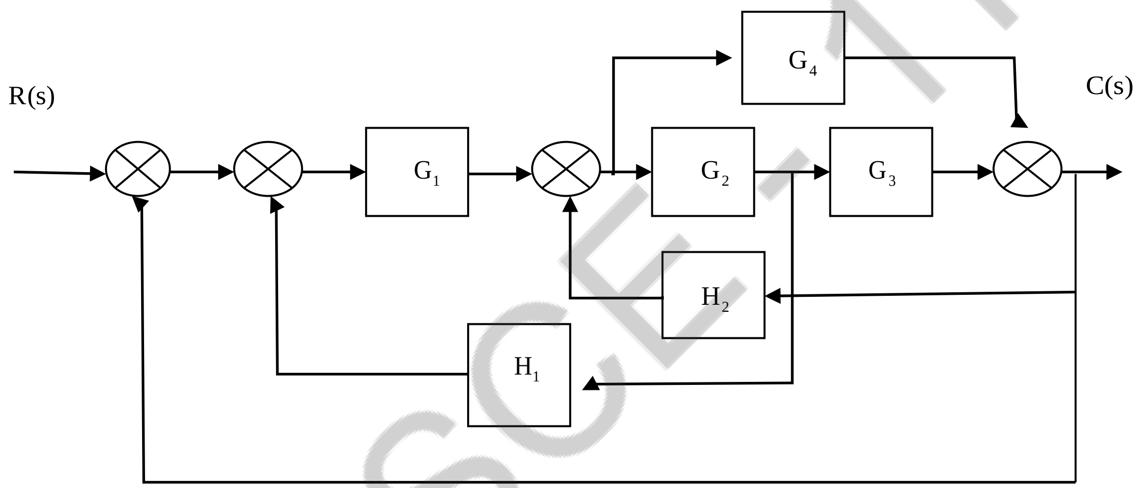


Proof

$$C = (R + CH)G$$
$$C = RG + CHG$$
$$C - CHG = RG$$
$$C(1 - GH) = RG$$
$$\frac{C}{R} = \frac{G}{1 - GH}$$

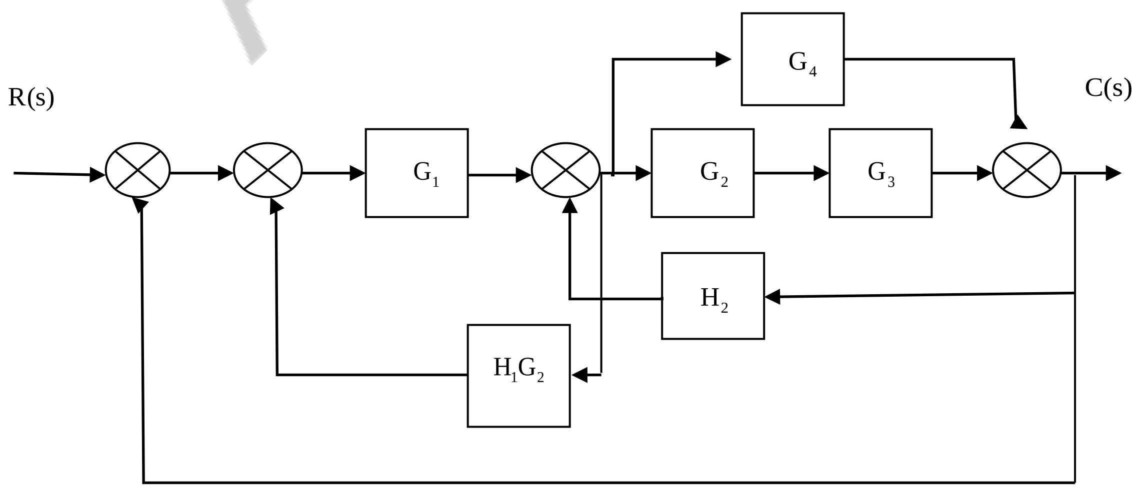
$$\frac{C(s)}{R(s)} = \frac{G_1G_3(G_2 + G_4)}{1 + G_3H_1 + G_1G_2G_3 + G_1G_3G_4}$$

2. Using the block diagram reduction technique. Find the closed loop transfer function of the system whose block diagram is shown in fig

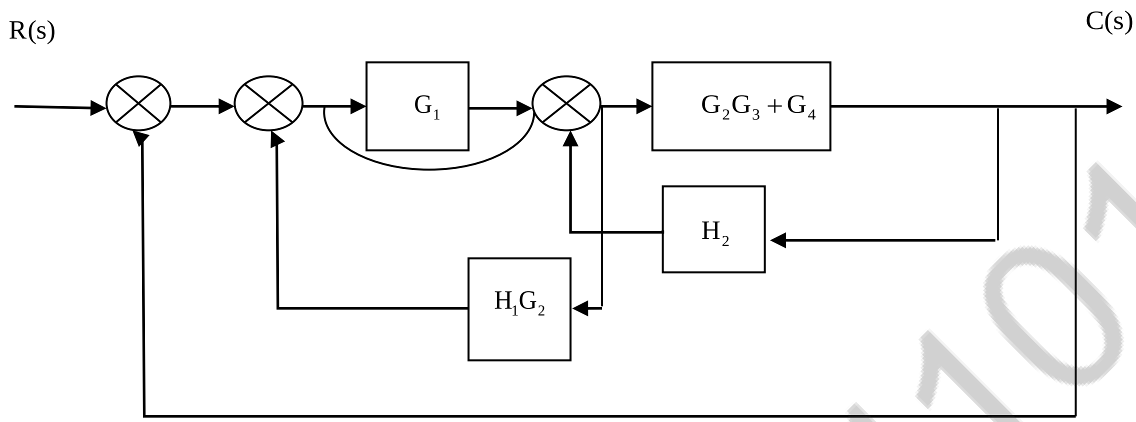


Sol

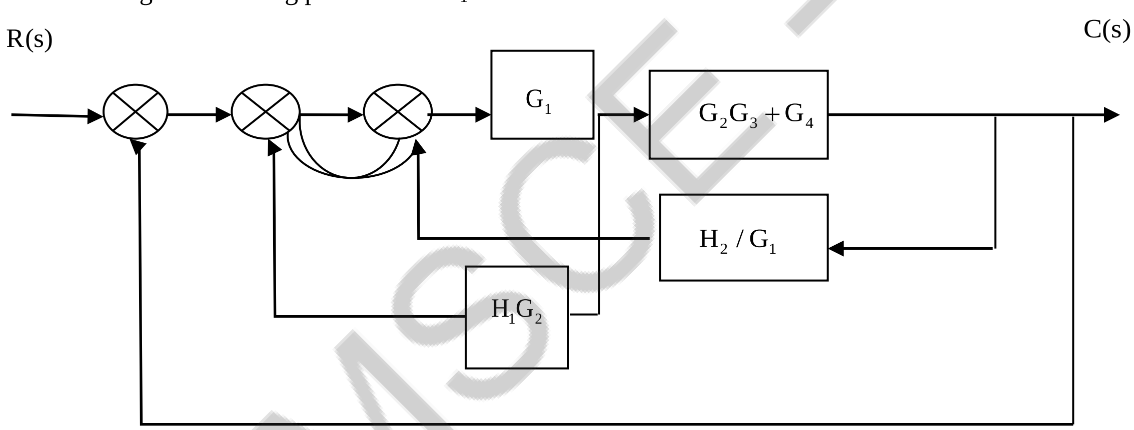
Moving the branch point before the block G_2



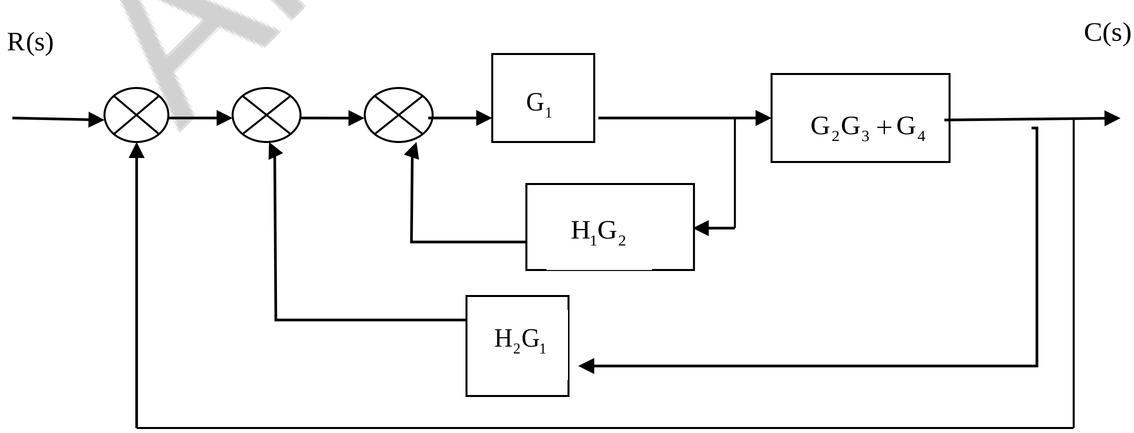
Combining the cascade blocks G_2 and G_3 parallel block G_1



Moving the summing point before G_1

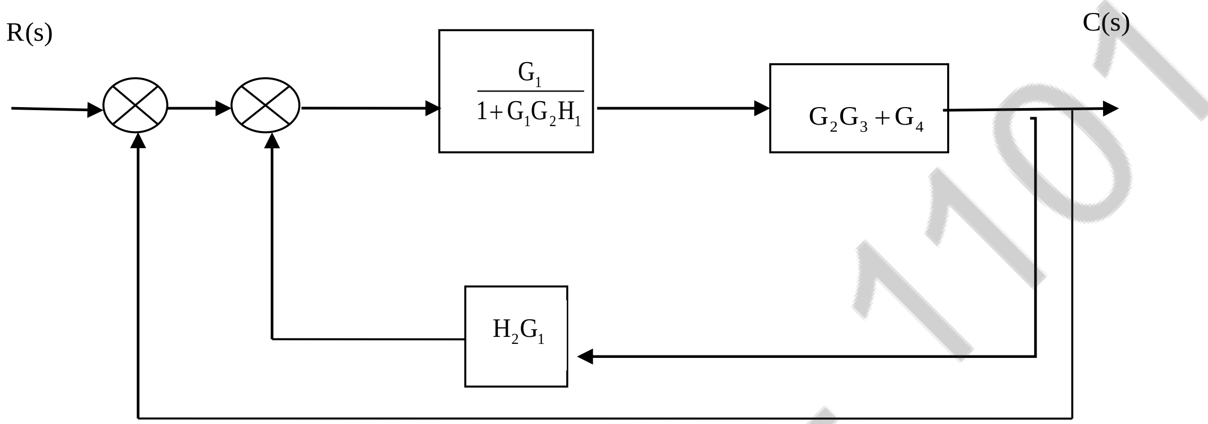


Interchanging the summing points

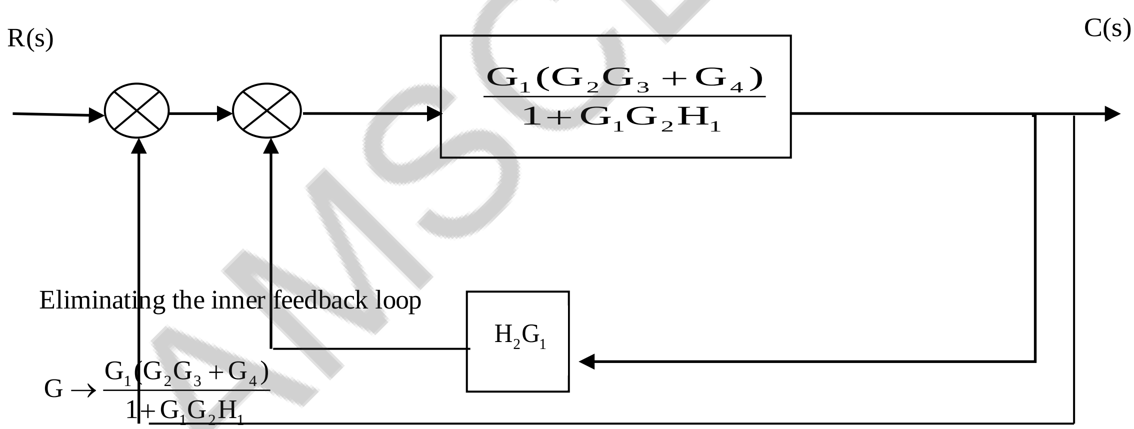


Eliminating the inner most negative feedback

$$G \rightarrow G_1$$
$$H \rightarrow H_1G_2$$
$$\frac{G}{1+GH} \rightarrow \frac{G_1}{1+G_1G_2H_1}$$



Combining cascade blocks

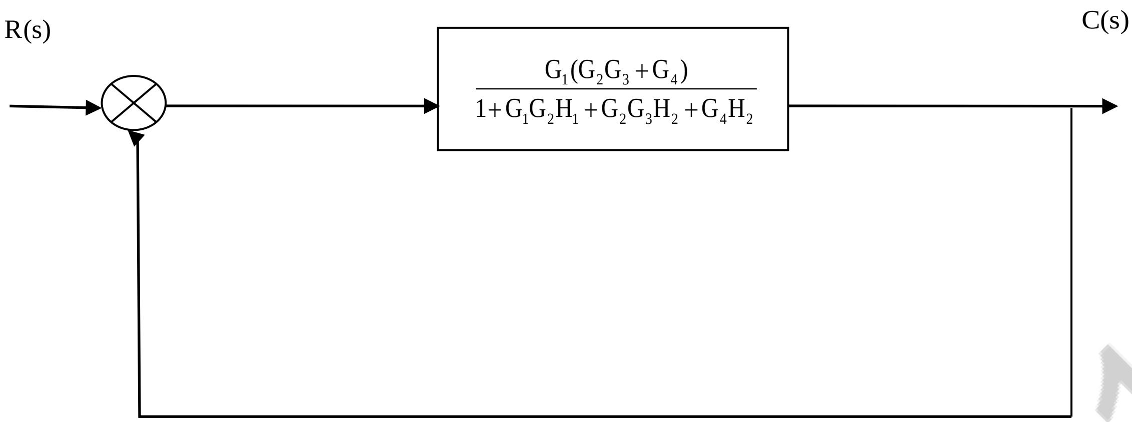


Eliminating the inner feedback loop

$$G \rightarrow \frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}$$
$$H \rightarrow H_2/G_1$$

$$\frac{G}{1+GH} = \frac{\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\frac{\cancel{G_1}(G_2G_3+G_4)}{1+G_1G_2H_1}\frac{H_2}{\cancel{G_1}}} = \frac{\frac{G_1(G_2G_3+G_4)}{\cancel{1+G_1G_2H_1}}}{\frac{1+G_1G_2H_2+G_2G_3H_2+G_4H_2}{\cancel{1+G_1G_2H_1}}}$$

$$= \frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$



Eliminating the negative, unity feedback

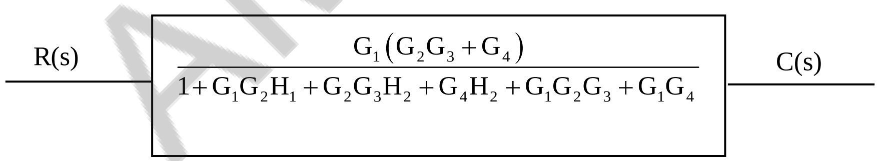
$$G \rightarrow \frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

$$H \rightarrow 1$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}$$

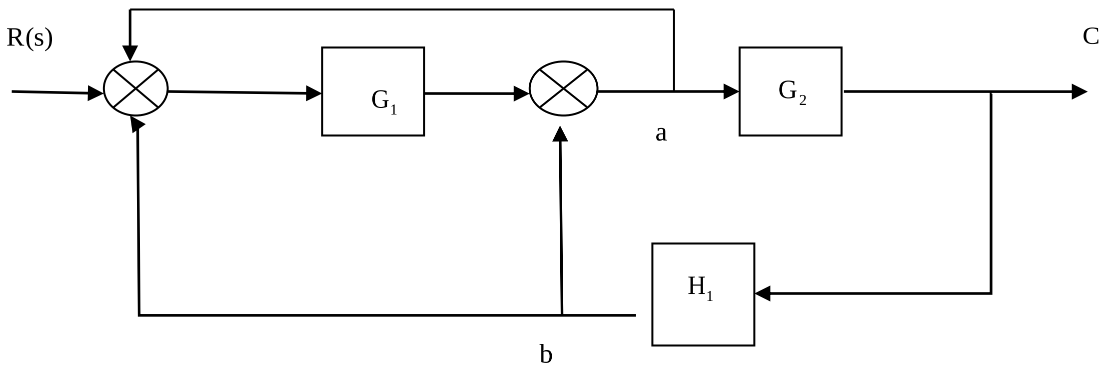
$$= \frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+\cancel{G_2G_3H_2}+G_4H_2}}{1+\dfrac{G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+\cancel{G_2G_3H_2}+G_4H_2}}$$

$$= \frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

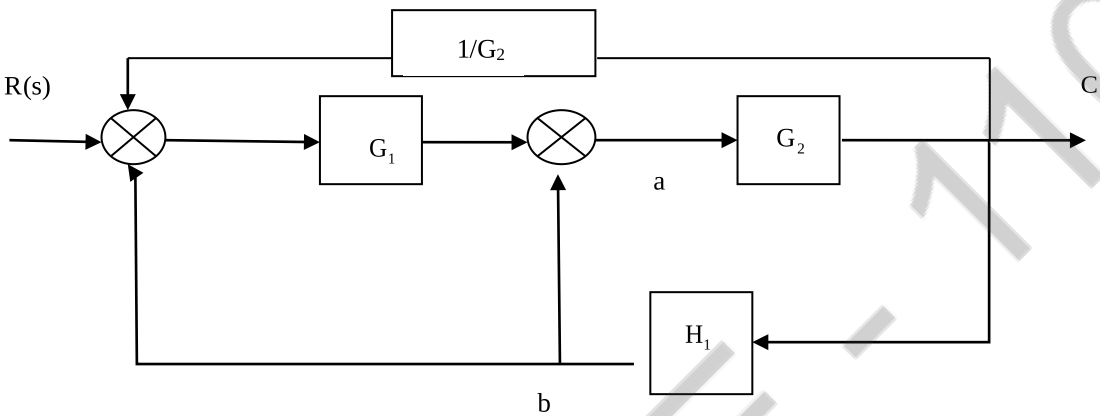


$$\frac{C(s)}{R(s)} = \frac{G_1\left(G_2G_3+G_4\right)}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

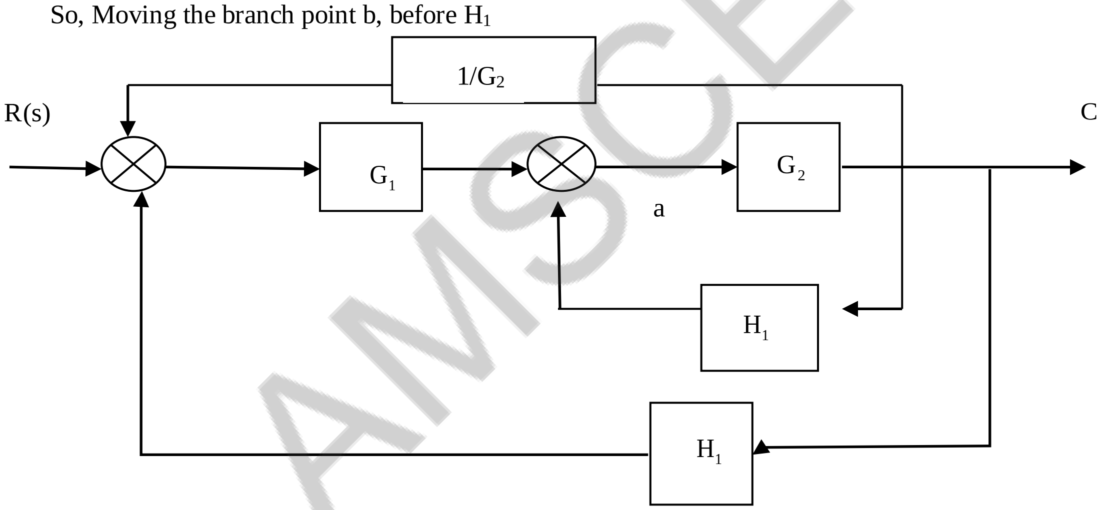
3.Using block diagram reduction technique, find C/R



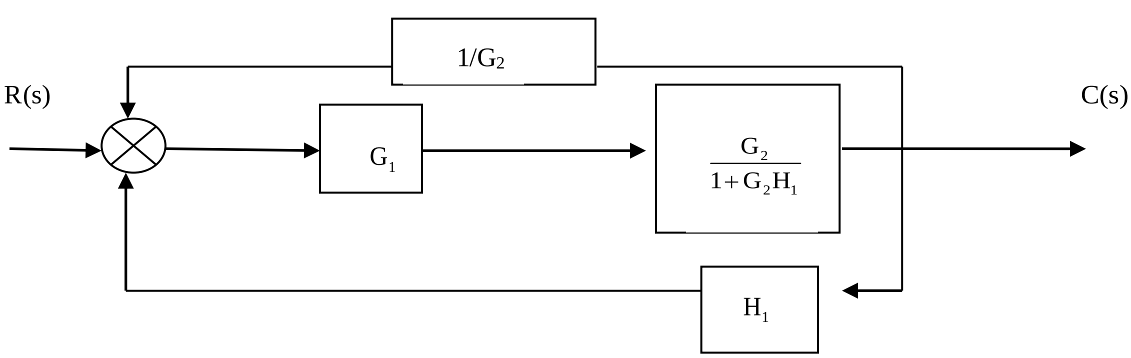
Sol : Moving th branch point a ahead of $G_{2,}$



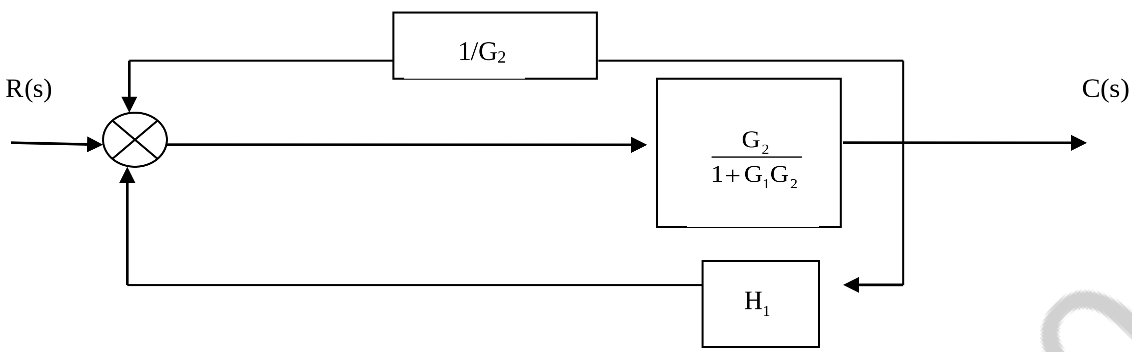
So, Moving the branch point b, before H_1



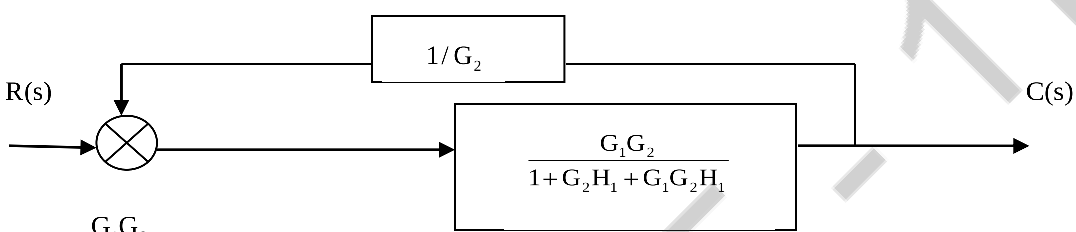
Eliminating the feedback H_1



Combining the cascade blocks

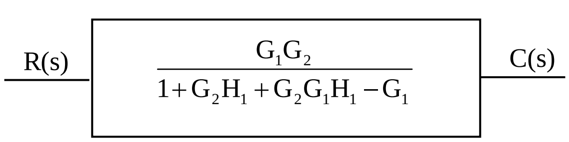


Eliminating the negative feedback, H_1



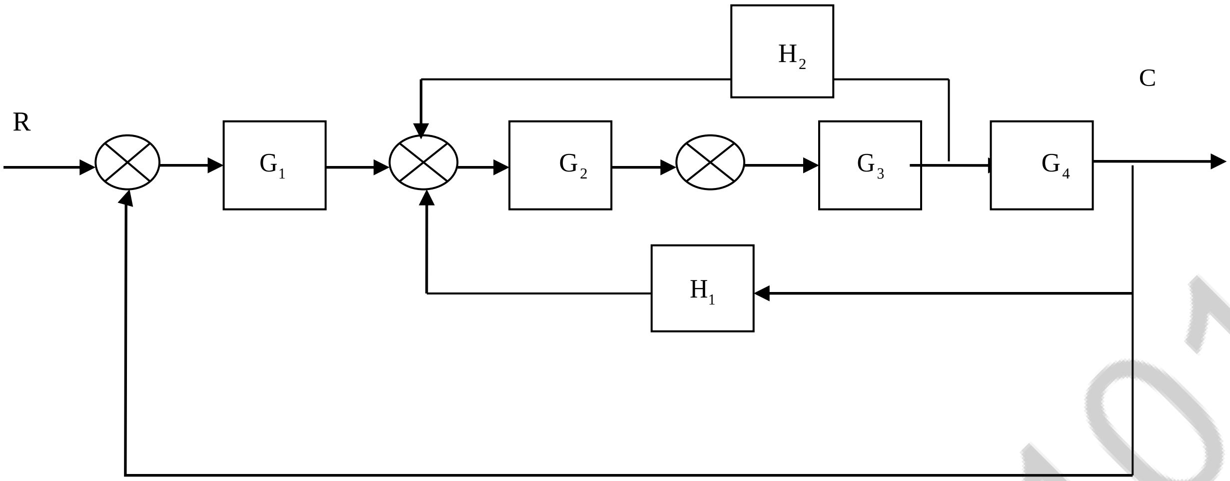
$$G=\frac{G_1G_2}{1+G_2H_1}$$
$$H=H_1$$
$$\frac{G}{1+GH}=\frac{\frac{G_1G_2}{1+G_2H_1}}{1+\frac{G_1G_2}{1+G_2H_1}H_1}$$
$$=\frac{G_1G_2}{1+G_2H_1+G_1G_2H_1}$$

$$G=\frac{G_1G_2}{1+G_2H_1+G_1G_2H_1},\ H\rightarrow 1/G_2$$
$$\frac{G}{1-GH}=\frac{\frac{G_1G_2}{1+G_2H_1+G_1G_2H_1}}{1-\frac{G_1\cancel{G_2}}{1+G_2+G_1G_2H_1}\frac{1}{\cancel{G_2}}}=\frac{G_1G_2}{1+G_2H_1+G_1G_2H_1-G_1}$$

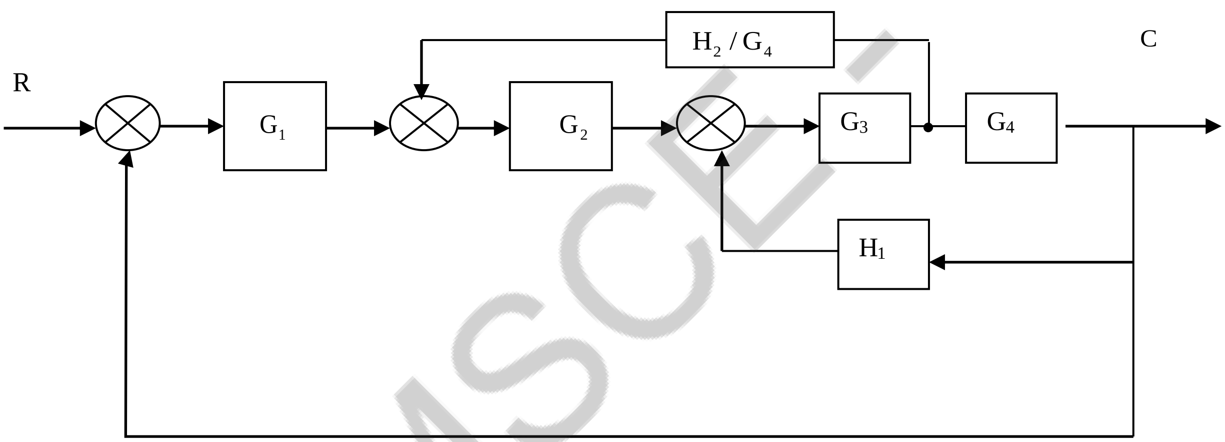


$$\frac{C(S)}{R(S)}=\frac{G_1G_2}{1+G_2H_1+G_1G_2H_1-G_1}$$

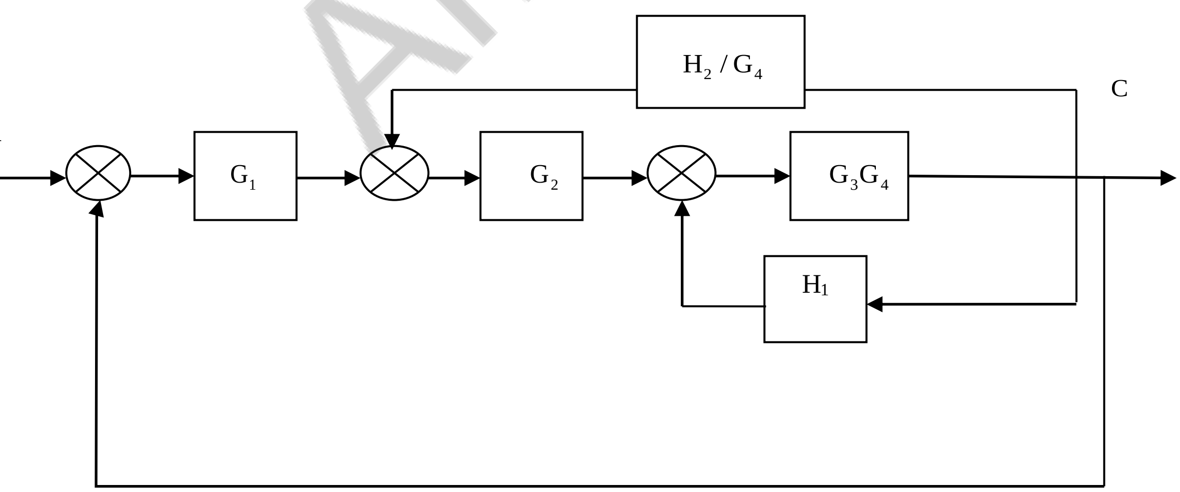
4.Using block diagram reduction technique find C/R NOV/DEC 2016



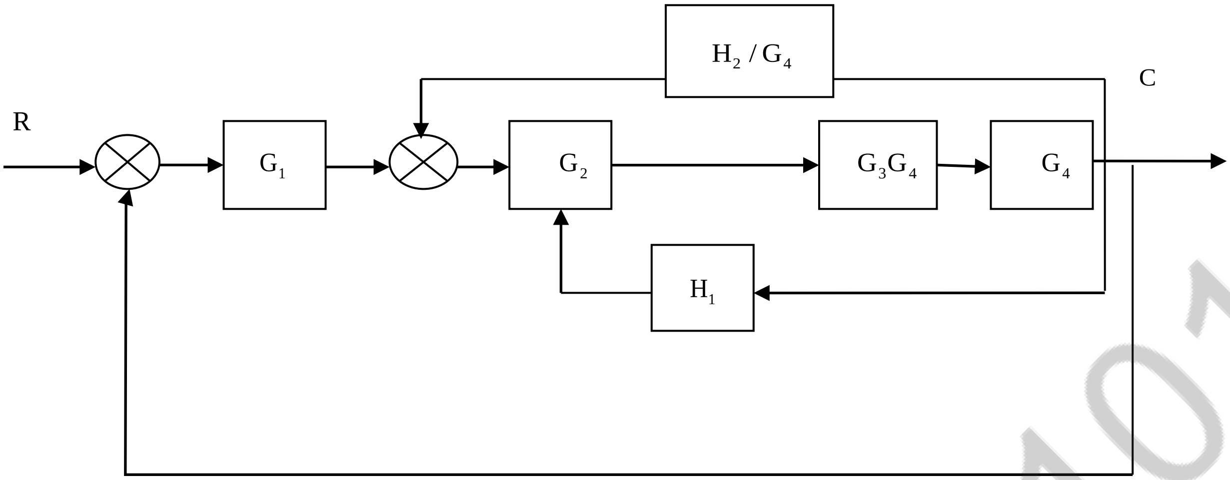
Sol: Moving the branch a ahead of G_4



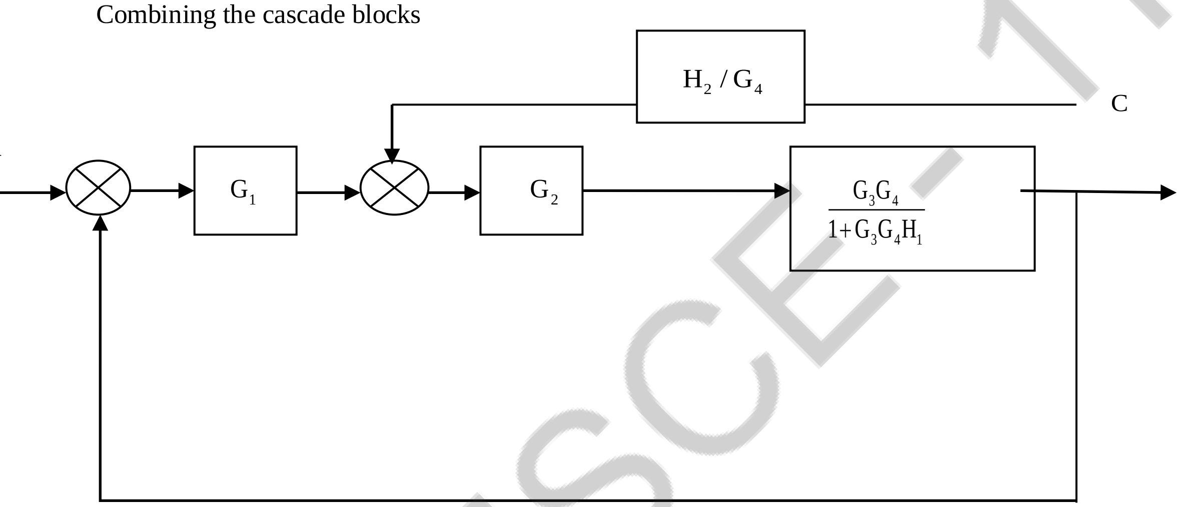
Combining the cascade blocks G_3 and G_4



Eliminating the feedback H_1



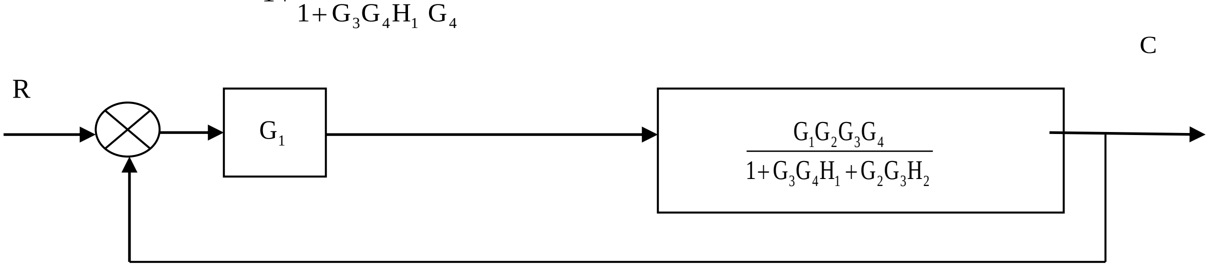
Combining the cascade blocks



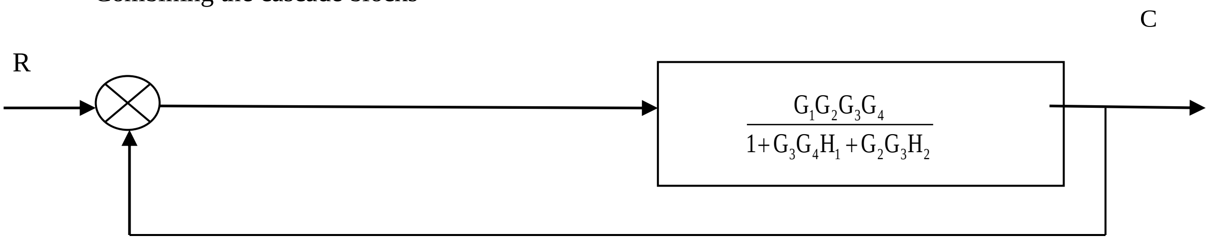
Eliminating the feedback, H_2 / G_4

$$G \rightarrow \frac{G_2G_3G_4}{1+G_3G_4H_1} \qquad H \rightarrow H_2 / G_4$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_2G_3G_4}{1+G_3G_4H_1}}{1+\dfrac{G_2G_3\cancel{G_4}}{1+G_3G_4H_1}\dfrac{H_2}{G_4}} \Rightarrow \frac{G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2}$$



Combining the cascade blocks



Eliminating the feedback (unity)

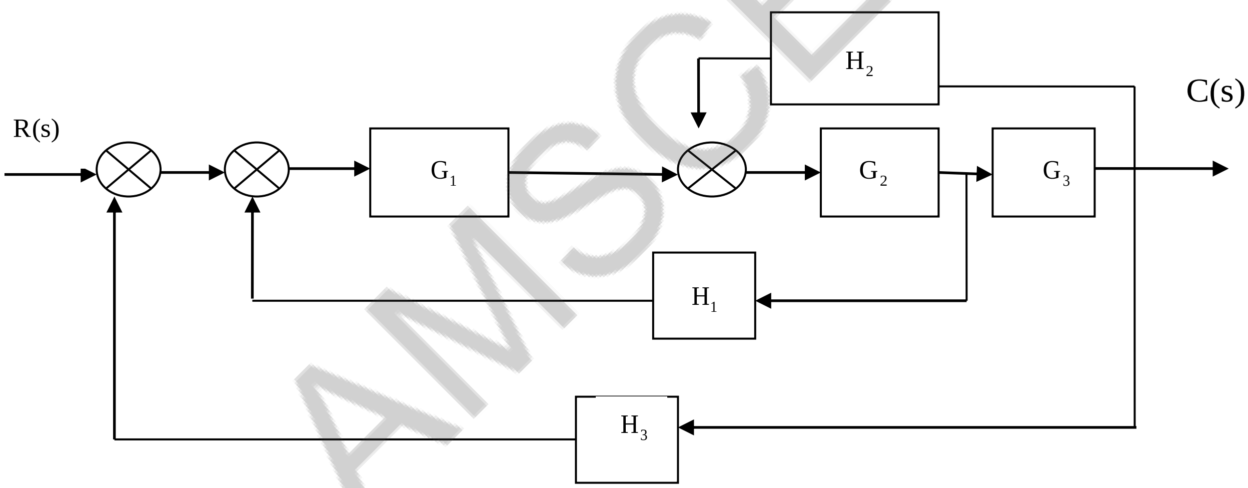
$$G \rightarrow \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2} \qquad H \rightarrow 1$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2}}{1+\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2}} \Rightarrow \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

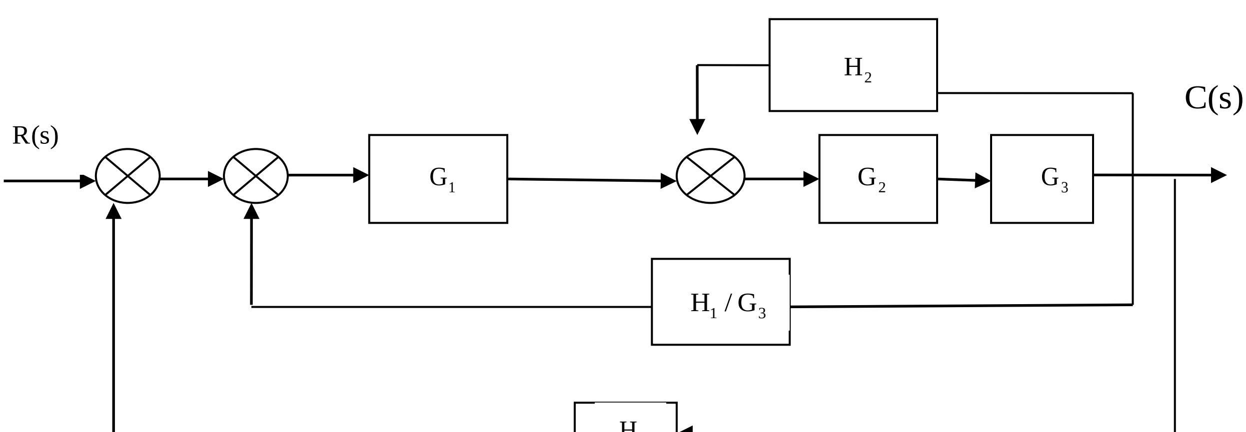
$$\underset{\rightarrow}{R}\boxed{\frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}}\underset{\rightarrow}{C}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

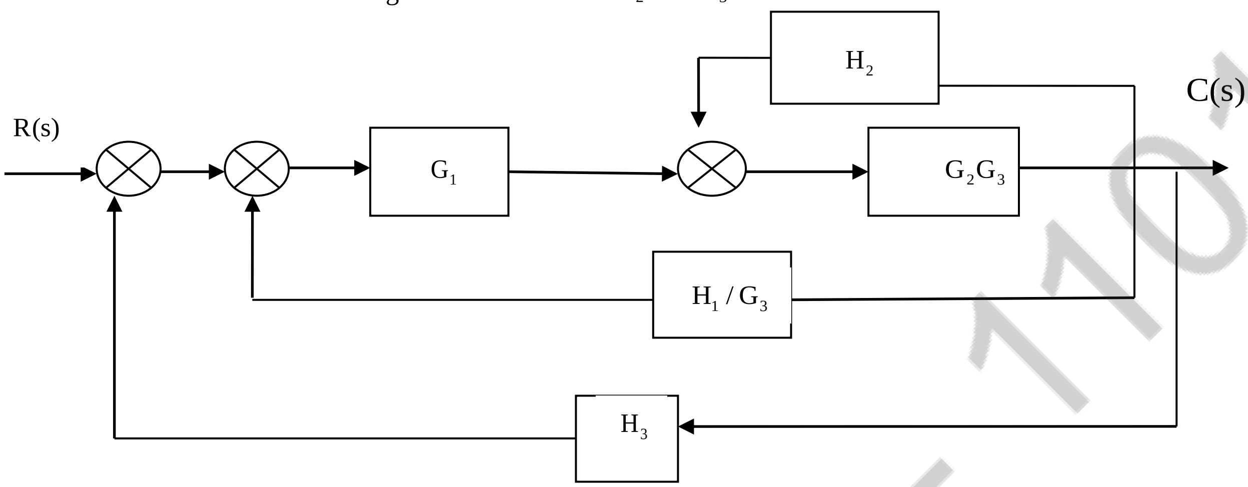
5.Using block diagram reduction technique, find C/R



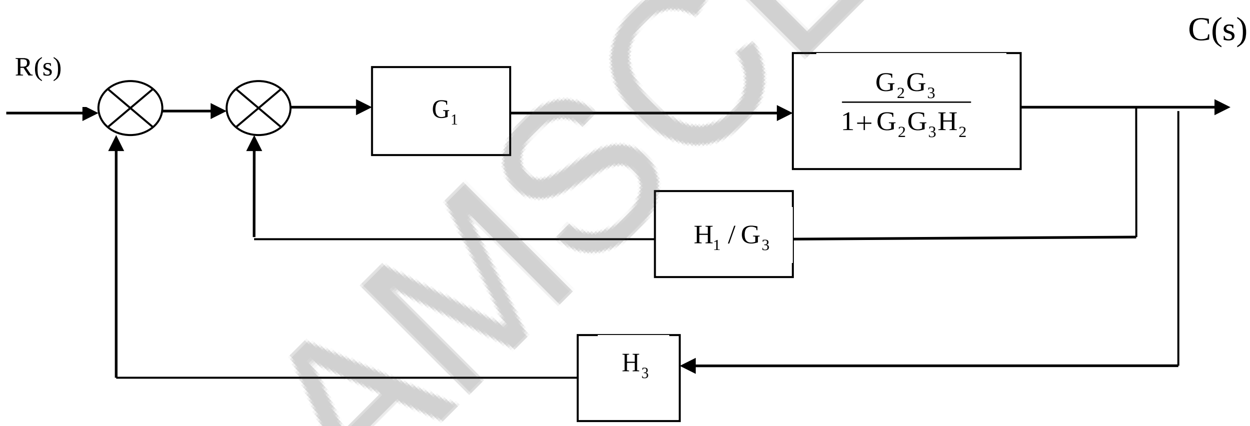
Sol: Moving the branch point a, ahead of G_3



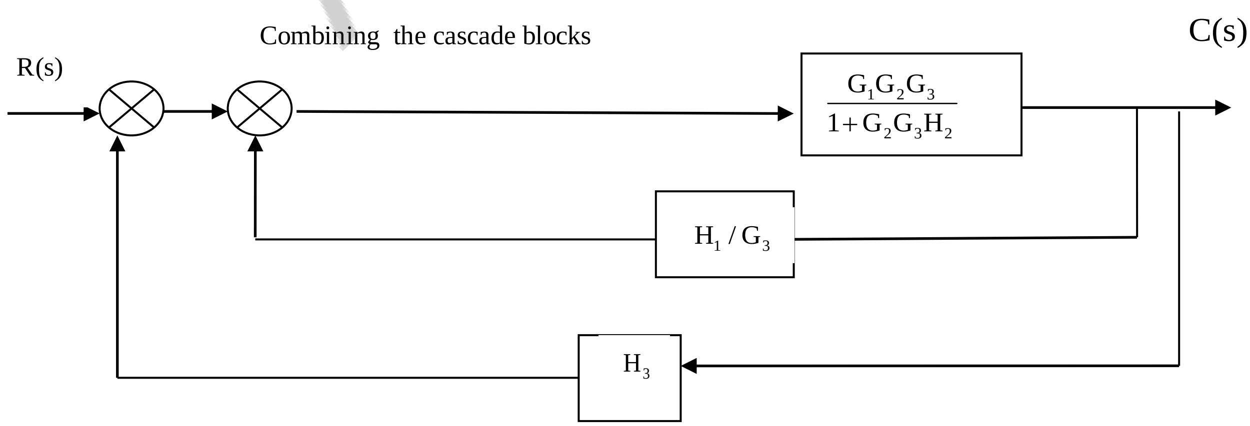
Combining the cascade blocks G_2 and G_3



Eliminating the feedback H_2

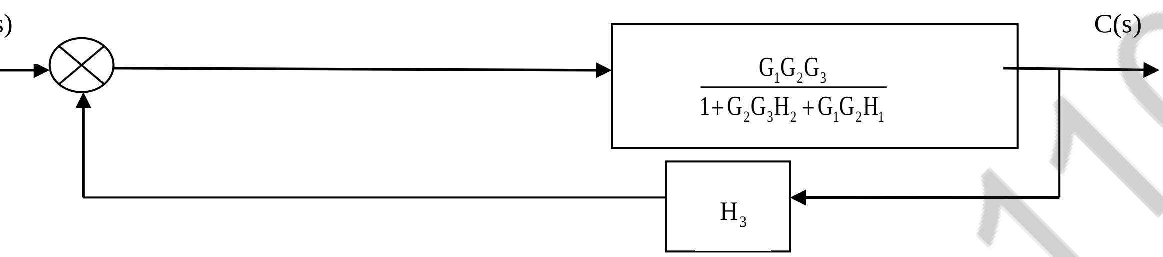


Combining the cascade blocks



Eliminating the feedback parts H_1 / G_3

$$G \rightarrow \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\dfrac{G_1G_2\cancel{G_3}}{1+G_2G_3H_2}\dfrac{H_1}{\cancel{G_3}}} \Rightarrow \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}$$



Eliminating the feedback path H_3,

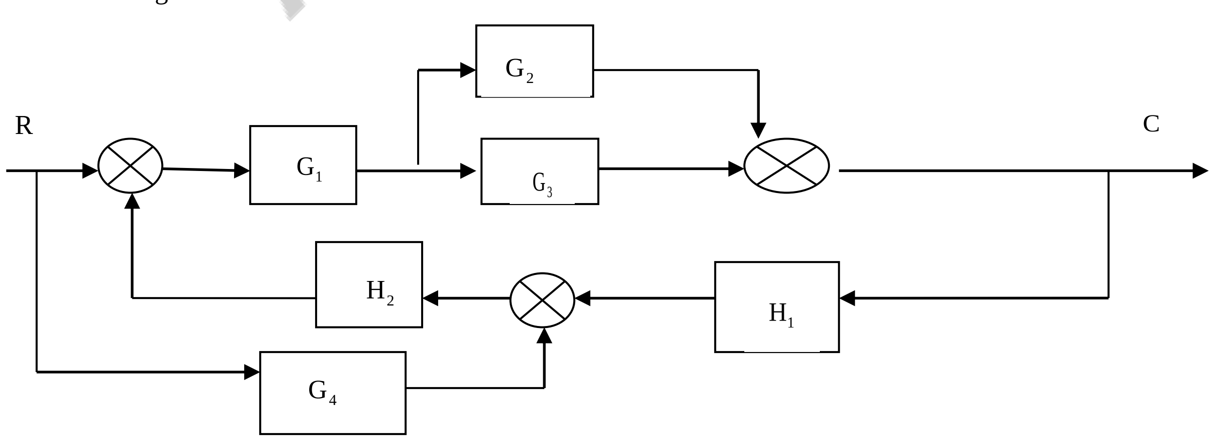
$$G \rightarrow \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1},\ H \rightarrow H_3$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}}{1+\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}H_3} \Rightarrow \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3H_3}$$

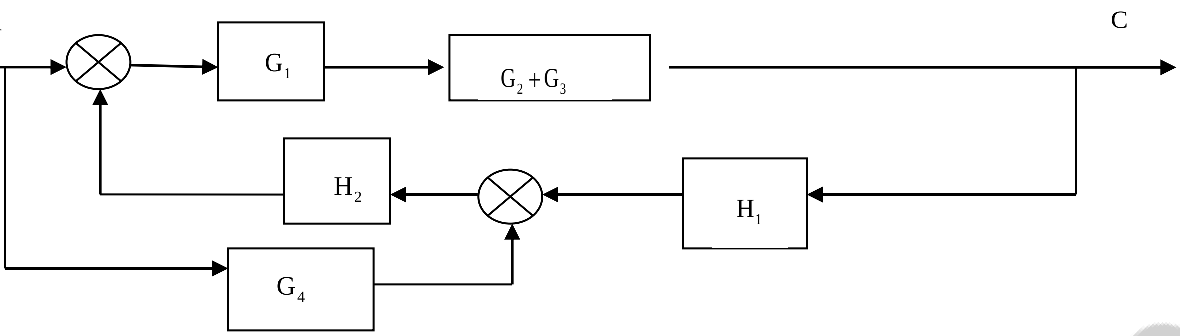
$$R(S)\xrightarrow{}\boxed{\frac{G_1G_2G_3}{1+G_2G_3H_2+G_2G_1H_1+G_1G_2G_3H_3}}\xrightarrow{}C(S)$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3H_3}$$

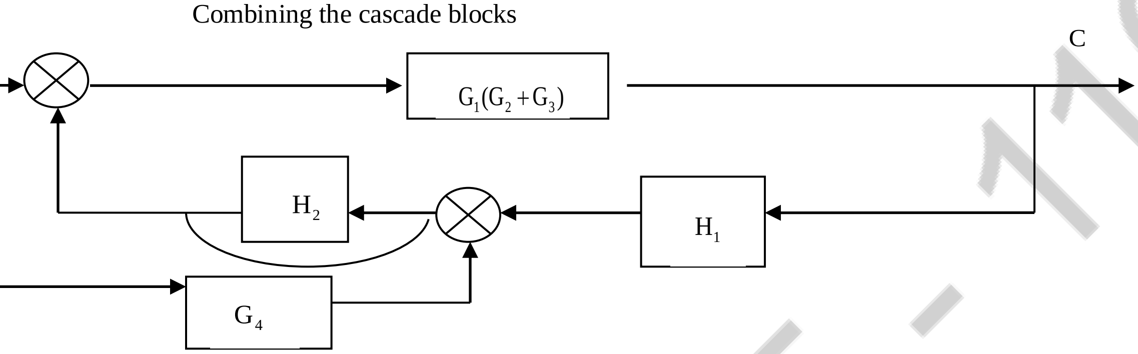
6.Using block diagram reduction technique, find the closed loop transfer function of a system, whose block diagram is shown in fig



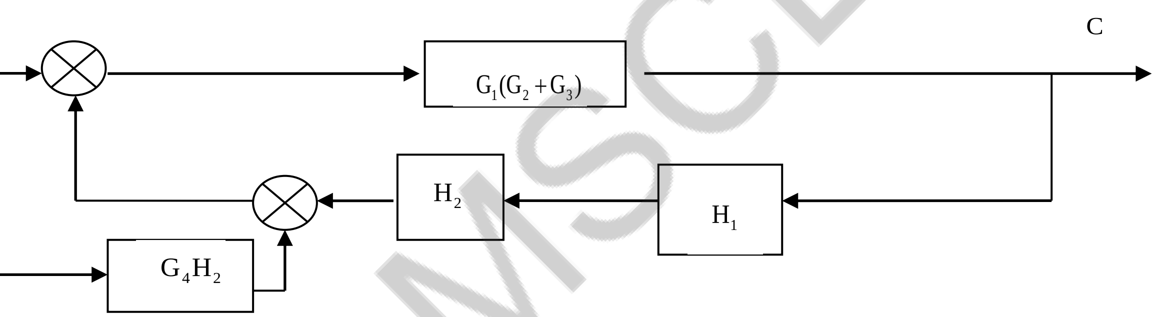
Sol: Combining the parallel blocks



Combining the cascade blocks

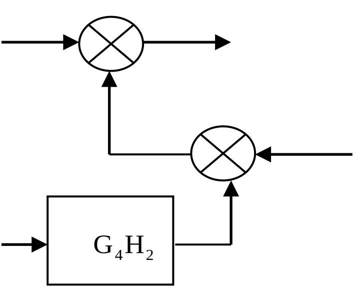


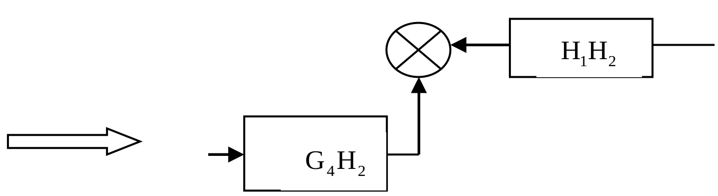
Moving the summing point ahead of H_2

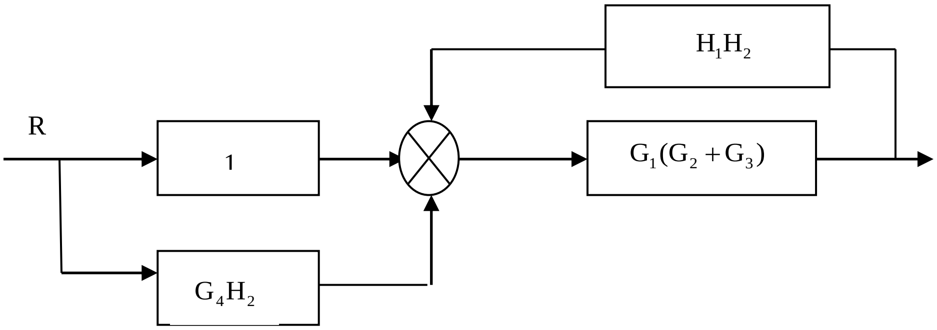


Cascading H_1 and H_2 combining the summing points.

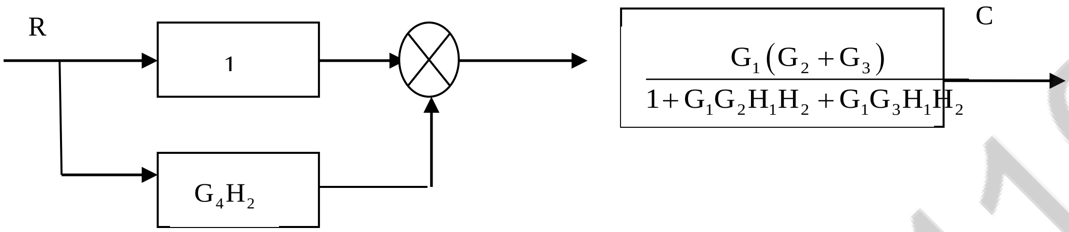
Elimination of summing point by multiply signs



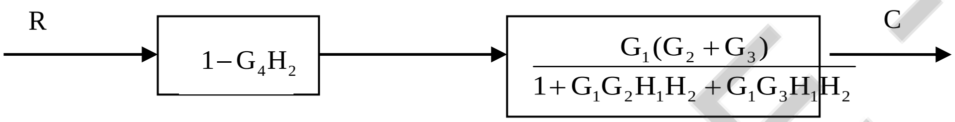




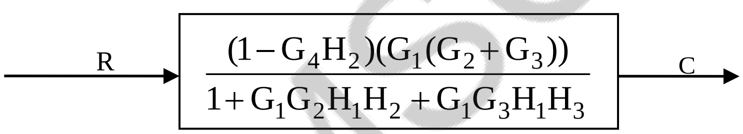
Feedback path H_1, H_2 elimination



Combining parallel path

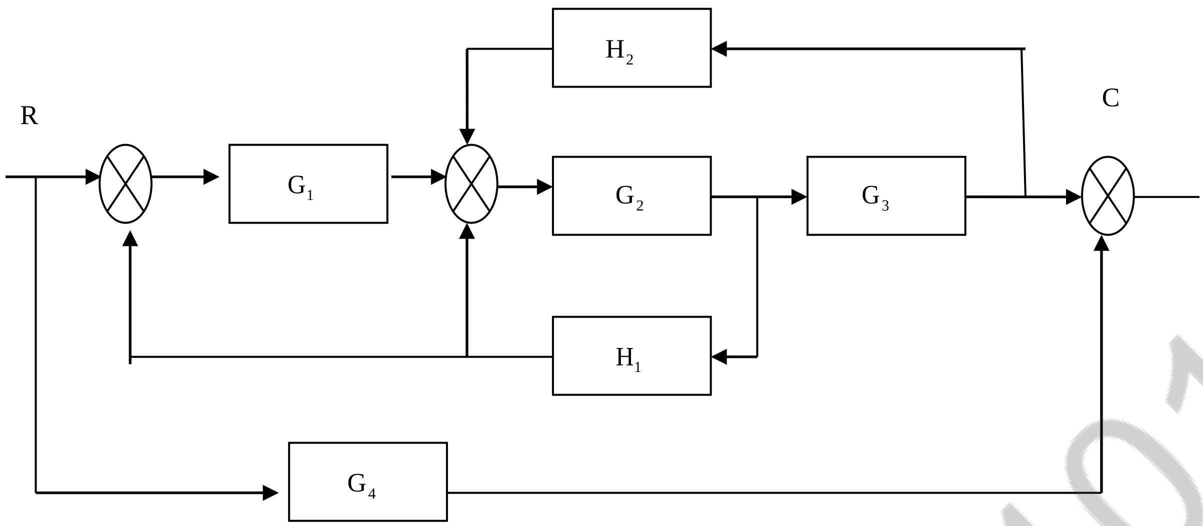


Combining cascade blocks

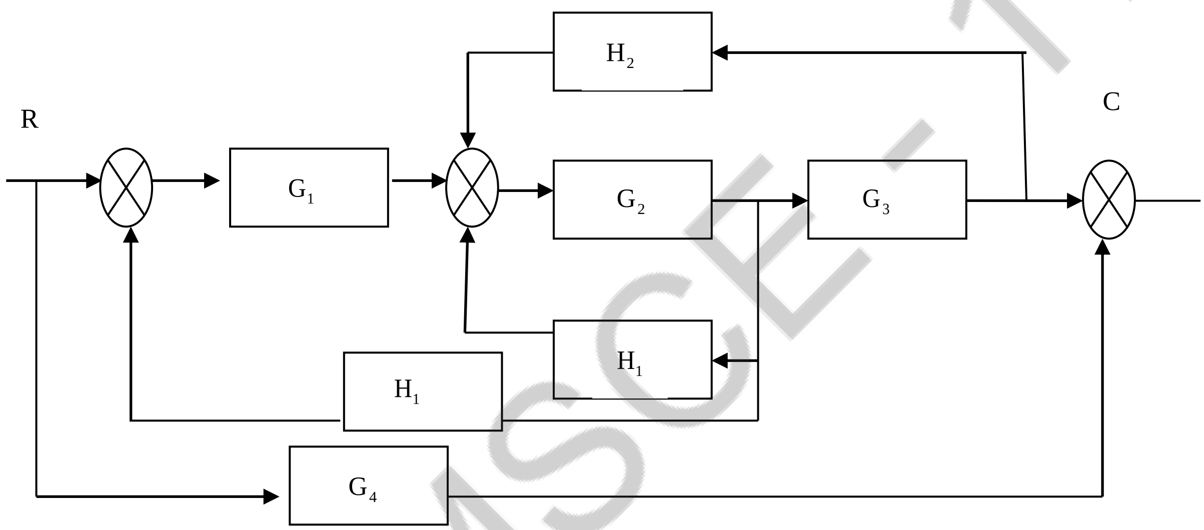


$$\frac{C}{R}=\frac{G_1(G_2+G_3)(1-G_4H_2)}{1+G_1G_2H_1H_2+G_1G_3H_1H_2}$$

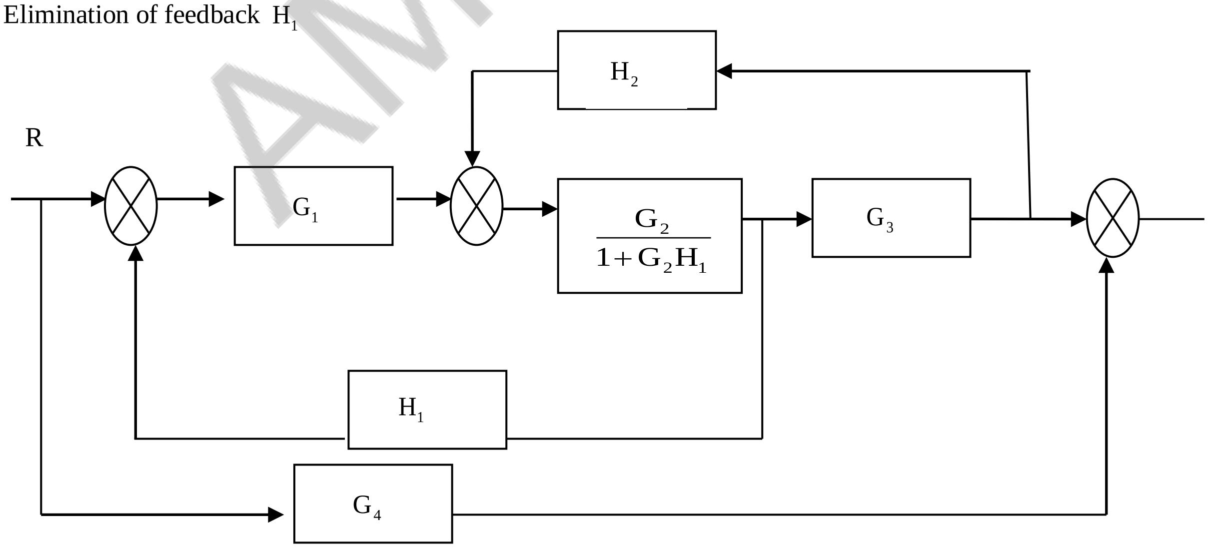
7.Using block diagram reduction technique, find C/R



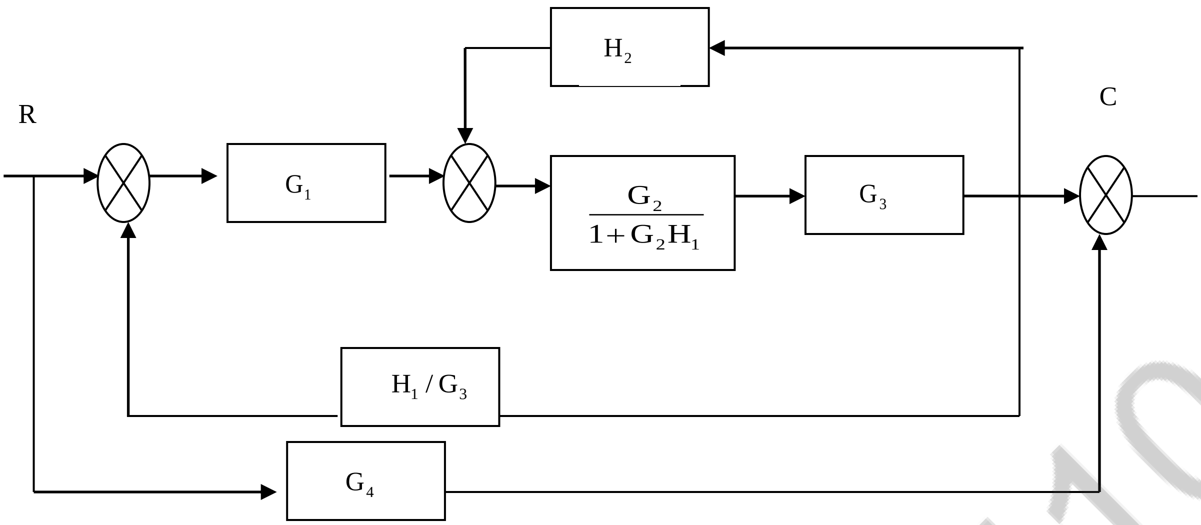
Sol: Moving the branch point a before H_1



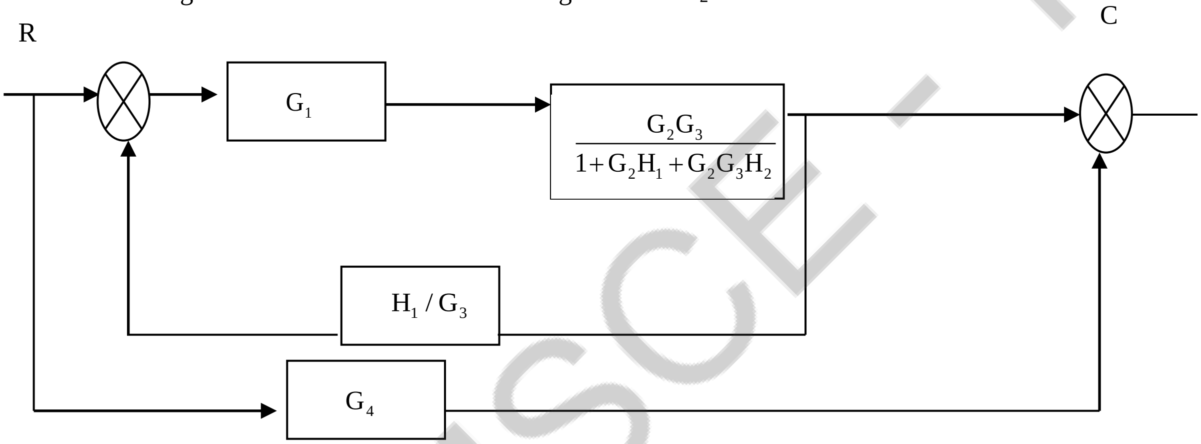
Elimination of feedback H_1



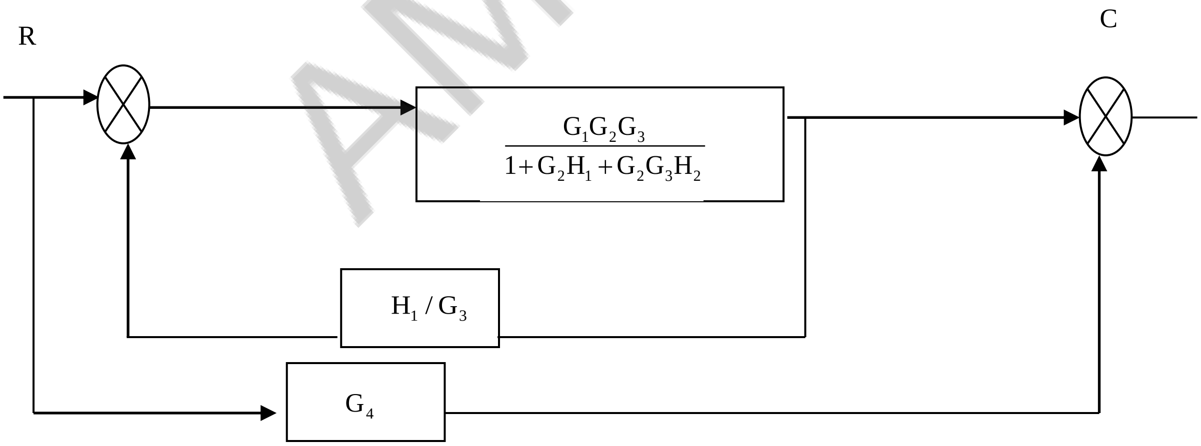
Moving the branch point ahead of G_3



Combining the cascade blocks and eliminating feedback H_2



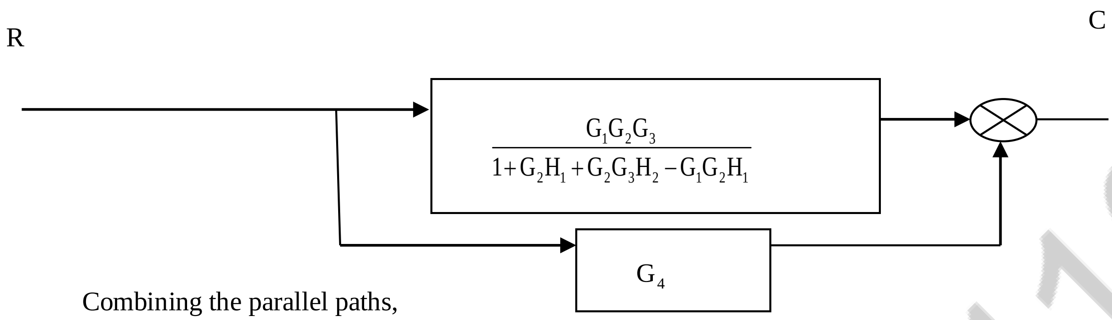
Combining the cascade blocks



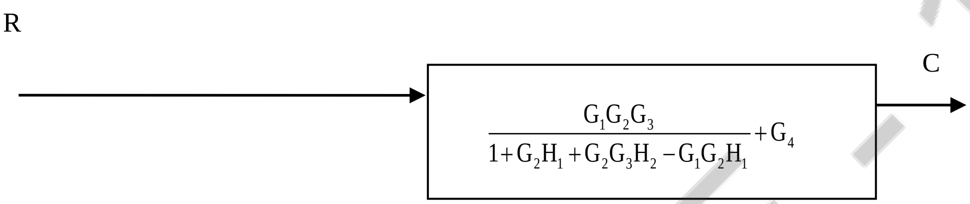
Eliminating the feedback path,

$$G \rightarrow \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2};\quad H \rightarrow \frac{H_1}{G_3}$$

$$\frac{G}{1-GH} = \frac{\dfrac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2}}{1-\dfrac{G_1G_2\cancel{G_3}}{1+G_2H_1+G_2G_3H_2}\dfrac{H_1}{G_3}} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1}$$

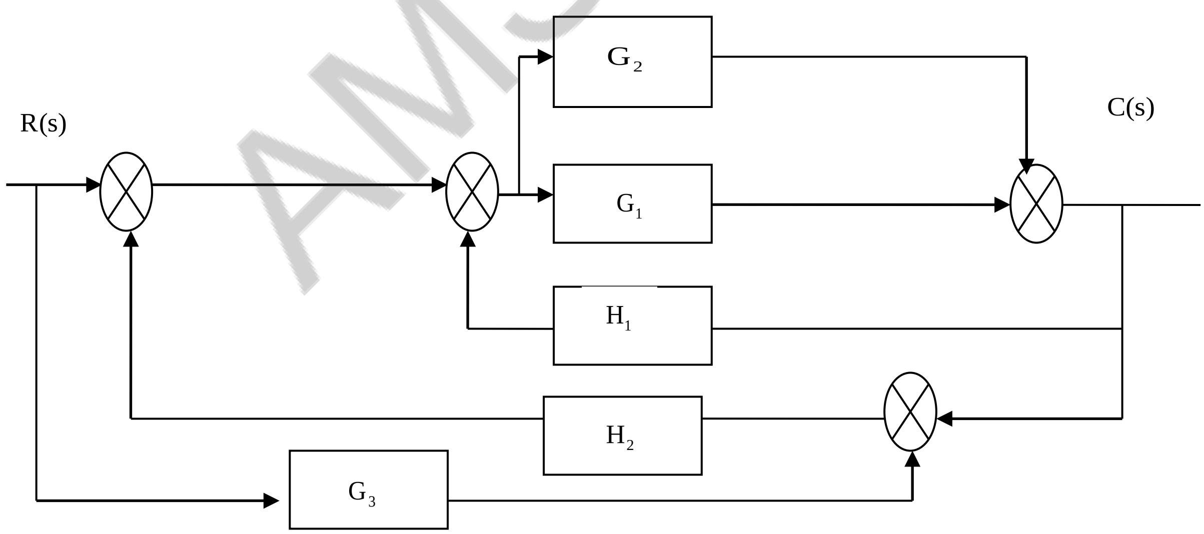


Combining the parallel paths,

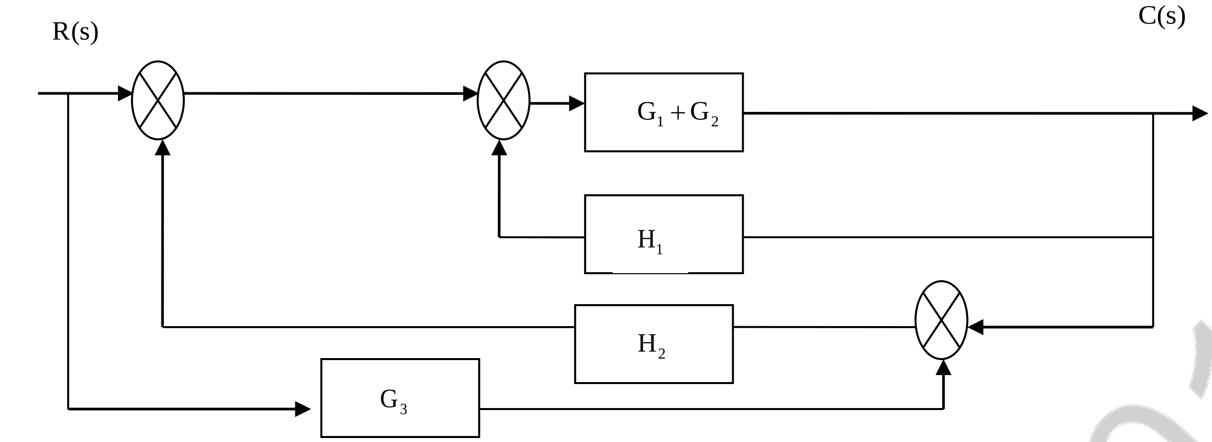


$$\frac{C}{R} = \frac{G_1G_2G_3+G_4(1+G_2H_1+G_2G_3H_2-G_1G_2H_1)}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1}$$

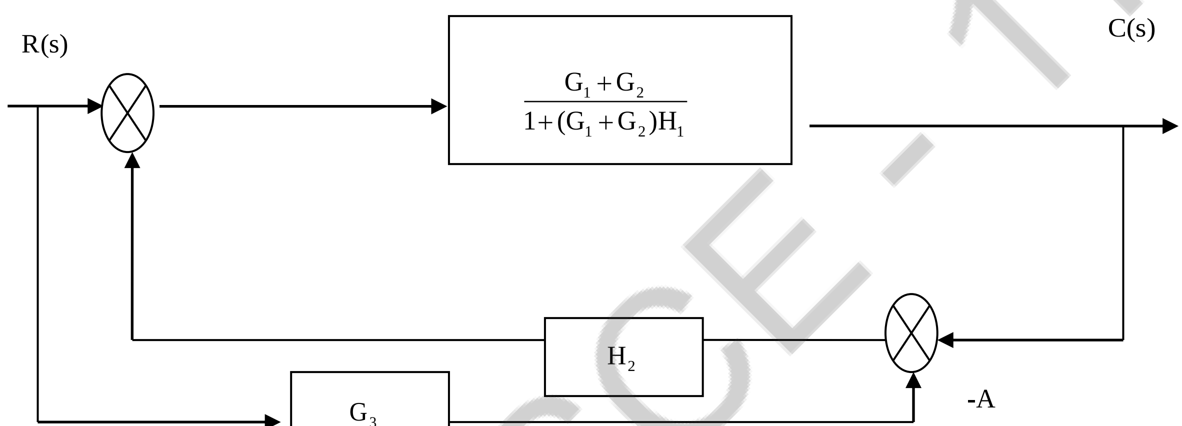
8.Find C(s)/R(s) of the system shown in fig using block diagram reduction technique



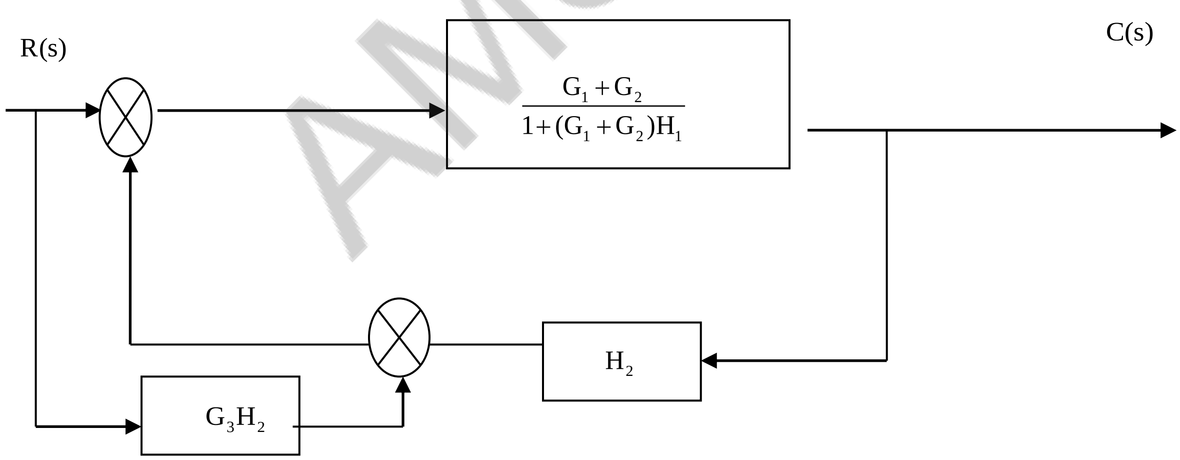
Sol: Combining the parallel blocks G_2 and G_1



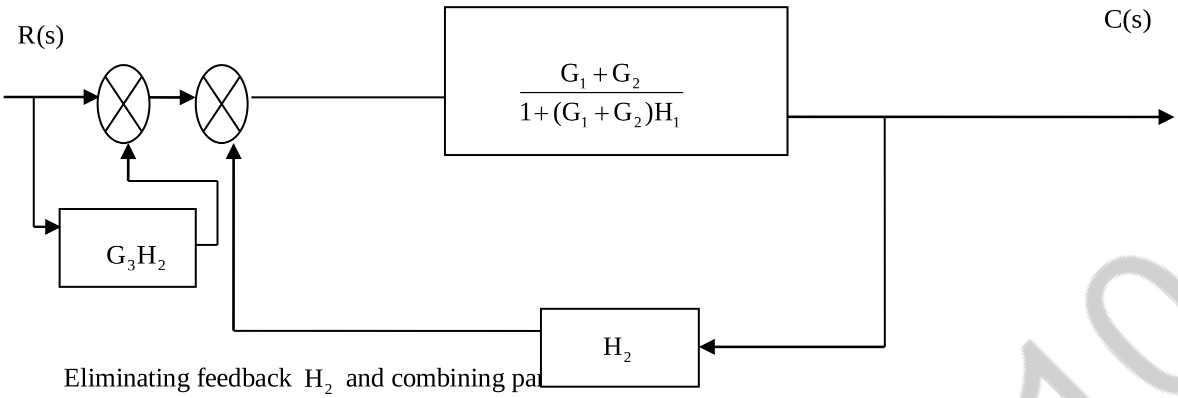
Eliminating the feedback H_1



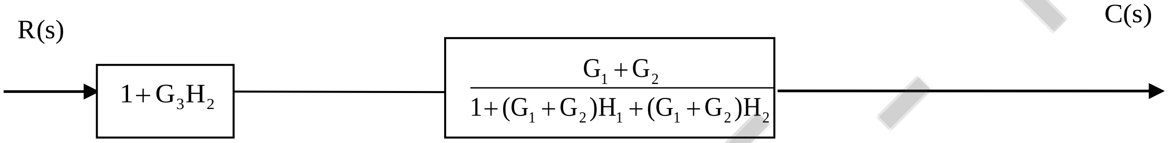
Moving the summing point (a) ahead of H_2



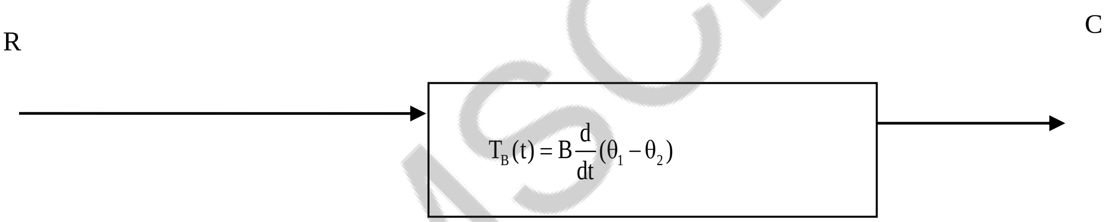
Eliminating the summing point by multiply signs



Eliminating feedback H_2 and combining pa

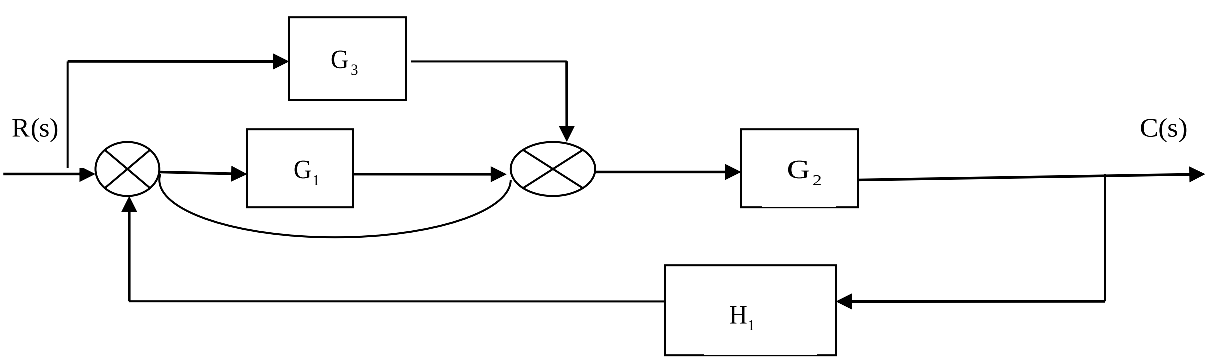


Combining the cascade blocks

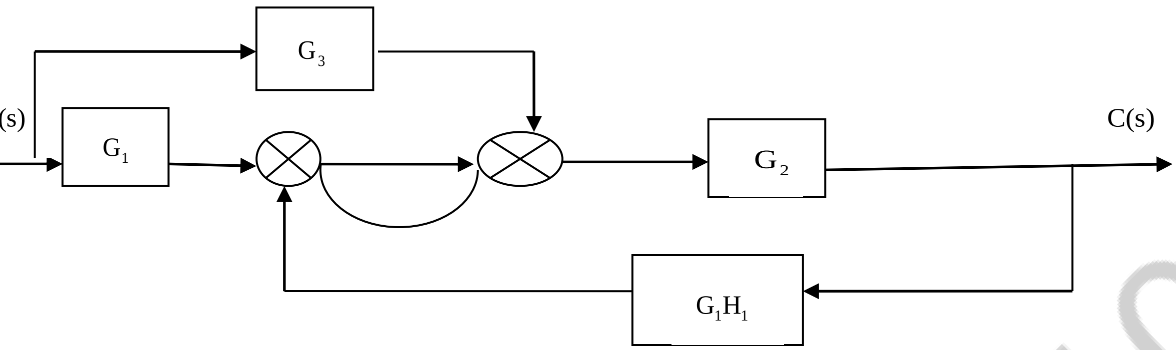


$$\frac{C(s)}{R(s)}=\frac{(1+G_3H_2)(G_1+G_2)}{1+(G_1+G_2)H_1+(G_1+G_2)H_2}$$

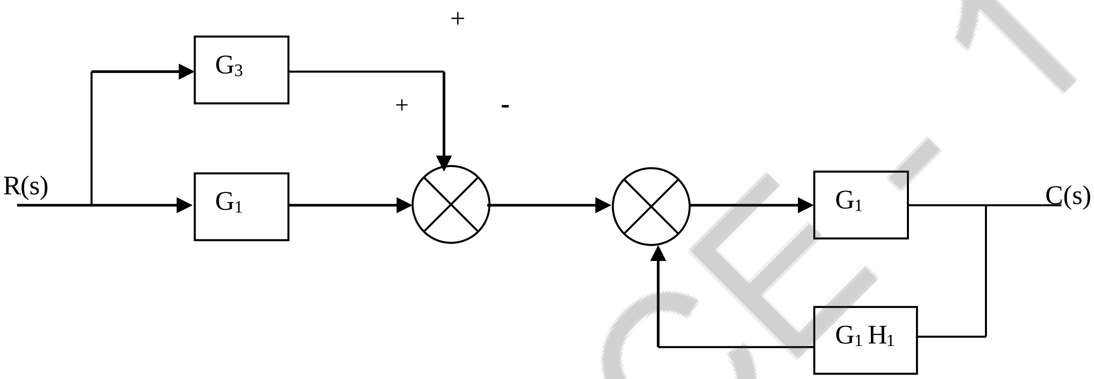
9. Find C(s)/R(s) of the system shown in fig. using block diagram reduction technique.



Sol: Moving the summing point a ahead of G_1

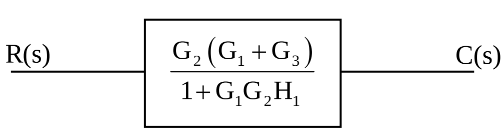


Inter changing the summing points,



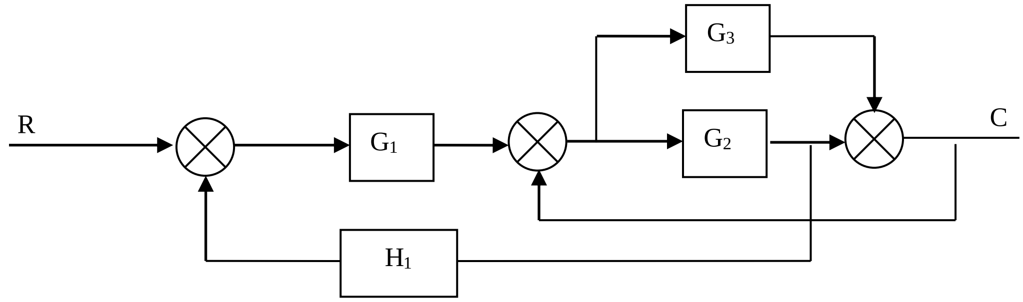
Combining the parallel blocks & Eliminating the feedback path G_1H_1.

Combining the cascade blocks

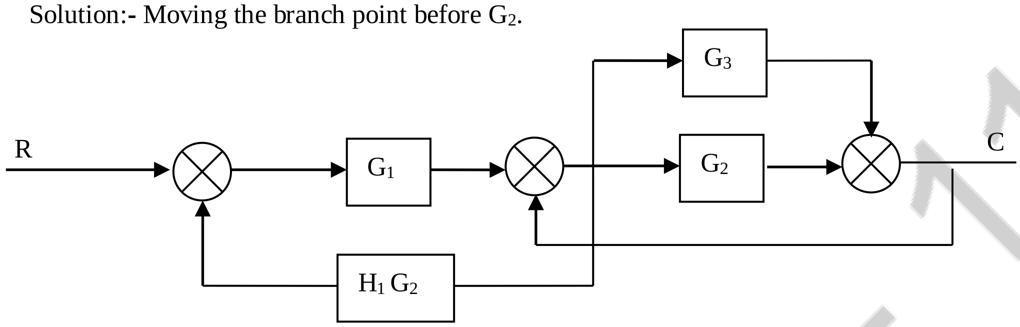


$$\frac{C(s)}{R(s)} = \frac{G_2(G_1 + G_3)}{1 + G_1G_2H_1}$$

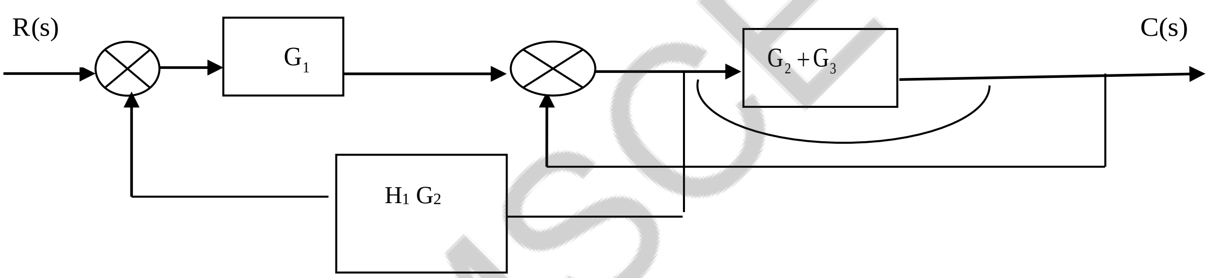
10. Determine the transfer function $\frac{C(s)}{R(s)}$ for the following block diagram



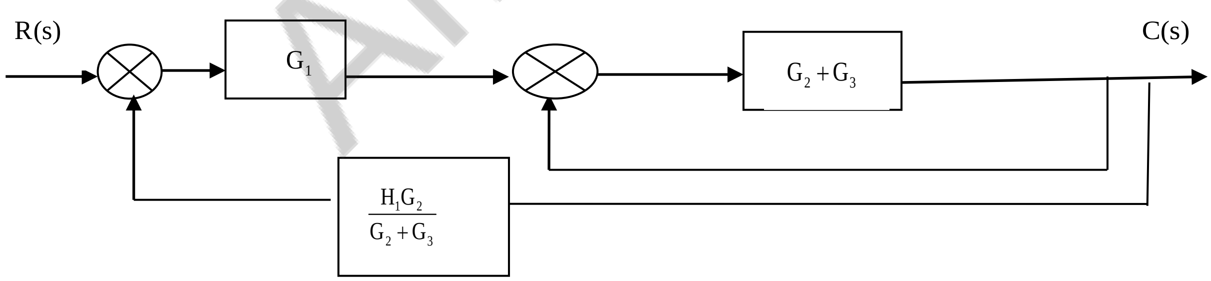
Solution:- Moving the branch point before G_2.



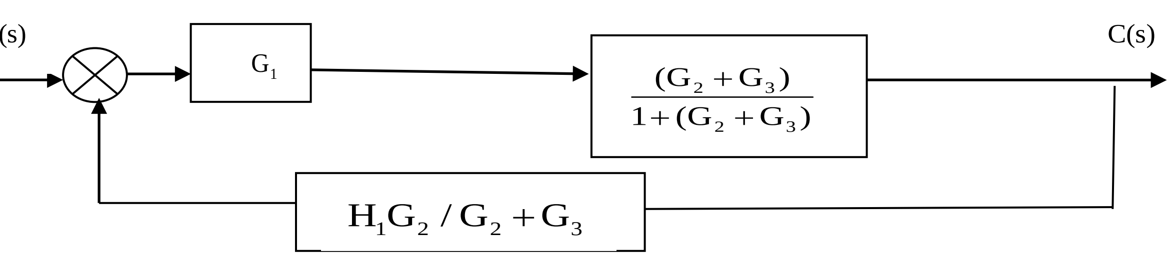
Combining the parallel blocks



Moving the branch point ahead of $(G_2 \text{and} G_3)$



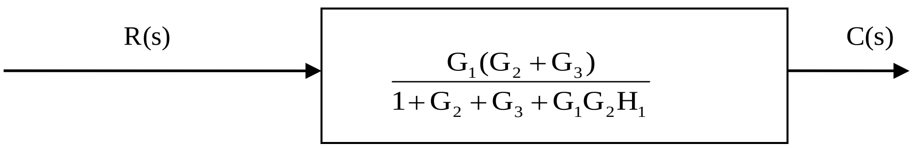
Eliminating the unity feedback path,



Combining the cascade blocks and eliminating the feedback

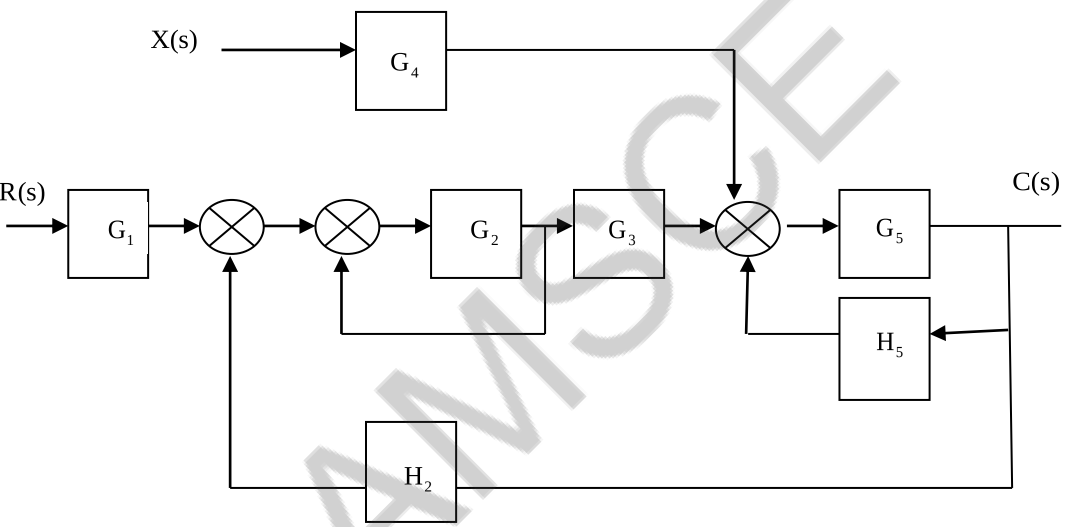
$$G \rightarrow \frac{G_1(G_2+G_3)}{1+(G_2+G_3)}, \quad H \rightarrow \frac{H_1G_2}{G_2+G_3}$$

$$\frac{G}{1+GH} = \frac{\dfrac{G_1(G_2+G_3)}{1+(G_2+G_3)}}{1+\dfrac{G_1(\cancel{G_2+G_3})}{1+(G_2+G_3)}\cdot\dfrac{H_1G_2}{\cancel{G_2+G_3}}} = \frac{G_1(G_2+G_3)}{1+G_2+G_3+G_1G_2H_1}$$

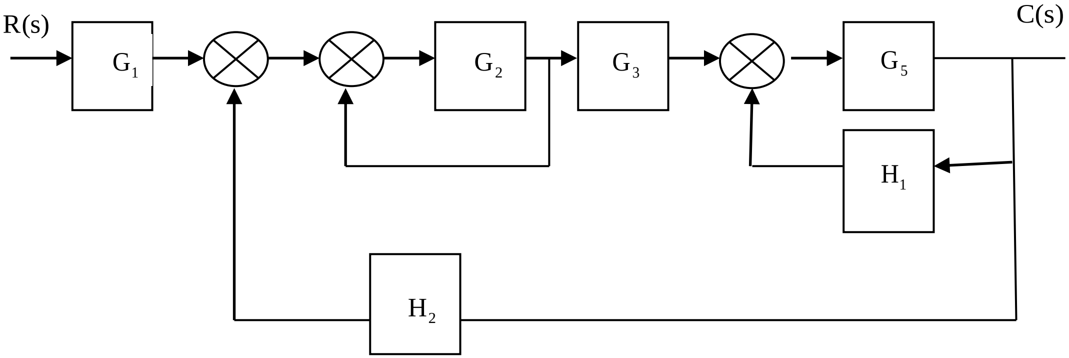


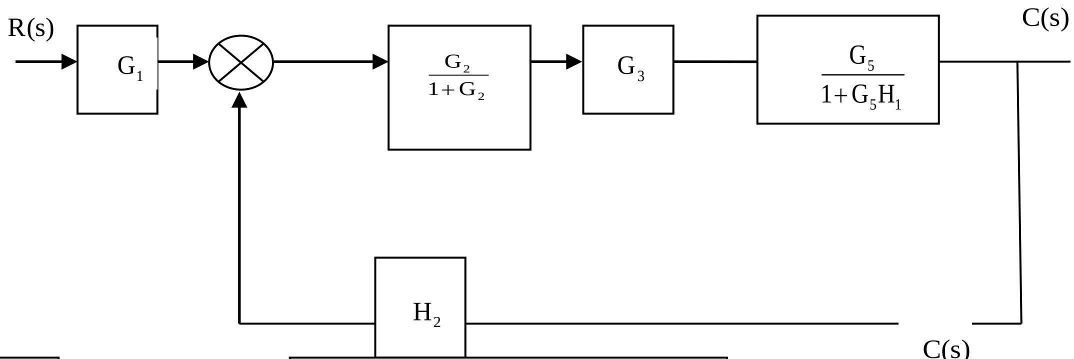
$$\frac{C(S)}{R(S)} = \frac{G_1(G_2+G_3)}{1+G_2+G_3+G_1G_2H_1}$$

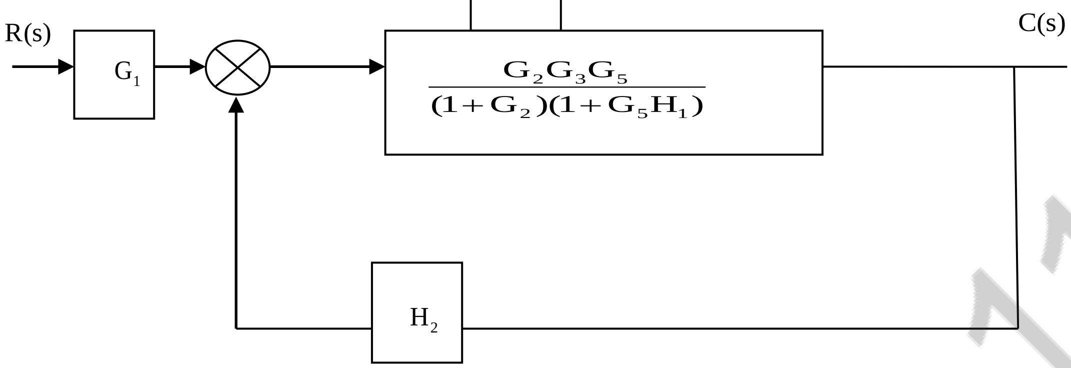
11. Using block diagram reduction technique find the transfer function from each input to the output C for the system shown in fig.

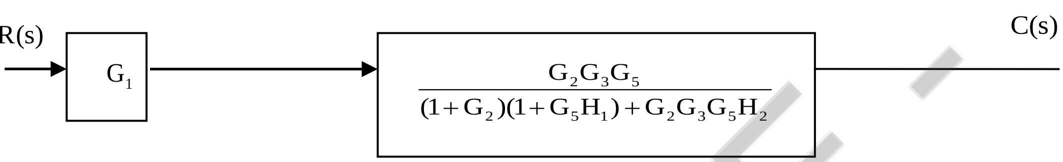


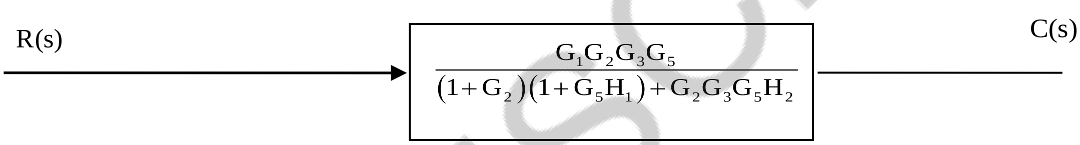
Sol: To find $\frac{C(s)}{R(s)} \rightarrow \text{put } X(s) = 0$





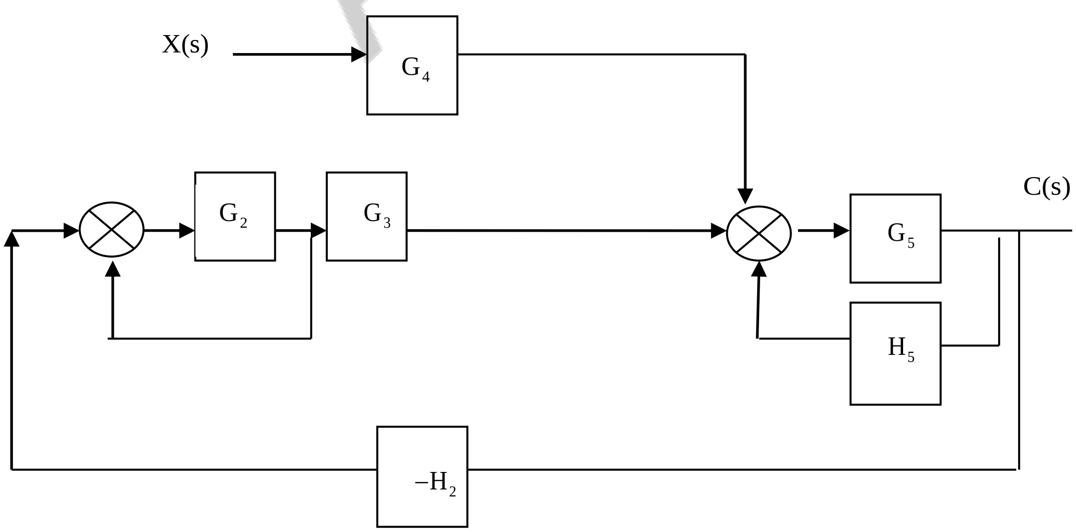


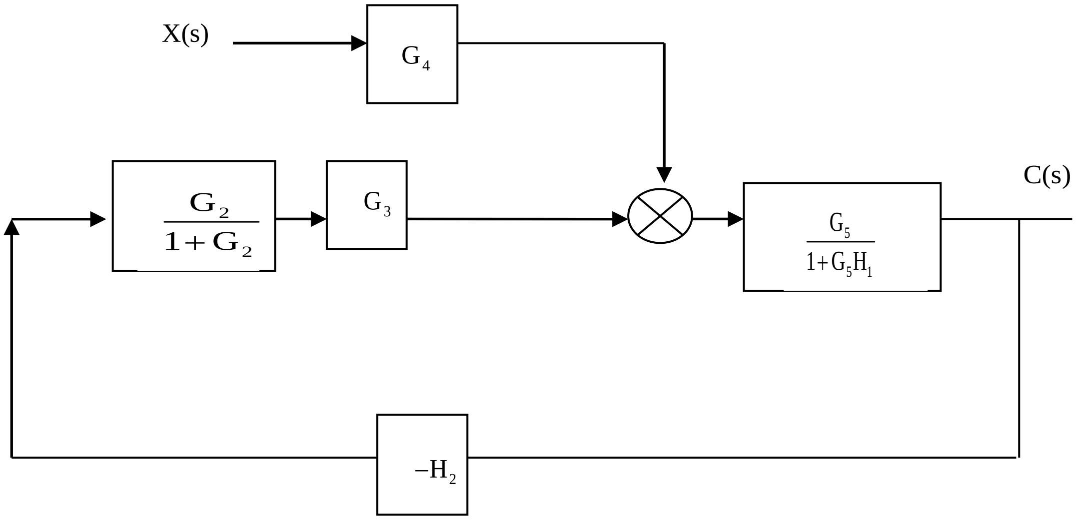


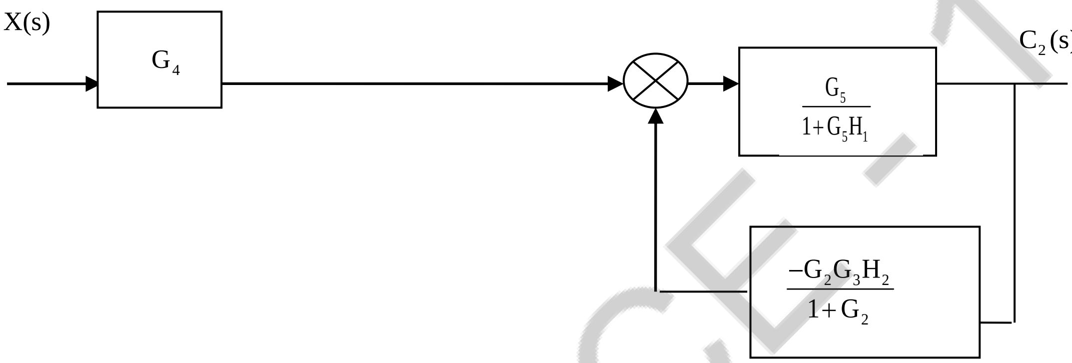


$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_5}{(1+G_2)(1+G_5H_1)+G_2G_3G_5H_2}$$

To find $\frac{C_2(s)}{X(s)}$ put R(s)=0,

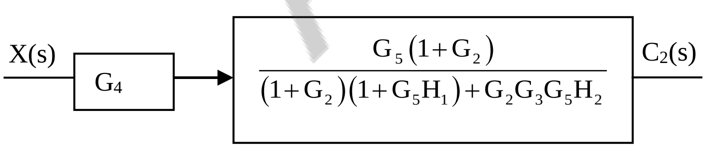


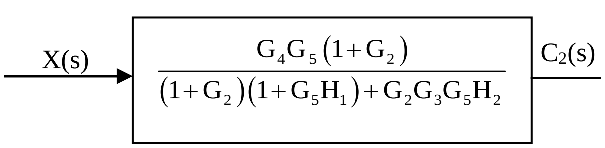




$$G \to \frac{G_5}{1+G_5H_1}, \quad H \to -\frac{G_2G_3H_2}{1+G_2}$$

$$\frac{G}{1+GH} = \frac{\frac{G_5}{1+G_5H_1}}{1+\frac{G_5}{1+G_5H_1}\frac{G_2G_3H_2}{(1+G_2)}} = \frac{G_5(1+G_2)}{(1+G_2)(1+G_5H_1)+G_2G_3G_5H_2}$$





$$\frac{C_2(s)}{X(s)} = \frac{G_4G_5\left(1+G_2\right)}{\left(1+G_2\right)\left(1+G_5H_1\right)+G_2G_3G_5H_2}$$

When both R(s) and X(s) are simultaneously present, the output

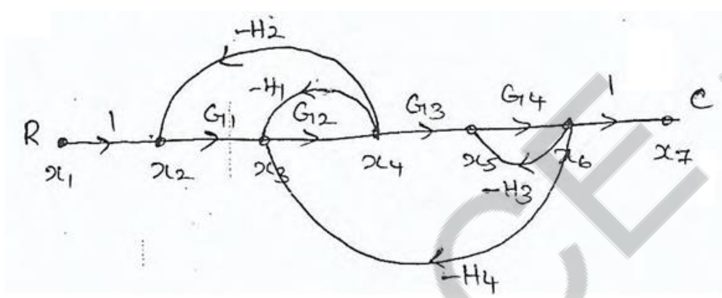
$C(s) = C_1(s) + C_2(s)$ as per superposition theorem

Hence $C(s) = \dfrac{R(s)G_1G_2G_3G_5 + X(s)G_4G_5\left(1+G_2\right)}{\left(1+G_2\right)\left(1+G_5H_1\right)+G_2G_3G_5H_2}$

SIGNAL FLOW GRAPH

PROBLEMS

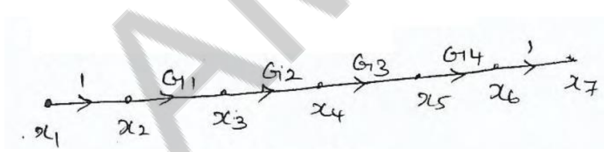
1. Obtain the overall transfer function of the following signal flow graph using Mason's gain formula



Sol:

Step 1:Forward path gains No. of forward path K=1

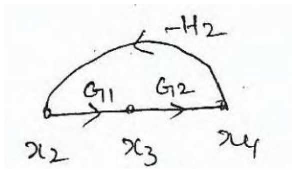
Forward path gain path $\rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow x_6 \rightarrow x_7$

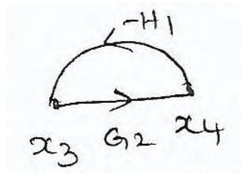


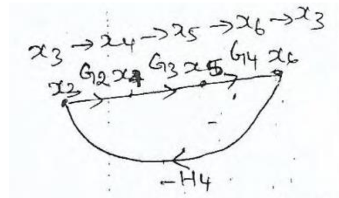
$P_1 = G_1G_2G_3G_4$

Step 2: Individual loop gains

$x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_2$ $\qquad$ $x_3 \rightarrow x_4 \rightarrow x_3$ $\qquad$ $x_3 \rightarrow x_2 \rightarrow x_5 \rightarrow x_6 \rightarrow x_3$

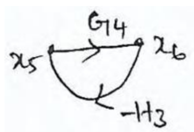






$P_{11} = -G_1G_2H_2$ $\quad$ $P_{21} = -G_2H_1$ $\quad$ $P_{31} = -G_2G_3G_4H_4$

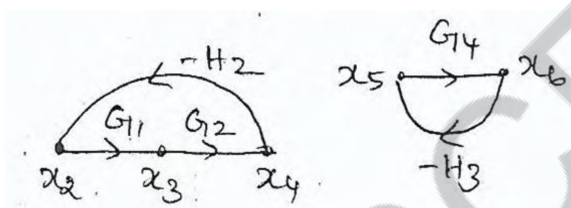
$x_5 \to x_6 \to x_5$



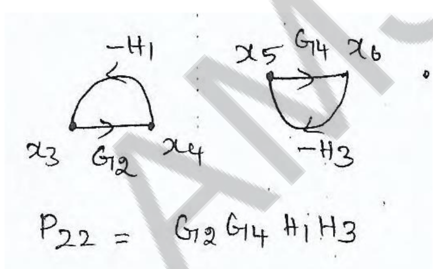
$P_{41} = -G_4H_3$

Step 3: Non touching loops →Gain products

There are two pairs of non touching loops



$P_{12} = G_1G_2G_4H_2H_3$



$P_{22} = G_2G_4H_1H_3$

Step 4: To find Δ and Δk

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41}] + [P_{12} + P_{22}]$$
$$= 1 + G_1G_2H_2 + G_2H_1 + G_4H_3 + G_2G_3G_4H_4 + G_1G_2G_4H_2H_3 + G_2G_4H_1H_3$$

$\Delta_1 = 1$ Since there in no part of the graph is not touching with first forward path

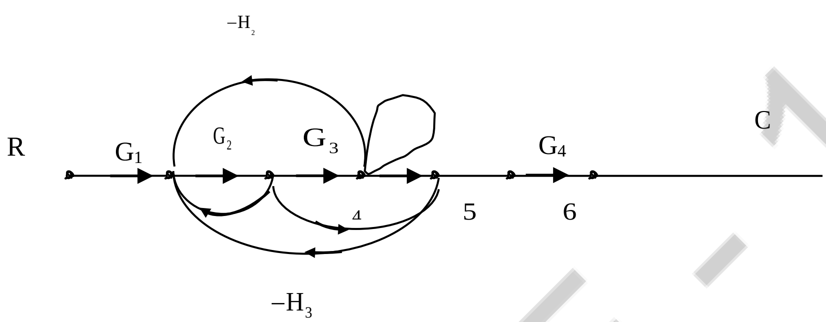
Step 5: Transfer function

By Mason's gain formula

$$T(s) = \frac{1}{\Delta}\sum_{k=1}^{n} P_k\Delta_k = \frac{1}{\Delta}\left(P_1\Delta_1\right)$$

$$T(s) = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_2 + G_2H_1 + G_4H_3 + G_2G_3G_4H_4 + G_1G_2G_4H_2H_3 + G_2G_4H_1H_3}$$

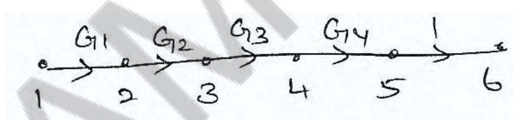
2.Find the overall gain of the system whose signal flow graph is shown in fig. Nov/Dec 2017



Sol: Step 1: Forward path gain No. Of forward path K=2

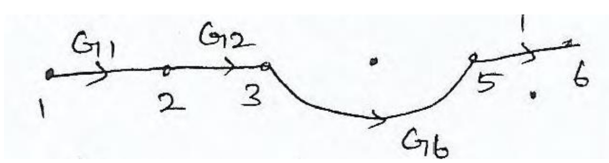
Forward path gain

Path 1 : 1→2→3→4→5→6



$P_1 = G_1G_2G_3G_4$

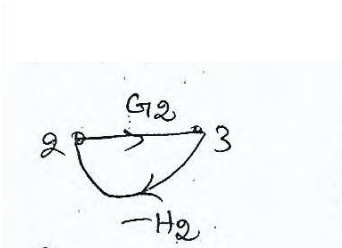
Path 2: 1→2→3→5→6

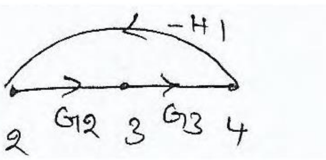


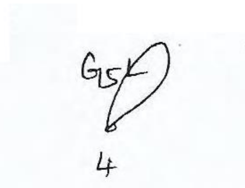
$P_2 = G_1G_2G_6$

Step 2: Individual loop gain

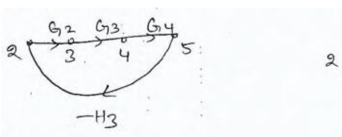
$2 \to 3 \to 2$ $\quad\quad$ $2 \to 3 \to 4 \to 2$ $\quad\quad$ $4 \to 4$

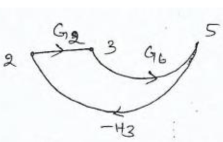






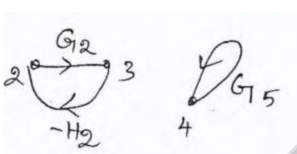
$P_{11} = -G_2H_2$ $\quad\quad$ $P_2 = -G_2G_3H_1$ $\quad\quad$ $P_{31} = G_5$

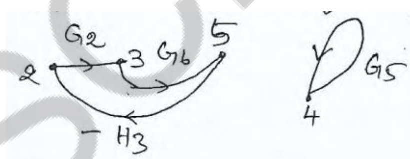




$P_{41} = -G_2G_3G_4H_3$ $\quad\quad$ $P_{51} = -G_2G_6H_3$

Step 3: Non touching loops – gain products. There are two pairs of non touching loops





$P_{12} = -G_2G_5H_2$ $\quad\quad$ $P_{22} = -G_2G_5G_6H_3$

Step 4: To find Δ and Δk

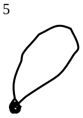
Δ=1-[$P_{11} + P_{21} + P_{31} + P_{41} + P_{51}$]+[$P_{12} + P_{22}$]

$$\Delta = 1 + G_2H_2 + G_2G_3H_1 - G_5 + G_2G_3G_4H_3 + G_2G_6H_3 - G_2G_5H_2 - G_2G_5G_6H_3$$

$\Delta_1 = 1 - 0 = 1$ Since there is no part of the graph is not touching with first forward path

$\Delta_2 = 1 - G_5 \to$ Since when forward path 2 being removed remaining part of the graph is as shown

G_5



3 4 5

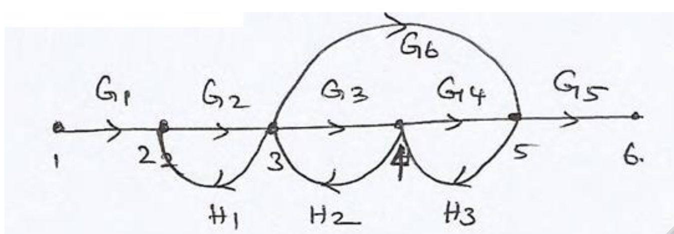
STEP 5: Transfer function:

By Mason's gain formula

$$T(S) = \frac{1}{\Delta}\sum_K P_K \Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2)$$

$$\therefore T(S) = \frac{G_1G_2G_3G_4 + G_1G_2G_6(1-G_5)}{1+G_2H_2+G_2G_3H_1-G_5+G_2G_3G_4H_3+G_2G_6H_3-G_2G_5H_2-G_2G_5G_6H_3}$$

3.The signal flow graph for a feedback control system is shown in fig. Determine the closed loop transfer function C(s)/R(s). Nov/Dec 2015

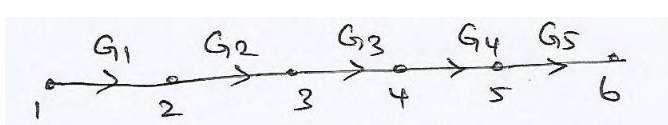


Sol:

Step : 1 forward path gains

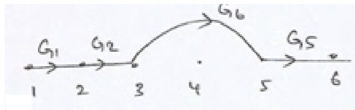
No. of forward path K=2

Path 1 1→2→3→4→5→6



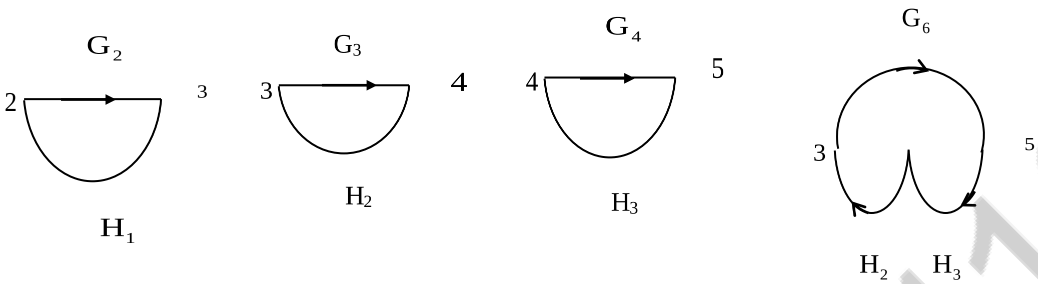
$$P_1 = G_1G_2G_3G_4G_5$$

Path 2 $1 \to 2 \to 3 \to 4 \to 5 \to 6$



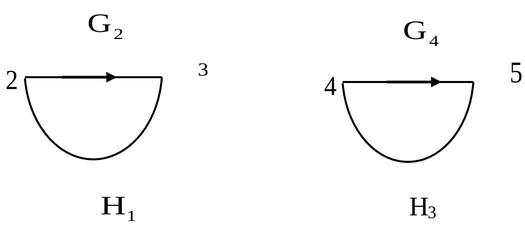
$$P_2 = G_1 G_2 G_5 G_6$$

Step 2: Individual loop gains



$P_{11} = G_2 H_1$ $P_{21} = G_3 H_2$ $P_{31} = G_4 H_3$ $P_{41} = G_6 H_2 H_3$

Step 3: Non touching loops gain products



$$P_{12} = G_2 G_4 H_1 H_2$$

Step : 4 To find $\Delta + \Delta_k$

$$\begin{aligned}
\Delta &= 1 - [P_{11} + P_{21} + P_{31} + P_{41}] + [P_{12}] \\
&= 1 - G_2 H_1 - G_3 H_2 - G_4 H_3 - G_6 H_2 H_3 + G_2 G_4 H_1 H_3 \\
\Delta_1 &= 1 - 0 = 1 \\
\Delta_2 &= 1 - 0 = 1
\end{aligned}$$

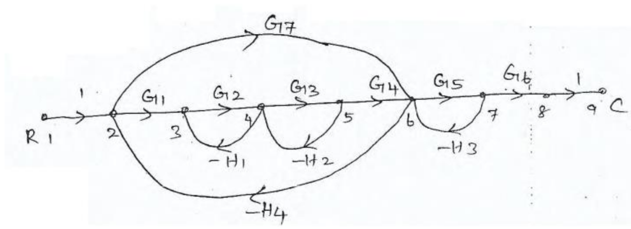
Since there is no part of the graph is not touching with first and second forward path respectively.

Step 5: Transfer function by Mason's gain formula

$$T(s) = \frac{1}{\Delta} \sum_k P_k \Delta_k - \frac{1}{\Delta}(P_1 \Delta_1 + P_2 \Delta_2)$$

$$\therefore T(s) = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_5 G_6}{1 - G_2 H_1 - G_3 H_2 - G_4 H_3 - G_6 H_2 H_3 + G_2 G_4 H_1 H_3}$$

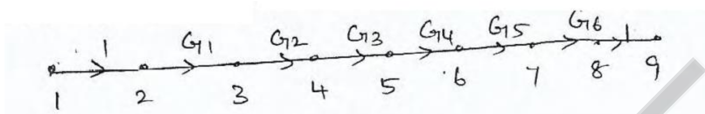
4.Obtain the overall transfer function of the following signal flow graph using Masons gain formula.



Sol: Step 1 forward path gain

No. of forward path K=2

Path 1→1→2→3→4→5→6→7→8→9



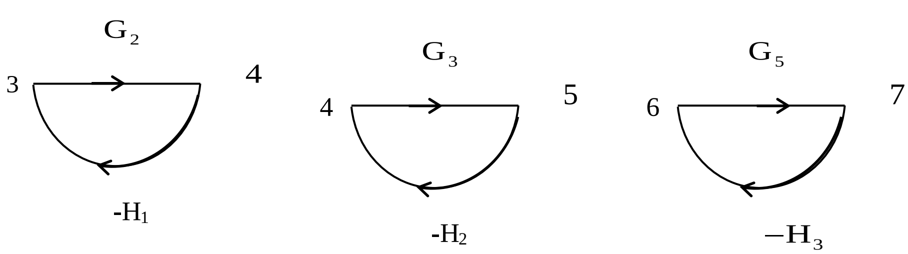
$P_1 = G_1G_2G_3G_4G_5G_6$

Path 2 →1→2→6→7→8→9

$P_2 = G_5G_6G_7$

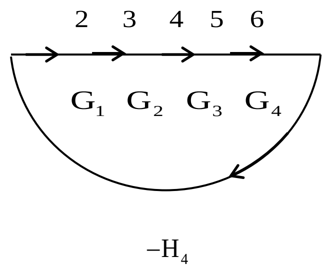
STEP:2 individual loop gains

3→4→3 4→5→4 6→7→6



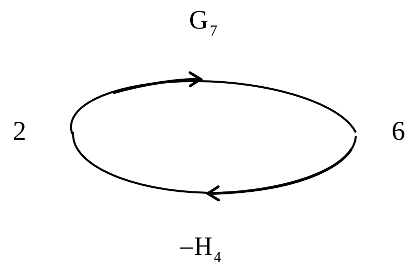
$\boxed{P_{11} = -G_2H_1}$ $\boxed{P_{21} = -G_3H_2}$ $\boxed{P_{31} = -G_5H_3}$

$2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 2$



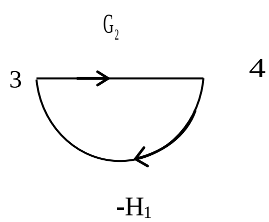
$\boxed{P_{41} = -G_1G_2G_3G_4H_4}$

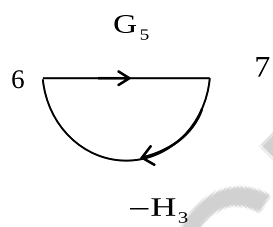
$2 \rightarrow 6 \rightarrow 2$



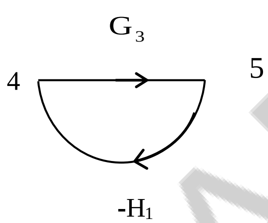
$\boxed{P_{51} = -G_7H_4}$

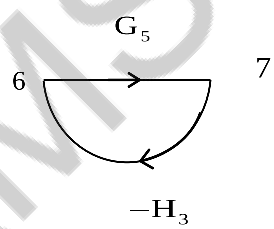
Step:3 Gain productions of non touching loops



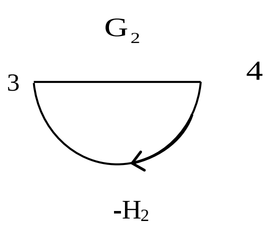


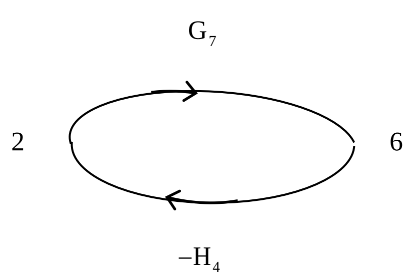
$P_{12} = G_2G_5H_1H_3$



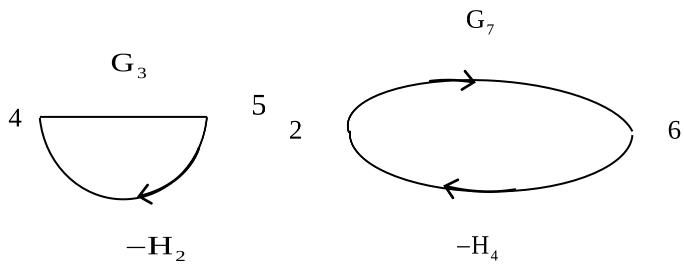


$P_{22} = G_3G_5H_2H_3$





$$P_{32} = G_2G_7H_1H_4$$



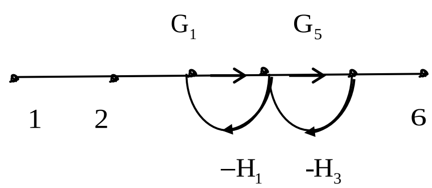
$$P_{42} = G_3G_7H_2H_4$$

Step:4 Determination of Δ & Δk

Δ_1=1; Since there is no part of the graph is not touching with first forward path

Δ_1=1-[L_1+ L_2]

$= 1 + G_1H_1 + G_5H_3$



Individual loops

$L_1 = -G_1H_1$

$L_2 = -G_5H_3$

$$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}] + [P_{12} + P_{22} + P_{32} + P_{42}]$$

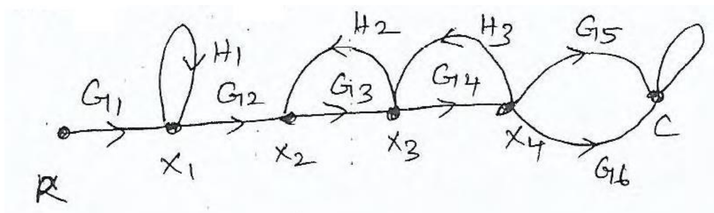
$$= 1 + G_2H_1 + G_3H_2 + G_5H_3 + G_1G_2G_3G_4H_4 + G_7H_4 + G_2G_5H_1H_3 + G_3G_5H_2H_3 + G_2G_7H_1H_4 + G_3G_7H_2H_4$$

Step:5 Transfer function by Hason's gain formula

$$T(s)=\sum_k \frac{P_k\Delta_k}{\Delta}=\frac{P_1\Delta_1+P_2\Delta_2}{\Delta}$$

$$=\frac{G_1G_2G_3G_4G_5G_6+G_5G_6G_7\left(1+G_1H_1+G_5H_3\right)}{1+G_2H_1+G_3H_2+G_5H_3+G_1G_2G_3G_4H_4+G_7H_4+G_2G_5H_1H_3+G_3G_5H_2H_3+G_2G_7H_1H_4+G_3G_7H_2H_4}$$

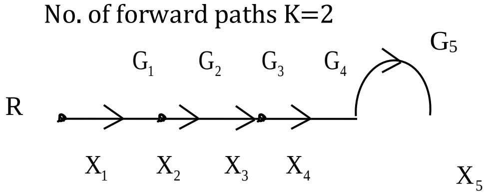
5.Determine the overall transfer function of SFG using Mason's gain formula



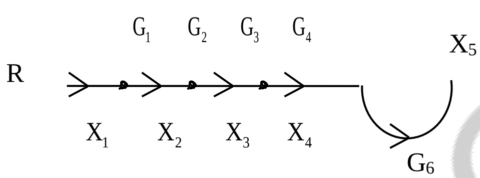
Sol

Step1: forward path gain

No. of forward paths K=2

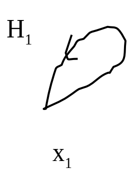


$P_1=G_1G_2G_3G_4G_5$

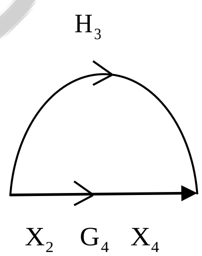


$P_1=G_1G_2G_3G_4G_6$

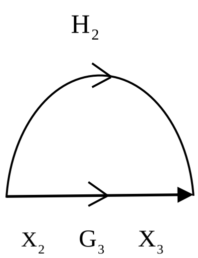
Step2: Individual loop gains



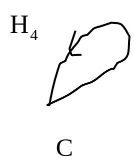
$P_{11}=H_1$



$P_{31}=G_4H_3$



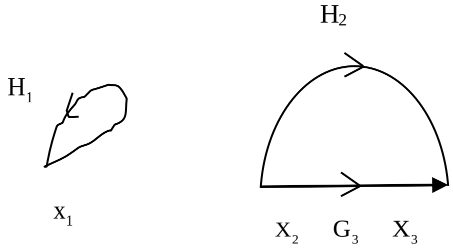
$P_{21}=G_3\,H_2$



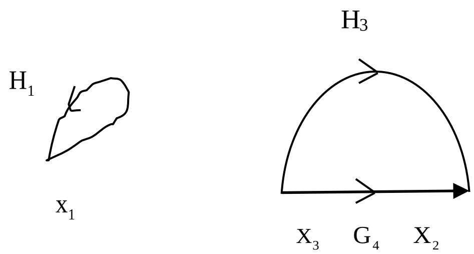
$P_{41}=H_4$

Step:3 Gain products of non touching loops Two non touching loops

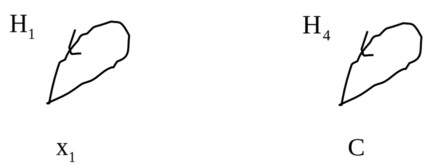
Two non touching loops



$P_{12} = H_1H_2G_3$

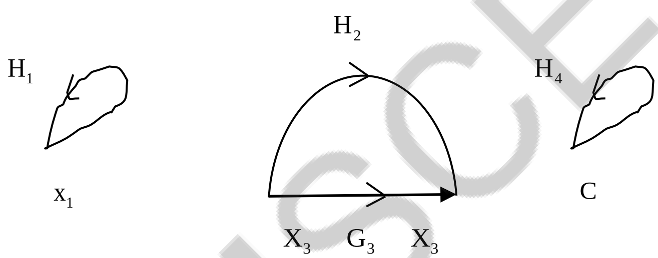


$P_{22} = H_1H_3G_4$

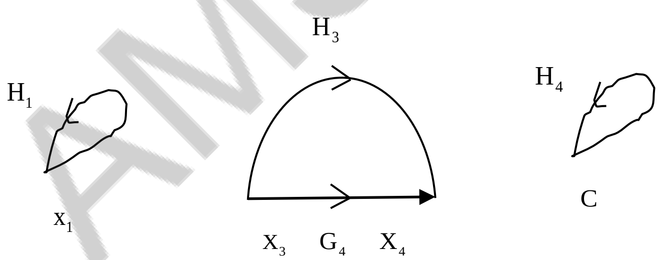


$P_{32} = H_1H_4$

Three non touching loops



$P_{13} = G_3H_1H_2H_4$



$P_{23} = G_4H_1H_3H_4$

Step :4 Determination of Δ and Δk

$$= 1 - [P_{11} + P_{21} + P_{31} + P_{41}] + [P_{12} + P_{22} + P_{32}] - [P_{13} + P_{23}]$$
$$= 1 - H_1 - G_3H_2 - G_4H_3 - H_4 + H_1H_2G_3 + H_1H_3G_4 + H_1H_4 - G_3H_1H_2H_4 - G_4H_1H_3H_4$$

$\Delta_1 = \Delta_2 = 1$ Since there is no part of the graph is not touching with forward path s

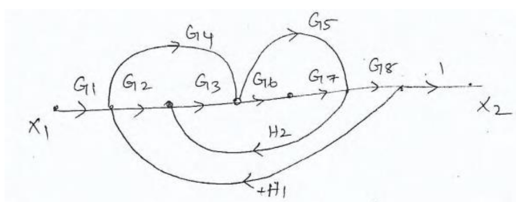
Step:5 Transfer function

By Mason's gain formula $T(s) = \sum_k \frac{p_k \Delta_k}{\Delta}$

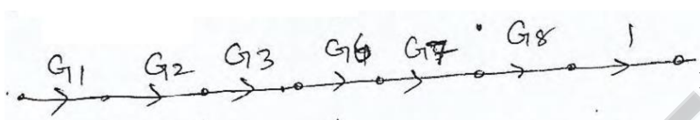
$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$T(s) = \frac{G_1G_2G_3G_4G_5 + G_1G_2G_3G_4G_6}{1 - H_1 - G_3H_2 - G_4H_3 - H_4 + H_1H_2G_3 + H_1H_3G_4 + H_1H_4 - G_3H_1H_2H_4 - G_4H_1H_3H_4}$$

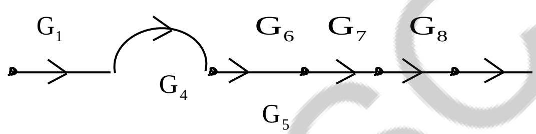
6. Using Mason's gain formula to find $\frac{x_2}{x_1}$



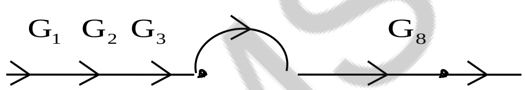
1. No. of forward paths k=4



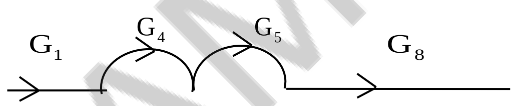
$P_1 = G_1G_2G_3G_6G_7G_8$



$P_2 = G_1G_4G_6G_7G_8$

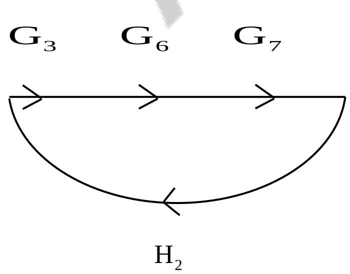


$P_3 = G_1G_2G_3G_5G_8$

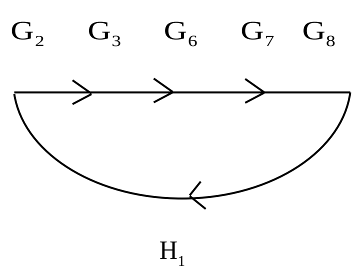


$P_4 = G_1G_4G_5G_8$

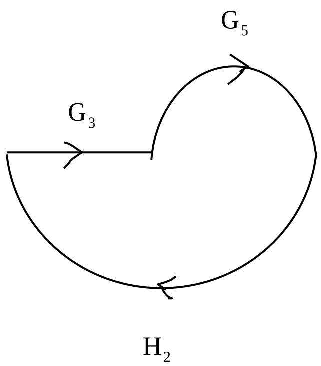
2. Individual loops and gains



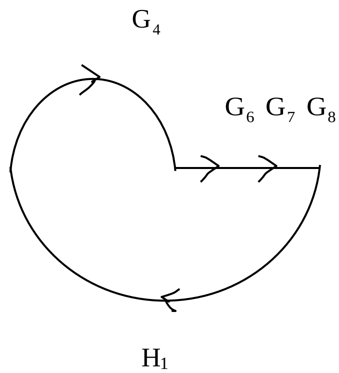
$P_{11} = G_2G_6G_7H_2$



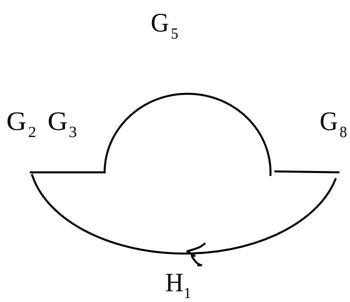
$P_{21} = G_2G_3G_6G_7G_8H_1$



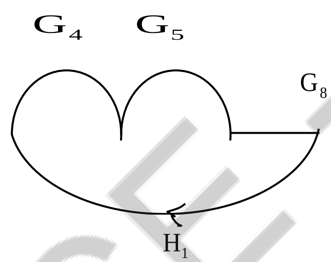
$P_{31} = G_3G_5H_2$



$P_{41} = G_4G_6G_7G_8H_1$



$P_{51} = G_2G_3G_5G_8H_1$



$P_{61} = G_4G_5G_8H_1$

Non touching loops $\rightarrow$Nil

Δ and Δk

Δ=1-$[P_{11} + P_{21} + P_{31} + P_{41} + P_{51} + P_{61}]$

=1-$G_2G_6G_7H_2 + G_2G_3G_6G_8H_1 - G_3G_5H_2 - G_4G_6G_7G_8H_1 - G_2G_3G_5G_8H_1 - G_4G_5G_8H_1$

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1$

$\Delta_4 = 1$

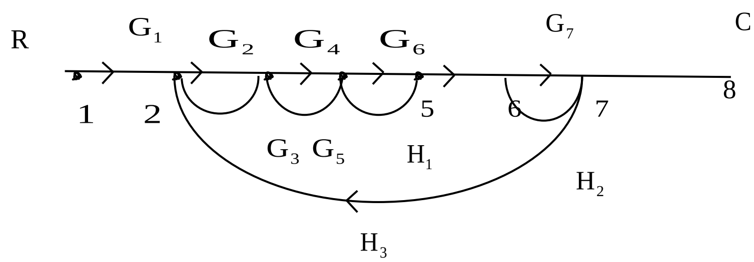
Since there is no part of the graph is not touching with forward paths

Transfer function by Mason's gain formula $T(s) = \sum_k \frac{P_k \Delta_k}{\Delta}$

$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

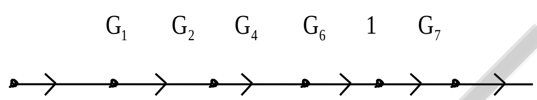
$$T(s) = \frac{G_1G_2G_3G_6G_7G_8 + G_1G_4G_6G_7G_8 + G_1G_2G_3G_5G_8 + G_1G_4G_5G_8}{1 - G_2G_6G_7H_2 - G_2G_3G_6G_7G_8H_1 - G_3G_5H_2 - G_4G_6G_7G_8H_1 - G_2G_3G_5G_8H_1 - G_4G_5G_8H_1}$$

7.Find L/R using Mason's gain formula

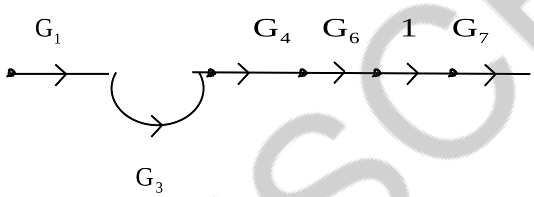


Sol :

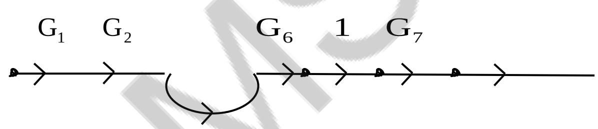
1. No. of forward path and forward path gains k=4.



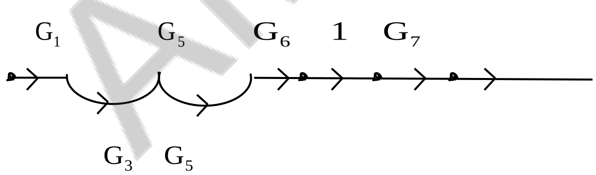
$P_1 = G_1G_2G_4G_6G_7$



$P_2 = G_1G_3G_4G_6G_7$

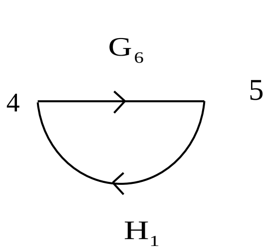


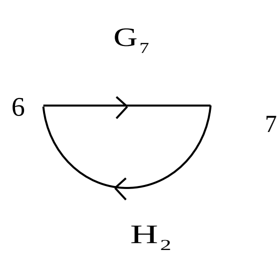
$P_3 = G_1G_2G_5G_6G_7$

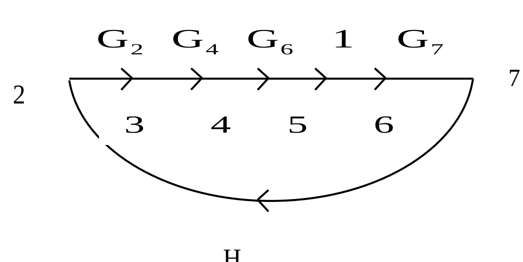


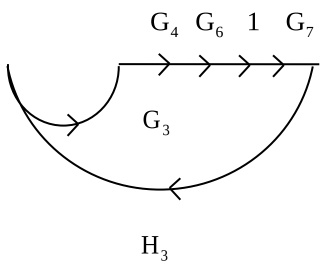
$P_4 = G_1G_3G_5G_6G_7$

2.Individual loops and gain

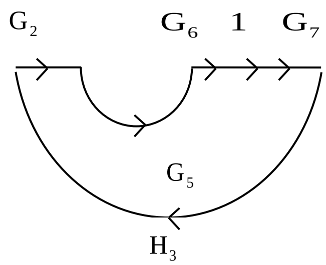




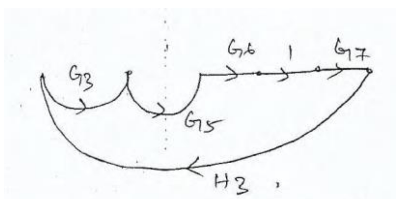




$P_{41} = G_3G_4G_6G_7H_3$

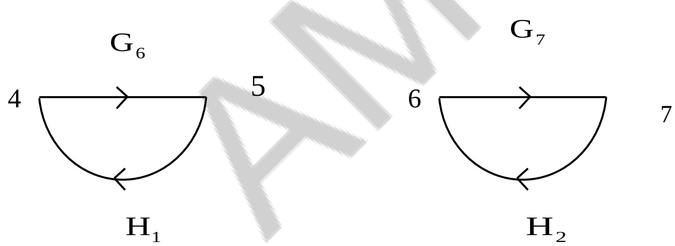


$P_{51} = G_2G_5G_6G_7H_3$



$P_{51} = G_3G_5G_6G_7H_3$

3.Non touching loops



$P_{12} = G_6G_7H_1H_2$

4. Δ and Δ_k

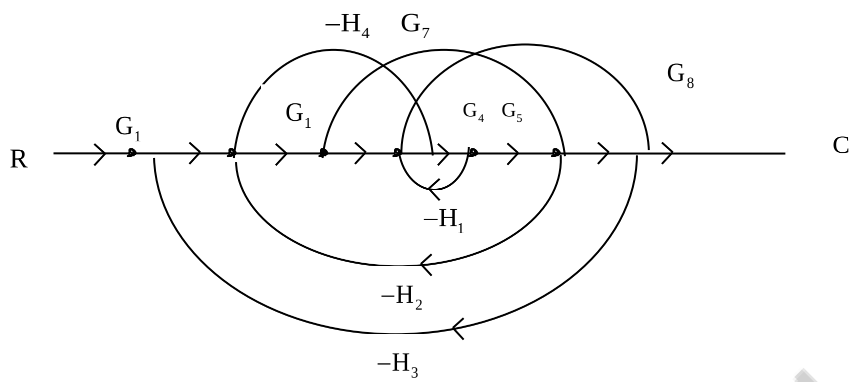
$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$ [Since there is no part of the graph is not touching the forward paths]

5.Transfer function by Mason's gain formula

$$T(S)=\sum_{K}\frac{p_K\Delta_K}{\Delta}=\frac{p_1\Delta_1+p_2\Delta_2+p_3\Delta_3+p_4\Delta_4}{\Delta}$$

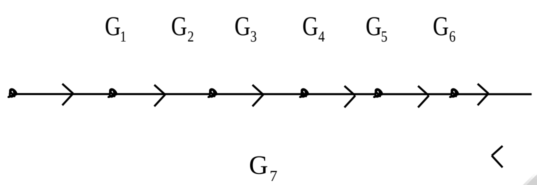
$$=\frac{G_1G_2G_4G_6G_7+G_1G_3G_4G_6G_7+G_1G_2G_5G_6G_7+G_1G_3G_5G_6G_7}{1-G_6H_1-G_7H_2-G_2G_4G_6G_7H_3-G_3G_4G_6G_7H_3-G_2G_5G_6G_7H_3-G_3G_5G_6G_7H_3+G_6G_7H_1H_2}$$

8. Find L/R using Mason's gain formula

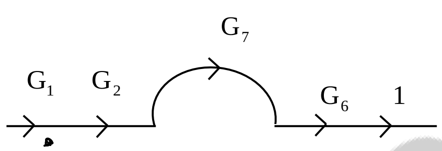


Sol

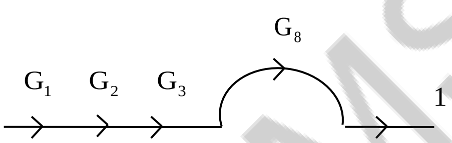
1. No. of forward path and gain K=3



$P_1=G_1G_2G_3G_4G_5G_6$

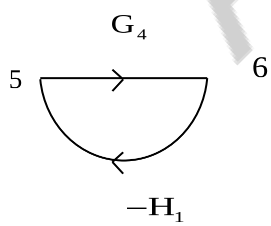


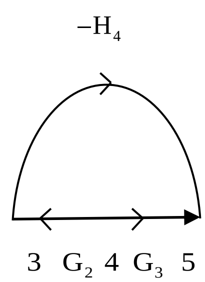
$P_2=G_1G_2G_7G_6$

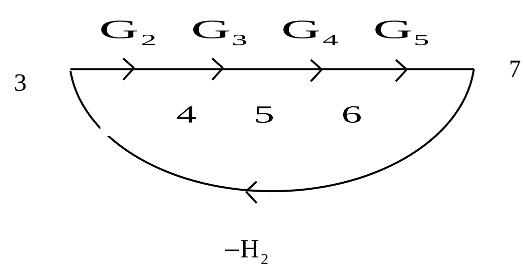


$P_3=G_1G_2G_3G_8$

2. Individual loops and gain



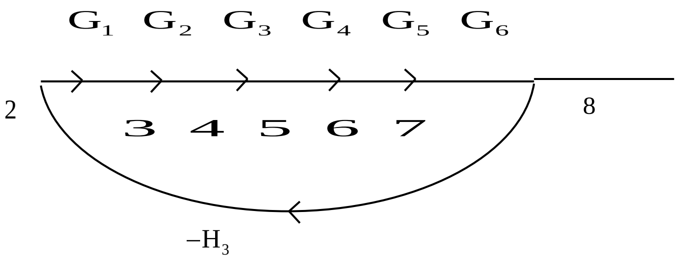




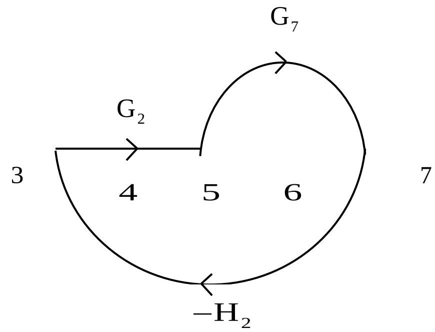
$P_{11}=-G_4H_1$

$P_{21}=-G_2G_3H_4$

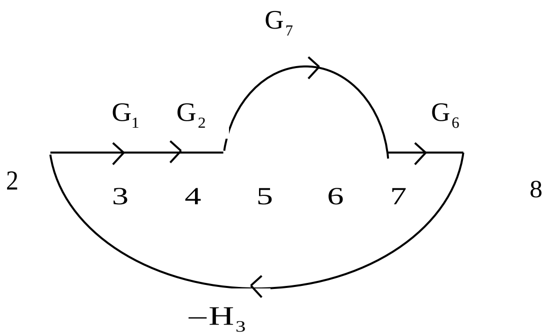
$P_{31}=-G_2G_3G_4G_5H_2$



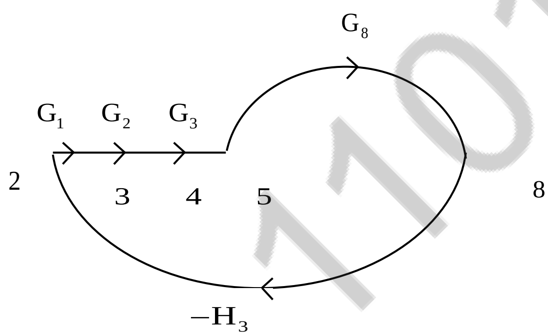
$P_{41} = -G_1G_2G_3G_4G_5G_6H_3$



$P_{51} = -G_2G_7H_2$

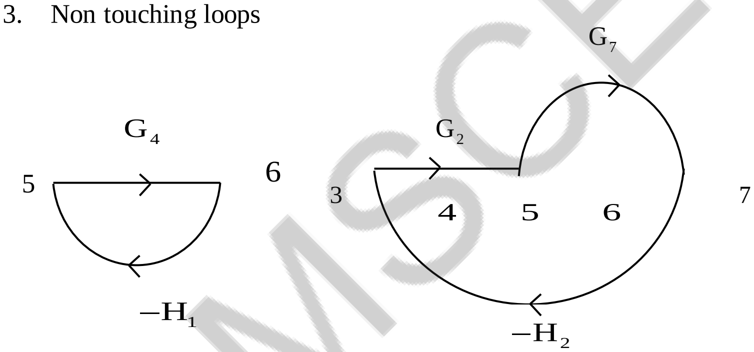


$P_{61} = -G_1G_2G_7G_6H_3$

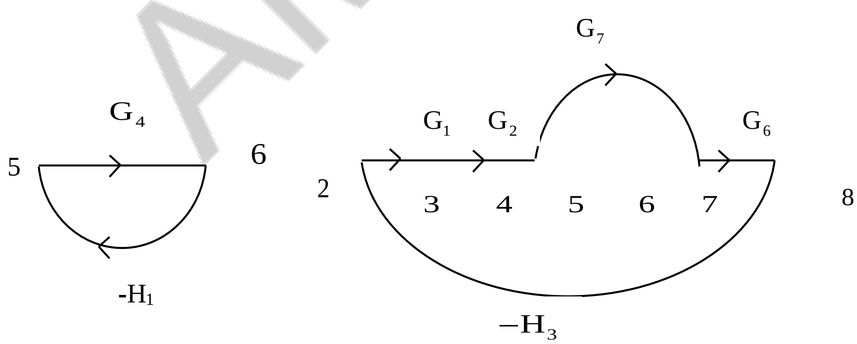


$P_{71} = -G_1G_2G_3G_8H_3$

3. Non touching loops



$P_{12} = G_4G_2G_7H_1H_2$



$P_{22} = G_1G_2G_4G_6G_7H_1H_3$

4. Δ and Δ_k

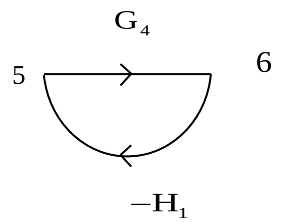
$\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51} + P_{61} + P_{71}] + [P_{12} + P_{22}\}$

$\Delta = 1 + G_4H_1 + G_2G_3H_4 + G_2G_3G_4G_5H_2 + G_1G_2G_3G_4G_5G_6H_3 + G_2G_7H_2 + G_1G_2G_7G_6H_3 + G_1G_2G_3G_8H_3 + G_2G_4G_7H_1H_2 + G_1G_2G$

$\Delta_1 = 1;$

$\Delta_3 = 1;$ since there is no part of the graph is not touching with forward path 1 and 3

$\Delta_2 = 1 + G_4H_1;$ when forward path 2 being removed, remaining part of the graph is as shown



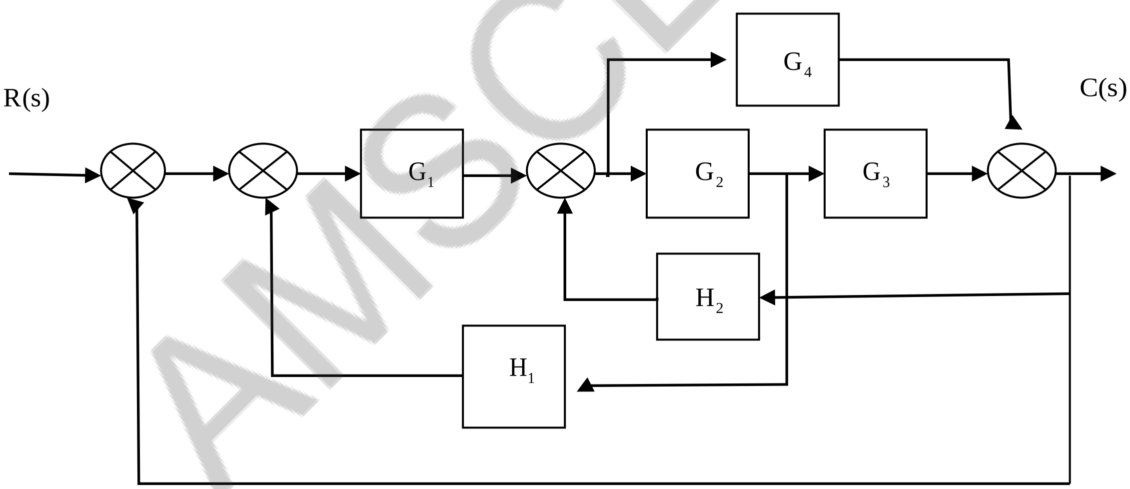
5.Transfer function By Mason's gain formula

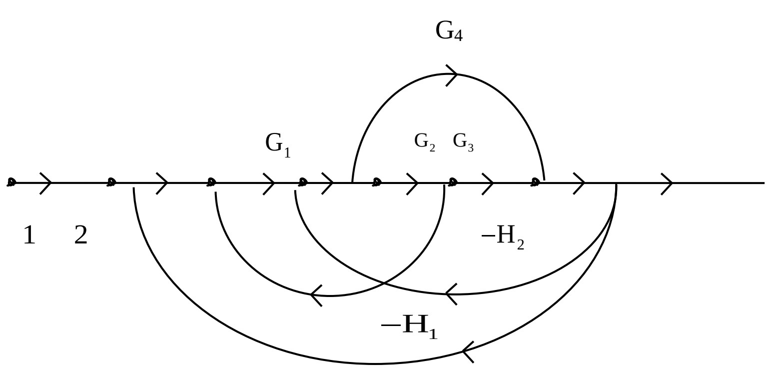
$$T(S) = \sum_K \frac{p_K\Delta_K}{\Delta} = \frac{p_1\Delta_1 + p_2\Delta_2 + p_3\Delta_3}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5G_6 + G_1G_2G_6G_7(1 + G_4H_1) + G_1G_2G_3G_8}{1 + G_4H_1 + G_2G_3H_4 + G_2G_3G_4G_5H_2 + G_2G_7H_2 + G_1G_2G_3G_4G_5G_6H_3 + G_1G_2G_3G_8H_8 + G_2G_4G_7H_1H_2 + G_1G_2G_4G_6G_7H_1H_3}$$

CONVERSION OF BLOCK DIAGRAM TO SIGNAL FLOW GRAPH

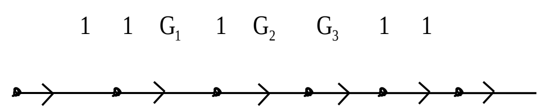
1. Convert the block diagram into signal flow graph and find the transfer function using Masons gain formula
   Procedure:
   1.Select a node for every branch point and summing point, input and output signal in block diagram
   2.for each block have a line segment on which its gain is written with direction



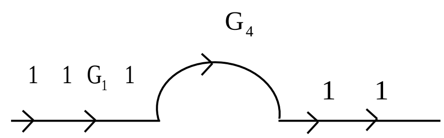


Sol :

1. No. of forward path and forward path gains k=2.

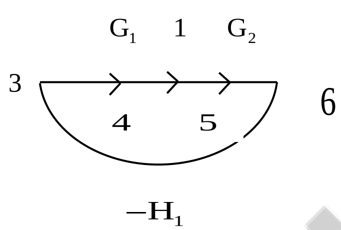


$P_1 = G_1G_2G_3$

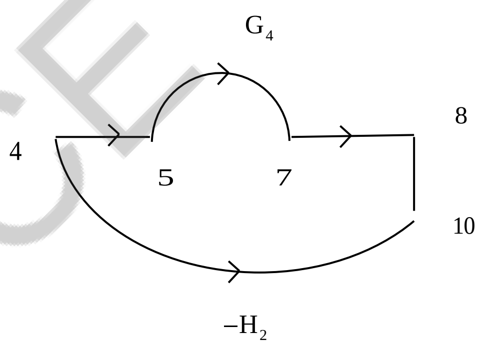


$P_2 = G_1G_4$

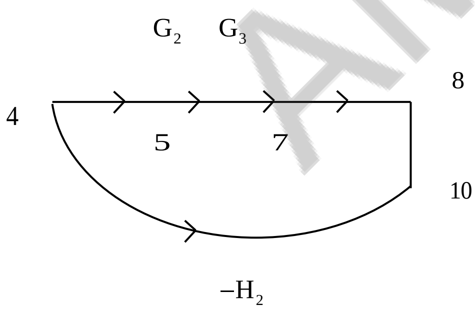
2.Individual loops and gain



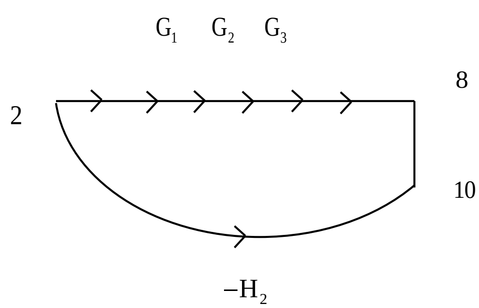
$P_{11} = -G_1G_2H_1$



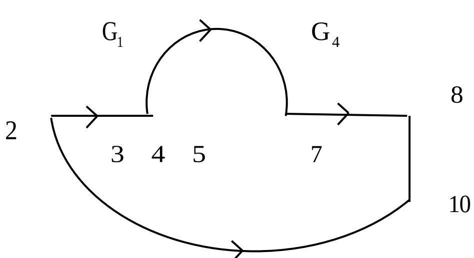
$P_{21} = -G_4H_2$



$p_{31} = -G_2G_3H_2$



$P_{41} = -G_1G_2G_3$



$P_{51} = -G_1G_4$

3.Non touching loops - NIL

Δ and Δk

4. $\Delta = 1 - [P_{11} + P_{21} + P_{31} + P_{41} + P_{51}]$

$= 1 + G_1G_2H_1 + G_4H_2 + G_2G_3H_2 + G_1G_2G_3 + G_1G_4$

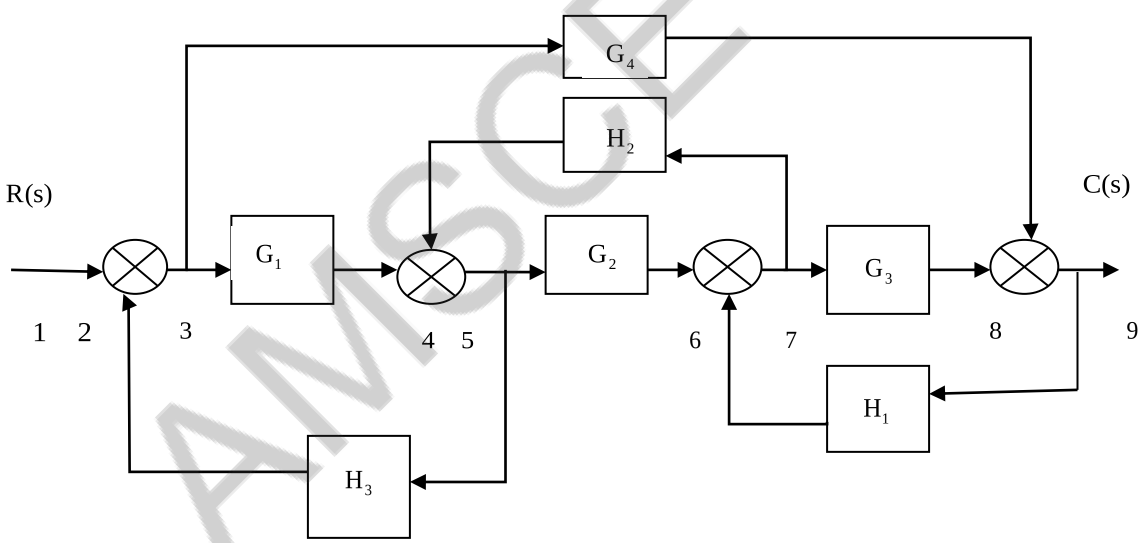
$\Delta_1 = \Delta_2 = 1$ [Since there is no part of the graph is not touching with the forward paths]

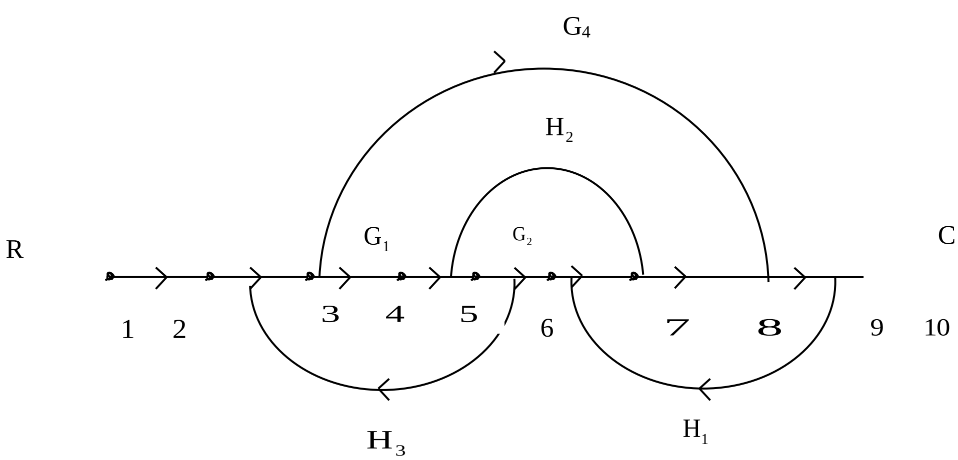
5. Transfer function by Mason's gain formula

$$T(s) = \sum_k \frac{P_k\Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_1G_2G_3 + G_4H_2}$$

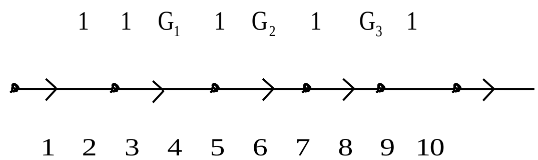
2.Convert block diagram to signal flow graph and find the transfer function using Mason's gain formula



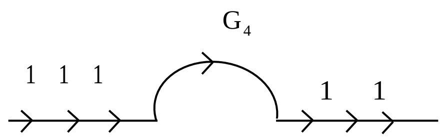


SOL:

1. No. of forward path and forward path gains k=2.

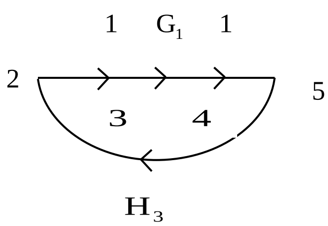


$P_1 = G_1G_2G_3$

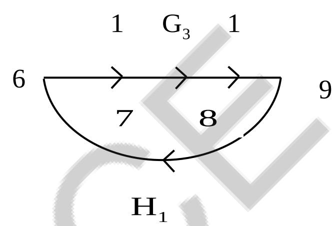


$P_2 = G_4$

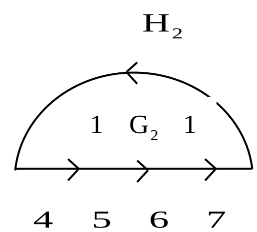
2.Individual loops and gain



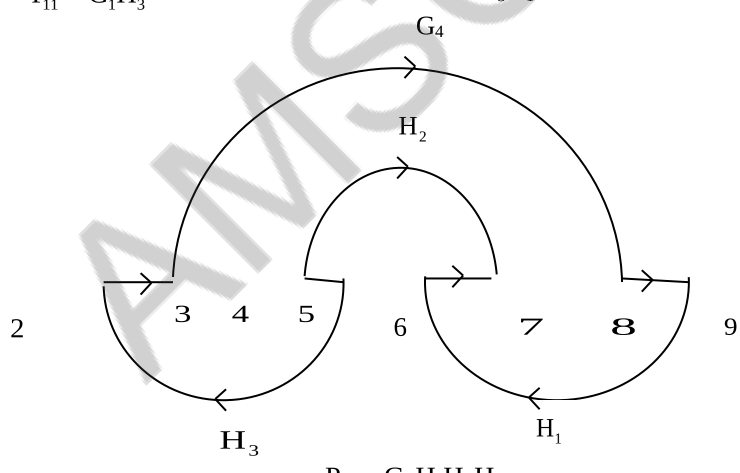
$P_{11} = G_1H_3$



$P_{21} = G_3H_1$

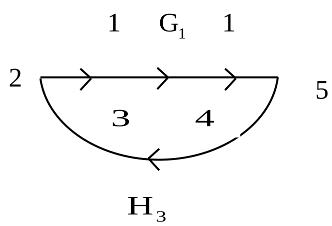


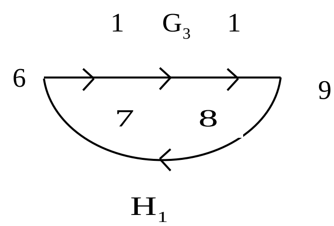
$P_{31} = G_2H_2$



$P_{41} = G_4H_1H_2H_3$

3.Non touching loops





$P_{12} = G_1G_3H_1H_3$

4. Δ and Δ_{ki}

$$\Delta = 1 - \left[P_{11} + P_{21} + P_{31} + P_{41}\right] + \left[P_{12}\right]$$
$$= 1 - G_1H_3 - G_3H_1 - G_2H_2 - G_4H_1H_2H_3G_1G_3H_1H_3$$

$\Delta_1 = 1$ [Since there is no part of the graph is not touching with the forward paths 1]

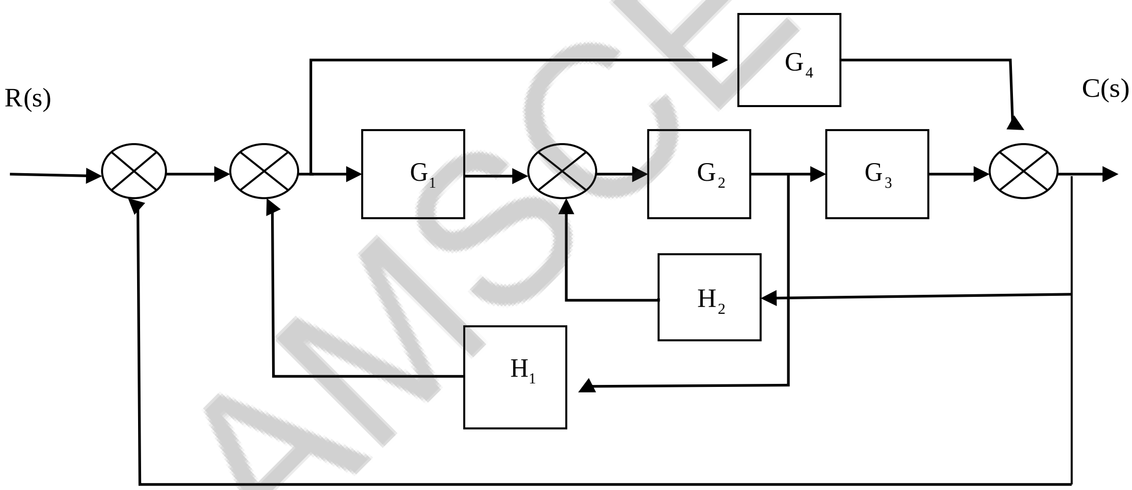
$\Delta_2 = 1 - G_2H_2$; When forward path 2 being removed, remaining part of the graph is as shown u Diagram

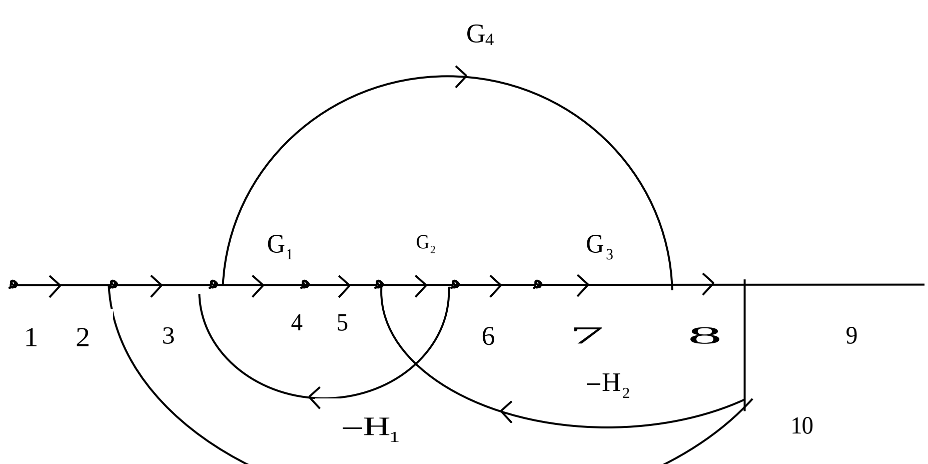
5. Transfer function By Masons gain formula

$$T(s) = \sum_k \frac{P_k\Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_4\left(1 - G_2H_2\right)}{1 + G_1H_3 + G_3H_1 - G_4H_1H_2H_3 + G_1G_3H_1H_3}$$

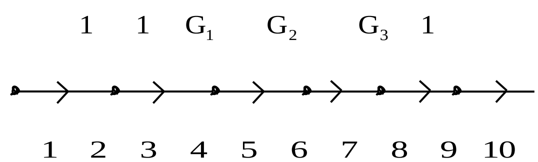
3.Construct an equivalent signal flow graph for the block diagram shown in fig and evaluate the transfer function



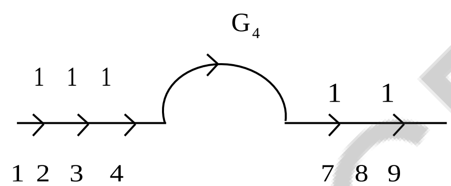


Sol

1. No. of forward path and forward path gains k=2.

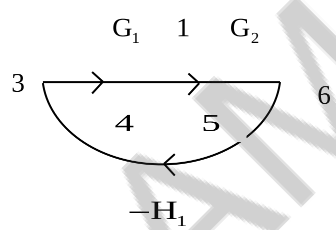


$P_1 = G_1G_2G_3$

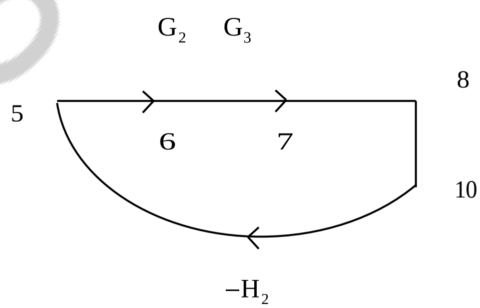


$P_2 = G_4$

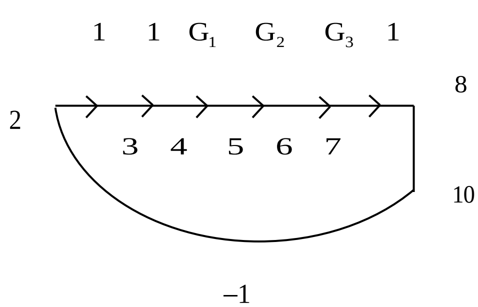
2. .Individual loops and gain



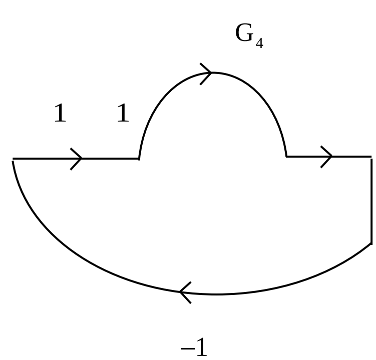
$P_{11} = -G_1H_1$



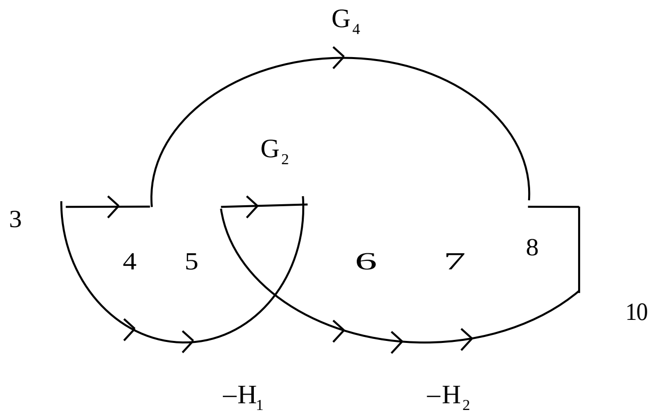
$P_{21} = -G_2G_3H_2$



$P_{31} = -G_1G_2G_3$



$P_{41} = -G_4$



$$P_{51} = G_2G_4H_1H_2$$

3 . Non touching loops - Nil

Δ and Δk

4. $\Delta = 1-[P_{11}+P_{21}+P_{31}+P_{41}+P_{51}]$

$= 1+G_1H_1+G_2G_3H_2+G_1G_2G_3+G_4-G_2G_4H_1H_2$

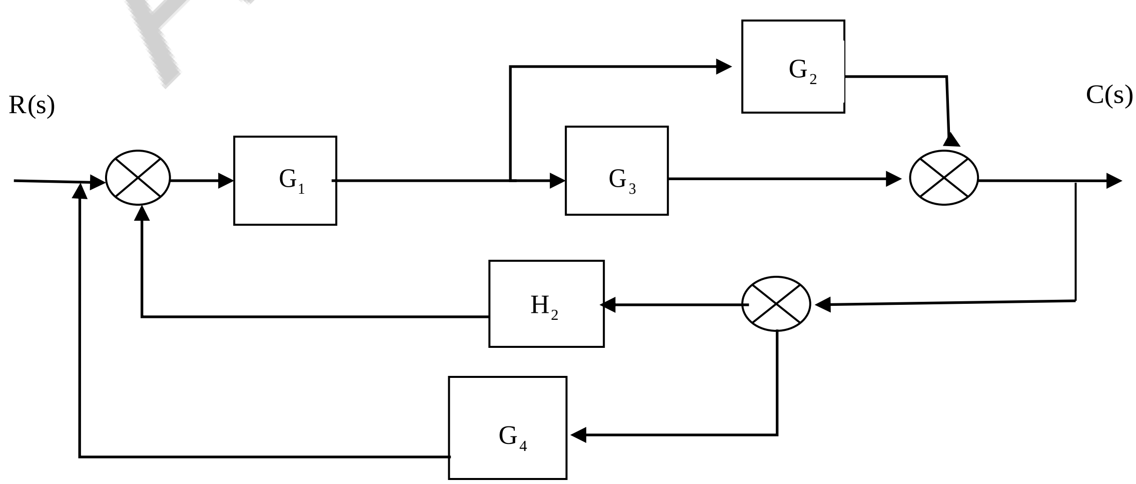
$\Delta_1 = \Delta_2 = 1$ [All the loops are touching the two forward paths]

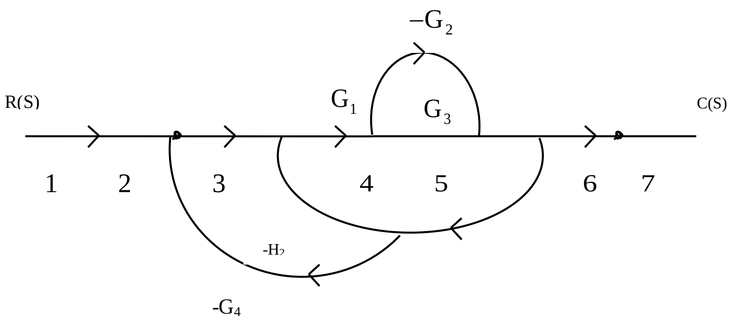
5.Transfer function By Masons gain formula

$$T(s) = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

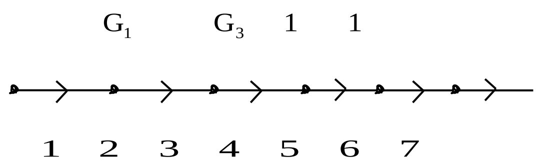
$$= \frac{G_1G_2G_3+G_4}{1+G_1G_2H_1+G_2G_3H_2-G_1G_2G_3+G_4-G_2G_4H_1H_2}$$

4.Draw the signal flow graph and evaluate the closed loop transfer of a system whose block diagram is shown in fig

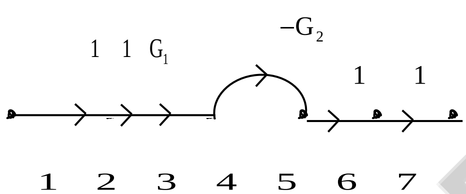




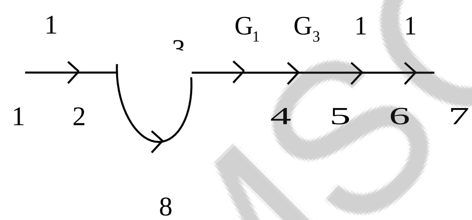
1. No. of forward path and forward path gains k=4.



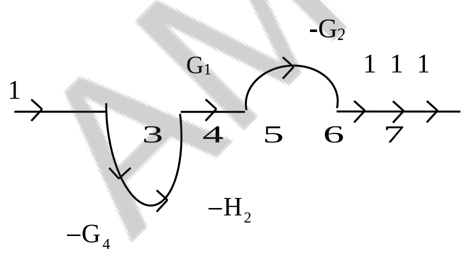
$P_1 = G_1G_3$



$P_2 = -G_1G_2$

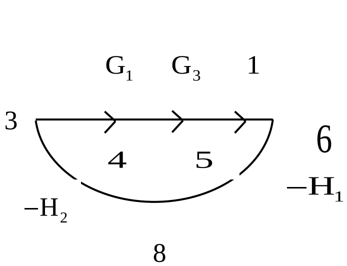


$P_3 = G_1G_3G_4H_2$

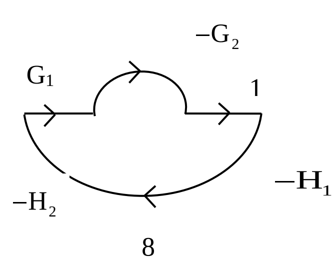


$P_4 = -G_1G_2G_4H_2$

2.Individual loops and gain



$P_{11} = G_1G_3H_2H_1$



$P_{21} = -G_1G_2H_1H_2$

3 . Non touching loops - Nil

4. Δ and Δk

$$\Delta = 1-[P_{11}+P_{21}] = 1+G_1G_2H_1H_2 - G_1G_3H_1H_2$$

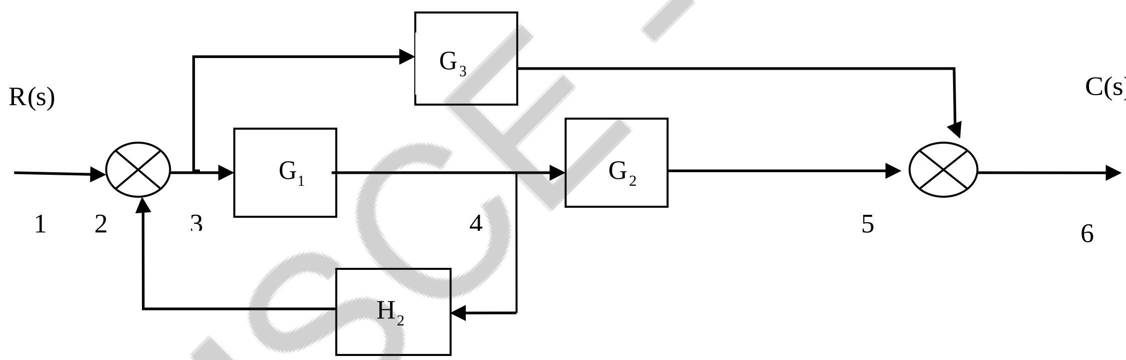
$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$ [All the loops are touching the two forward paths]

5.Transfer function By Masons gain formula

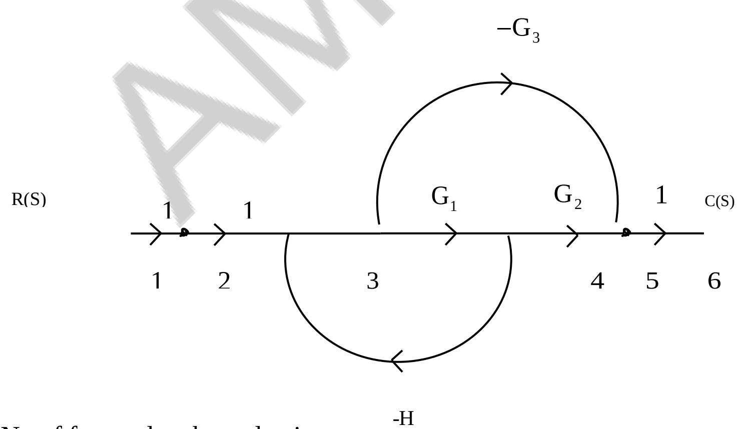
$$T(s) = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_3 + G_1G_3G_4H_2 - G_1G_2 - G_1G_2G_4H_2}{1+G_1G_2H_1H_2 - G_1G_3H_1H_2}$$

5.Convert the block diagram shown in fig, to signal flow graph and find the transfer function, using Mason's gain formula. Verify with block diagram approach MAY/JUNE 2016

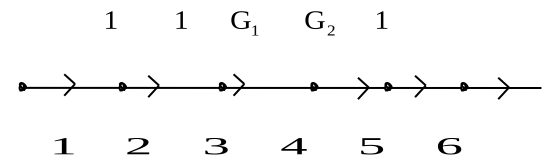


SOL

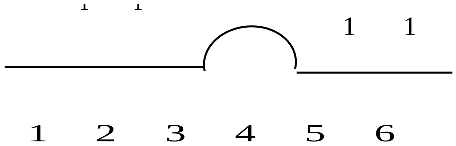


Sol

1. No of forward paths and gain

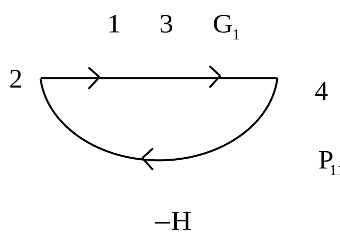


$P_1 = G_1 G_2$



$P_2 = -G_3$

2. Individual loops and loop gain



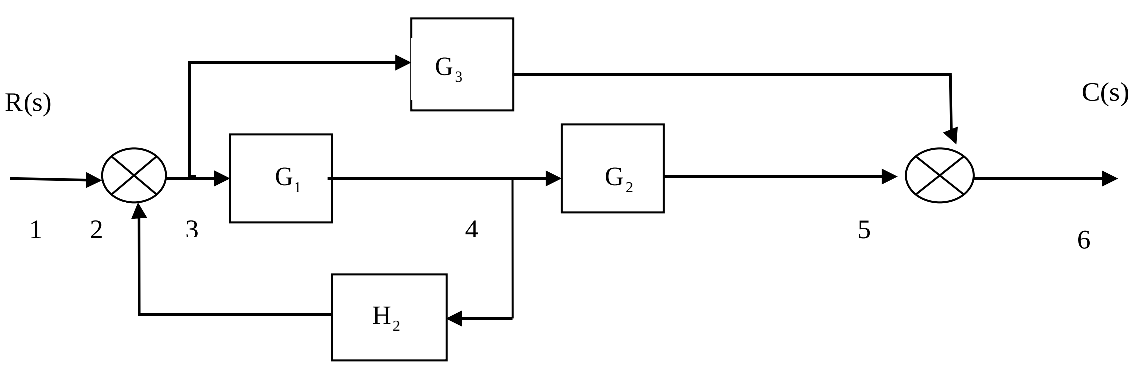
$P_{11} = -G_1 H_1$

3. Non touching loops – Nil
   Δ and Δk
4. $\Delta = 1 - P_{11} = 1 + G_1 H$
   $\Delta_1 = \Delta_2 = 1$
   As all the loops are touching the forward paths

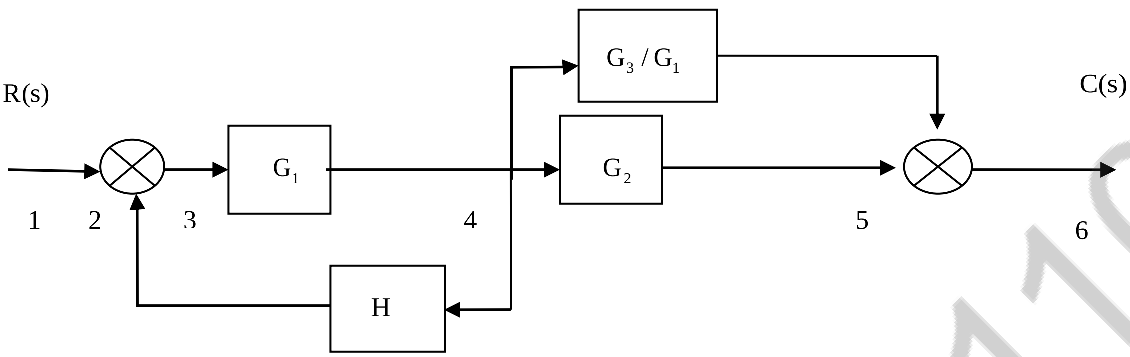
5.Transfer function By Masons gain formula

$$T(s) = \frac{1}{\Delta} \sum_k P_k \Delta_k = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$
$$= \frac{G_1 G_2 - G_3}{1 + G_1 H_1}$$

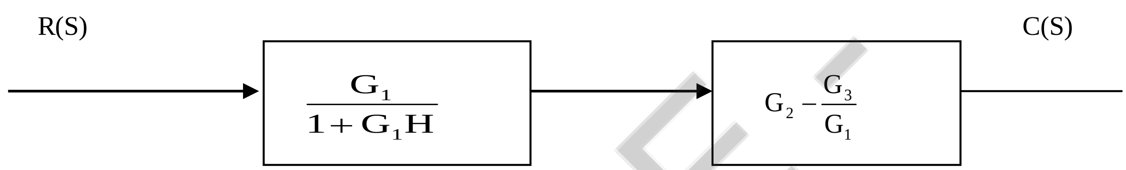
Verification by block diagram reduction technique

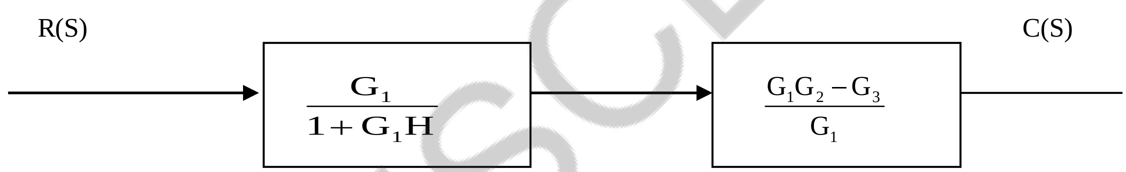


Step :1 Moving a branch point ahead of block G_1

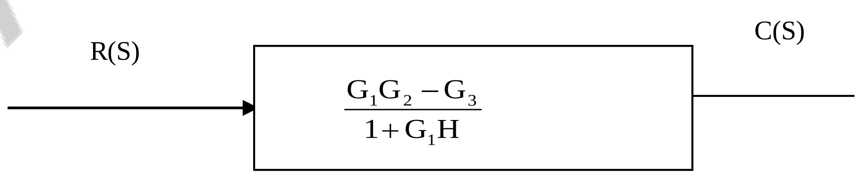


Step:2 Eliminating –ve feedback and combining the parallel path





Step:3 Combining the cascade blocks

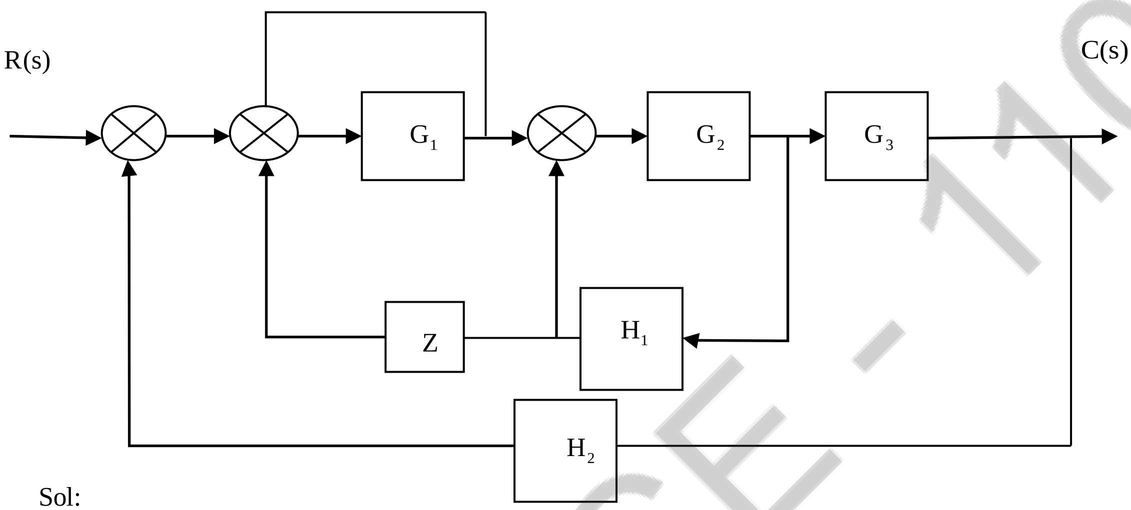


$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2 - G_3}{1+G_1H} \qquad (2)$$

Eqn (1) and (2) are equal. Hence Verified.

6.Find the transfer function of the system shown un fig. by block diagram reduction technique and signal flow graph technique

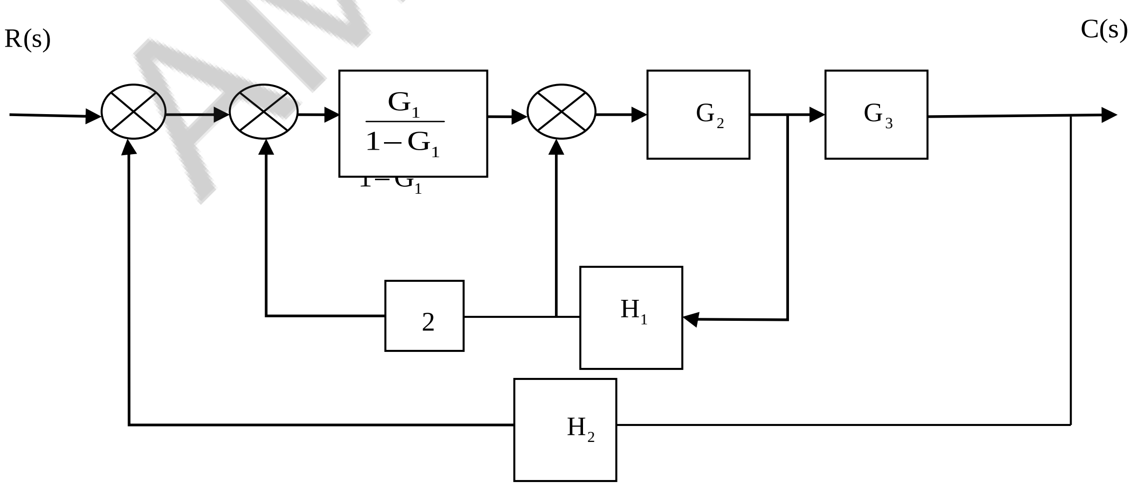
April/ May 2015



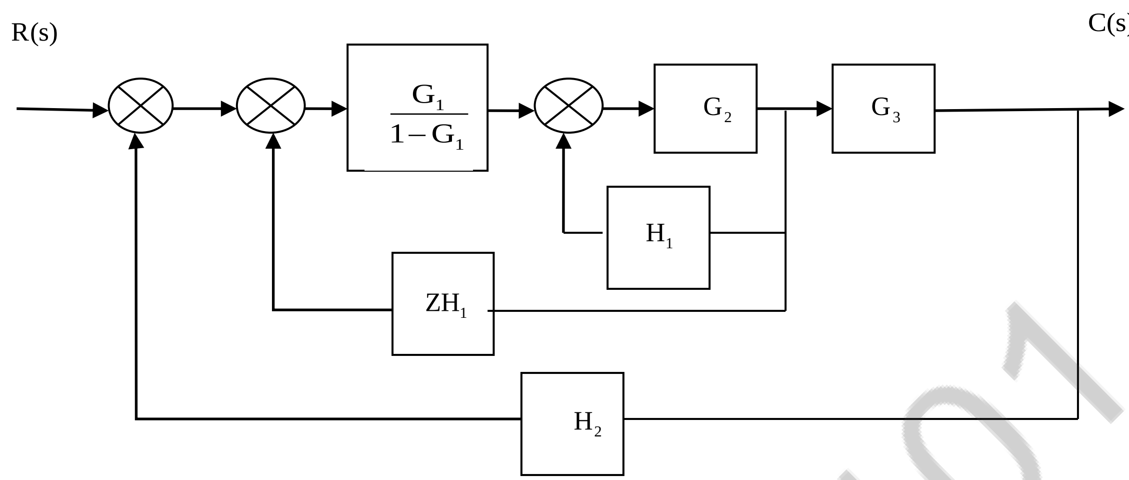
Sol:

By block diagram reduction technique

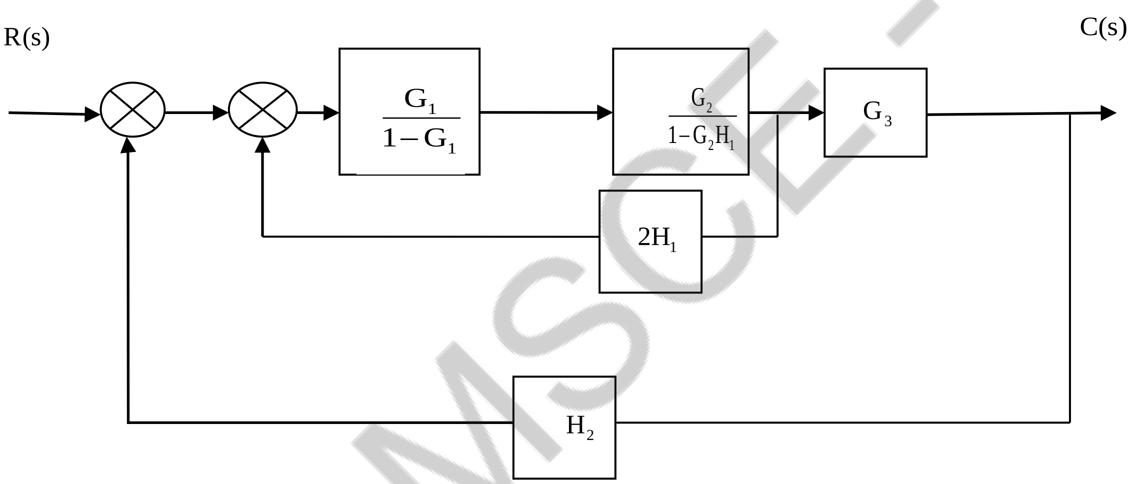
Step:1 Removing Unity feedback



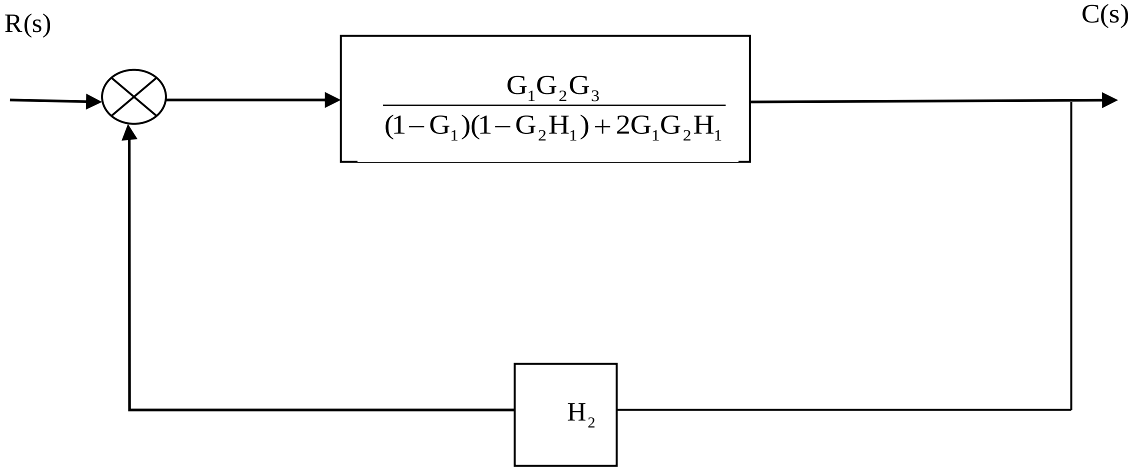
Step:2 Moving the branch point before H_1



Eliminating of feedback H_1



Combining the cascade blocks combining the cascade blocks



Eliminating the feedback path H_2

$$G \rightarrow \frac{G_1G_2G_3}{(1-G_1)(1+G_2H_1)+2G_1G_2H_1} \qquad H \rightarrow H_2$$

$$\frac{G}{1+GH} \rightarrow \frac{\dfrac{G_1G_2G_3}{(1-G_1)(1+G_2H_1)+2G_1G_2H_1}}{1+\dfrac{G_1G_2G_3H_2}{(1-G_1)(1+G_2H_1)+2G_1G_2H_1}}$$

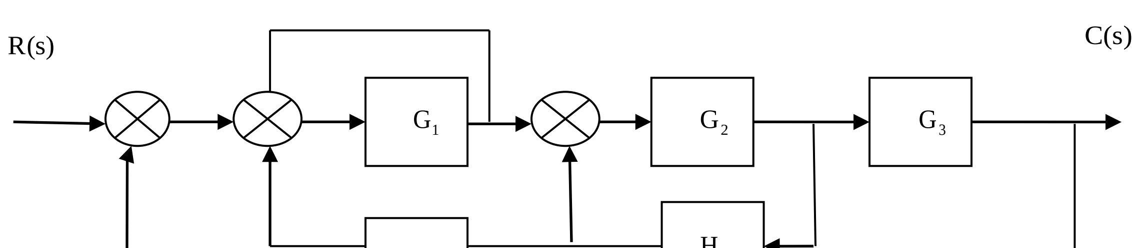
$$= \frac{G_1G_2G_3}{(1-G_1)(1+G_2H_1)+2G_1G_2H_1+G_1G_2G_3H_2}$$

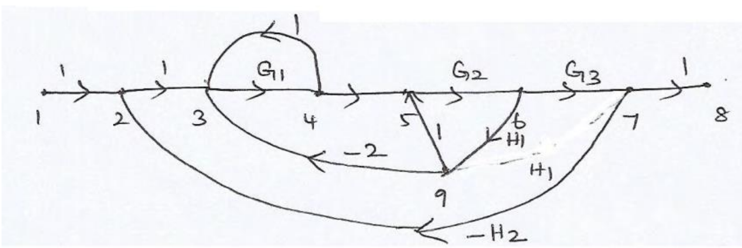
$$= \frac{G_1G_2G_3}{1-G_1-G_2H_1+3G_1G_2H_1+G_1G_2G_3H_2}$$

$$R(s) \longrightarrow \boxed{\frac{G_1G_2G_3}{1-G_1-G_2H_1+3G_1G_2H_1+G_1G_2G_3H_2}} \longrightarrow C(s)$$

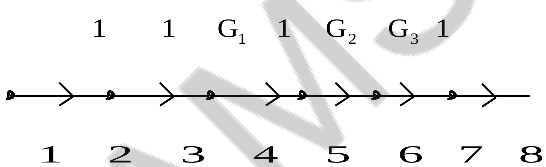
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1-G_1-G_2H_1+3G_1G_2H_1+G_1G_2G_3H_2}$$

(ii) by signal flow graph technique



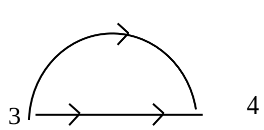


Step:1No of forward path and forward path gain K=1

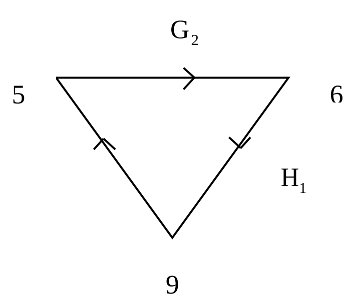


$P_1 = G_1G_2G_3$

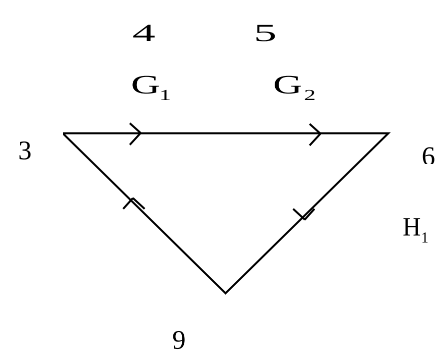
Step:2 Individual loops and individual loop gains



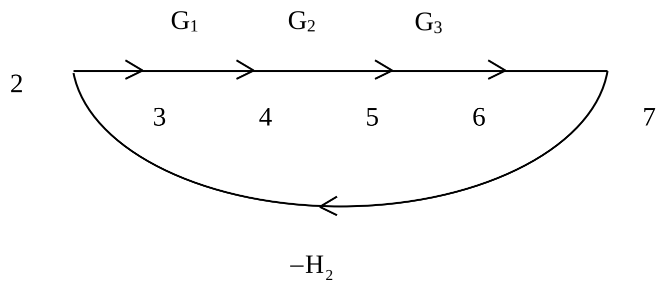
$P_{11} = G_1$



$P_{21} = G_2H_1$

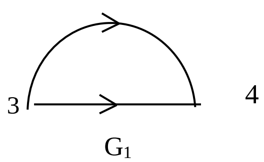


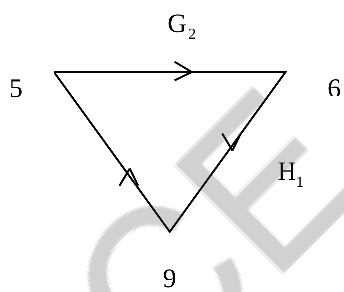
$P_{31} = 2G_1G_2H_1$



$P_{41} = -G_1G_2G_3H_2$

Step:3 Non touching loops – gain products





$P_{12} = G_1G_2H_1$

Step:4 To find

Δ and Δk

$\Delta = 1 - \left[P_{11} + P_{21} + P_{31} + P_{41}\right] + P_{12}$

$= 1 - G_1 - G_2H_1 + 2G_1G_2H_1 + G_1G_2G_3H_2 + G_1G_2H_1$

$= 1 - G_1 - G_2H_1 + 3G_1G_2H_1 + G_1G_2G_3H_2$

$\Delta_1 =$ as all the loops are touching the forward paths

Step:5 Transfer function by Mason's gain formula,

$$T(S) = \frac{1}{\Delta}\sum_{K} p_K \Delta_K = \frac{p_1\Delta_1}{\Delta}$$

$$= \frac{G_1G_2G_3}{1 - G_1 - G_2H_1 + 3G_1G_2H_1 + G_1G_2G_3H_2}$$

$$\therefore T(s) = \frac{G_1G_2G_3}{1 - G_1 - G_2H_1 + 3G_1G_2H_1 + G_1G_2G_2H_2} \qquad \text{Hence verified}$$

CONSTRUCTION OF SIGNAL FLOW GRAPH FROM THE SYSTEM EQUATIONS

STEPS:

1.Obtain the system equations by writing differential equations governing the system

2.Represent each variable by a separate node.

3.Use the property that value of the variable represented by a node is an variable represented by a node is an algebraic sum of all the signals entering at that node, to simulate the equations

4. Coeffcients of the variables in the equations are to be represented as the brancg gain, joining the nodes in signal flow graph.

5.Show the input and output variables separately to completely signal flow graph

1. Construct signal flow graph for the set of linear equations and determine overall transfer for using Mason's gain formula

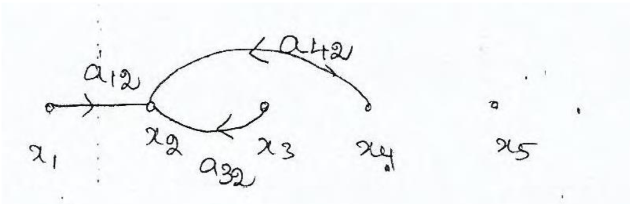
$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 \qquad (1)$$
$$x_3 = a_{23}x_2 \qquad (2)$$
$$x_4 = a_{24}x_2 + a_{34}x_3 + a_{44}x_4 \qquad (3)$$
$$x_5 = a_{25}x_2 + a_{45}x_4 \qquad (4)$$

Step:1

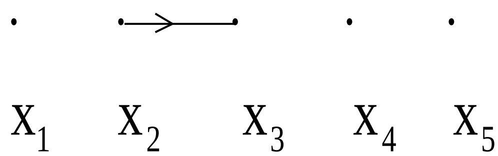
. . . . .

x_1 x_2 x_3 x_4 x_5

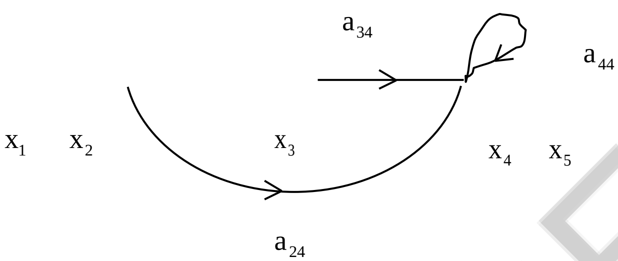
Eqn 1



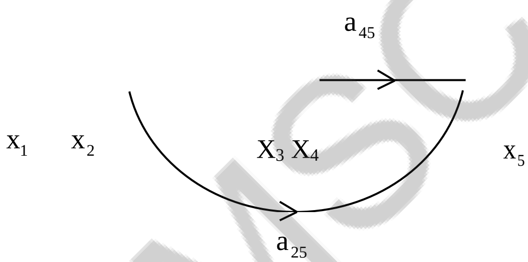
Eqn 2



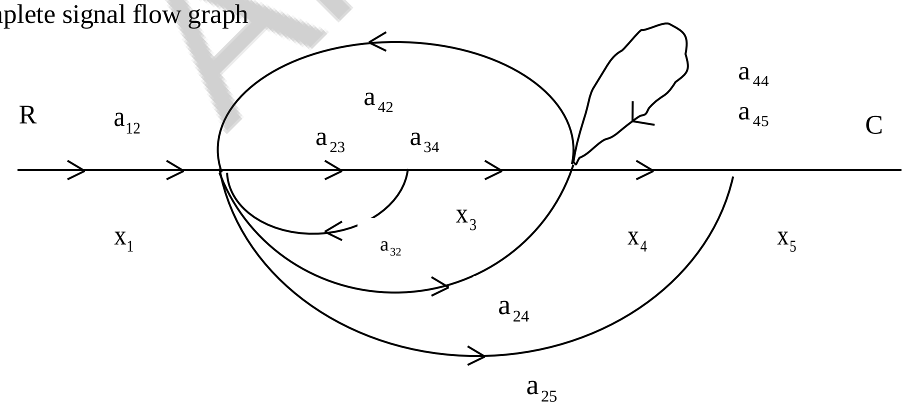
Eqn 3



Eqn 4



Complete signal flow graph



1. No. of forward paths and forward path gain k=3

$P_1 = a_{12}a_{23}a_{34}a_{45}\left[x_1 \to x_2 \to x_3 \to x_4 \to x_5\right]$

$P_2 = a_{12}a_{24}a_{45}\left[x_1 \to x_2 \to x_4 \to x_5\right]$

$P_3 = a_{12}a_{25}\left[x_1 \to x_2 \to x_5\right]$

2. Individual loops and gain

$P_{11} = a_{23}a_{32}\left[x_2 \to x_3 \to x_2\right]$

$P_{21} = a_{23}a_{34}a_{42}\left[x_2 \to x_3 \to x_4 \to x_2\right]$

$P_{31} = a_{24}a_{42}\left[x_2 \to x_4 \to x_2\right]$

$P_{41} = a_{44}\left[x_4\right]$

3. Non touching loops

$P_{12} = P_{11,}P_{41} = a_{32}a_{23}a_{44}$

4.

Δ and Δk

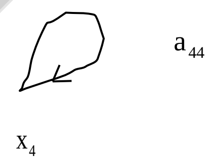
$\Delta = 1 - \left[P_{11} + P_{21} + P_{31} + P_{41}\right] + P_{12}$

$= 1 - \left[a_{32}a_{23} + a_{23}a_{34}a_{42} + a_{24}a_{42} + a_{44} + a_{23}a_{32}a_{44}\right]$

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1 - a_{44}$



5. Transfer function by Mason's gain formula

$$T(s) = \sum_k \frac{P_k\Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

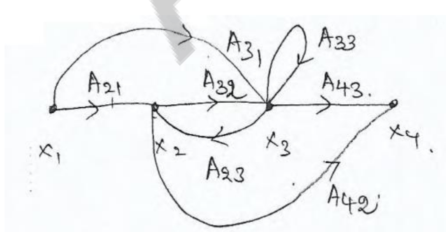
$$T(s) = \frac{a_{12}a_{23}a_{34}a_{45} + a_{12}a_{24}a_{45} + a_{12}a_{25}(1 - a_{44})}{1 - [a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{24}a_{42} + a_{44}] + a_{23}a_{32}a_{44}}$$

2.Construct the signal flow graph for the following set of simultaneous equations

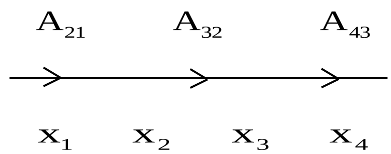
$X_2 = A_{21}X_1 + A_{23}X_3;\ \ X_3 = A_{31}X_1 + A_{32}X_2 + A_{33}X_3;$

$X_4 = A_{42}X_2 + A_{43}X_3$

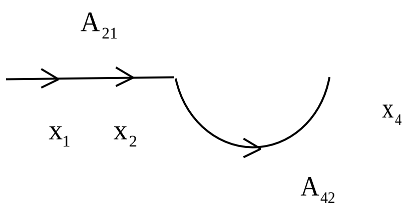
And obtain the overall transfer function using Mason's gain formula



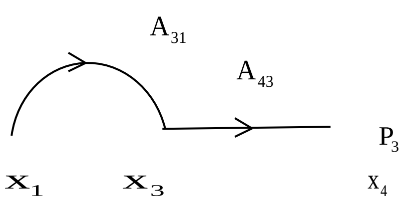
1. No of forward paths and gain



$P_1 = A_{21}A_{32}A_{43}$

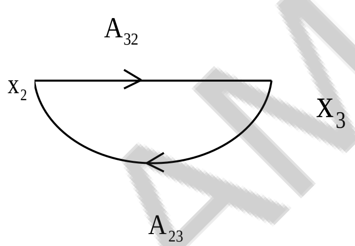


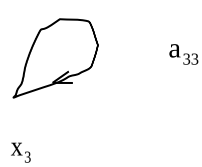
$P_2 = A_{21}A_{42}$



$P_3 = A_{31}A_{43}$

2. Individual loops and gain





3. Non touching loops = Nil

$P_{21} = A_{33}$

$P_{11} = A_{23}A_{32}$

4. Δ and Δk

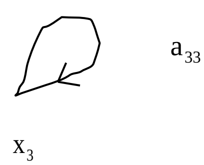
$\Delta = 1 - [P_{11} + P_{21}]$

$= 1 - [A_{32}A_{23} + A_{33}]$

$\Delta_1 = 1$

$\Delta_2 = 1 - A_{33}$

$\Delta_3 = 1$



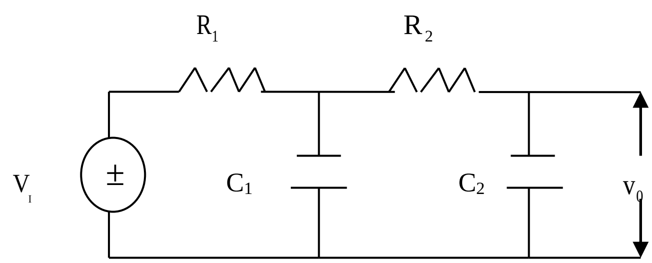
5. Transfer function By Mason's gain formula

$$T(s) = \sum_k \frac{P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

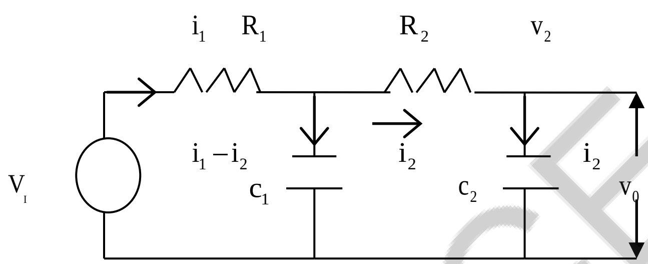
$$\frac{A_{21}A_{32}A_{43} + A_{21}A_{42}(1 + A_{33}) + A_{31}A_{43}}{1 - [A_{32}A_{23} + A_{33}]}$$

V_0 / V_1

3.For the network shown below, draw the signal flow graph and find transfer function Mason;s gain formula.



Sol



Consider current through series element and voltage across the shunt element.

$V_i, i_1, V_1, i_2, V_2, V_o$

$i_1, v_1, i_2, v_2 \rightarrow$ mixed nodes

Nodes/Variables are

$V_i \rightarrow$ input node

$V_0 \rightarrow$ output node

Current through R_1

$$i = \frac{V_i - V_1}{R_1}$$

$$= \frac{V_i}{R_1} - \frac{V_1}{R_1}$$

$$I_1(s) = \frac{1}{R_1} V_i(s) - \frac{1}{R_1} V_1(s) \qquad (1)$$

Voltage across C_1

$$V_1 = \frac{1}{C_1}\int (i_1 - i_2)dt$$

$$V_1(s) = \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right]$$

$$V_1(s) = \frac{1}{C_1 s} I_1(s) - \frac{1}{C_1 s} I_2(s) \qquad (2)$$

current through R_2

$$i_2 = \frac{V_1 - V_2}{R_2}$$

$$I_2(s) = \frac{V_1(s) - V_2(s)}{R_2}$$

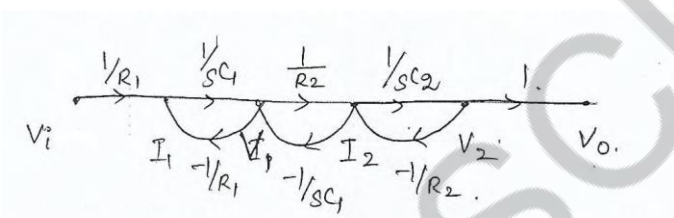
$$I_2(s) = \frac{1}{R_2} V_1(s) - \frac{1}{R_2} V_2(s) \qquad (3)$$

Voltage across C_2

$$V_2 = \frac{1}{C_2}\int i_2 dt$$

$$V_2 = \frac{1}{C_2 s} I_2(s) \qquad (4)$$

Construction of signal flow graph is as follows



$$-\frac{1}{R_1} \qquad -\frac{1}{SC_1} \qquad -\frac{1}{R_2}$$

$$-\frac{1}{R_1} \qquad -\frac{1}{SC_1} \qquad -\frac{1}{R_2}$$

1. No. of forward path and gains

$$V_i \rightarrow I_1 \rightarrow V_1 \rightarrow I_2 \rightarrow V_2 \rightarrow V_0$$

$$P_1 = \frac{1}{R_1}.\frac{1}{sC_1}.\frac{1}{R_2}.\frac{1}{sC_2} = \frac{1}{R_1 R_2 C_1 C_2 s^2}$$

K=1

2. Individual loops and gain

$$I_1 \to V_2 \to I_1 \qquad P_{11} = \frac{-1}{R_1C_1s}$$

$$V_2 \to I_2 \to V_2 \qquad P_{21} = \frac{-1}{R_2C_2s}$$

$$I_2 \to V_3 \to I_2 \qquad P_{31} = \frac{-1}{R_2C_2s}$$

3. Non touching loops

P_{11} and P_{31} are non touching loop pair

$$P_{11} = P_{11}P_{31} = \frac{1}{R_1R_2C_1C_2s^2}$$

Δ and Δk

$$\Delta = 1 - \left[P_{11} + P_{21} + P_{31}\right] + P_{12}$$

4. $$= 1 + \frac{1}{R_1C_1s} + \frac{1}{R_2C_1s} + \frac{1}{R_2C_2s} + \frac{1}{R_1R_2C_1C_2s^2}$$

$$\Delta = \frac{\left[R_1R_2C_1C_2s^2 + \left(R_2C_2 + R_1C_1 + R_1C_2\right)s + 1\right]}{R_1R_2C_1C_2s^2}$$

$$\Delta_1 = 1$$

5. Transfer function by Mason's gain formula,

$$T(s) = \frac{1}{\Delta}\sum_k P_k\Delta_k$$

$$T(s) = \frac{V_0(s)}{V_1(s)} = \frac{\dfrac{1}{R_1R_2C_1C_2s^2}}{\dfrac{\left[R_1R_2C_1C_2s^2 + \left(R_2C_2 + R_1C_1 + R_1C_2\right)s + 1\right]}{R_1R_2C_1C_2s^2}}$$

$$\frac{V_0(s)}{V_1(s)} = \frac{1}{R_1R_2C_1C_2s^2 + \left(R_2C_2 + R_1C_1 + R_1C_2\right)s + 1}$$

*** INCLUDE THIS**

1. Explain about DC Servo Motor

A DC servo motor is used as an actuator to drive a load. It is usually a DC motor of low power rating. DC servo motors have a high ratio of starting torque to inertia and therefore they have a faster dynamic response.

- DC motors are constructed using rare earth permanent magnets which have high residual flux density and high coercively.

- As no field winding is used, the field copper losses is zero and hence, the overall efficiency of the motor is high.

- The speed torque characteristic of this motor is flat over a wide range, as the armature reaction is negligible.

- Moreover speed in directly proportional to the armature voltage for a given torque. Armature of a DC servo motor is specially designed to have low inertia.

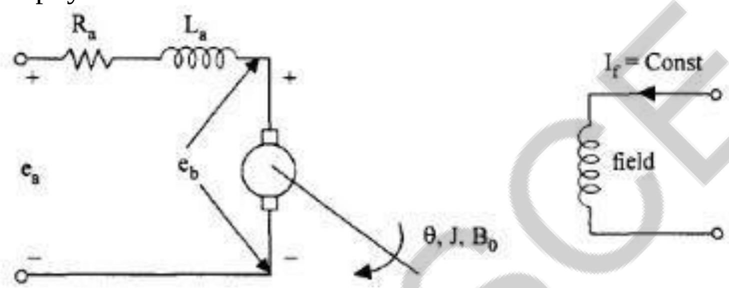
- In some application DC servo motors are used with magnetic flux produced by field windings.

- The speed of PMDC motors can be controlled by applying variable armature voltage. These are called armature voltage controlled DC servo motors.

- Wound field DC motors can be controlled by either controlling the armature voltage or controlling rho field current. Let us now consider modelling of these two types or DC servo motors.

(a) Armature controlled DC servo motor

The physical model of an armature controlled DC servo motor is given in



The armature winding has a resistance R_a and inductance L_a.

The field is produced either by a permanent magnet or the field winding is separately excited and supplied with constant voltage so that the field current I_f is a constant. When the armature is supplied with a DC voltage of e_a volts, the armature rotates and produces a back e.m.f e_b

The armature current i_a depends on the difference of e_b and e_n. The armature has a moment of inertiaJ, frictional coefficient B_o

The angular displacement of the motor is θ. The torque produced by the motor is given by

$$T = K_T\, i_a$$

Where K_T is the motor torque constant.

The back emf is proportional to the speed of the motor and hence

$$e_b = K_b\ \dot{\theta}$$

The differential equation representing the electrical system is given by

$$R_a\, i_a + L_a \frac{di_a}{dt} + e_b = e_a$$

Taking Laplace transform of equation from above equation

$$T(s) = K_T\ I_a(s)$$
$$E_b(s) = K_b\ s\ \theta(s)$$
$$(R_a + s\ L_a)\ I_a(s) + E_b(s) = E_a(s)$$
$$I_a(s) = \frac{E_a(s) - K_b s\,\theta(s)}{R_a + sL_a}$$

The mathematical model of the mechanical system is given by

$$J\ \frac{d^2\theta}{dt^2} + B_0 \frac{d\theta}{dt} = T$$

Taking Laplace transform

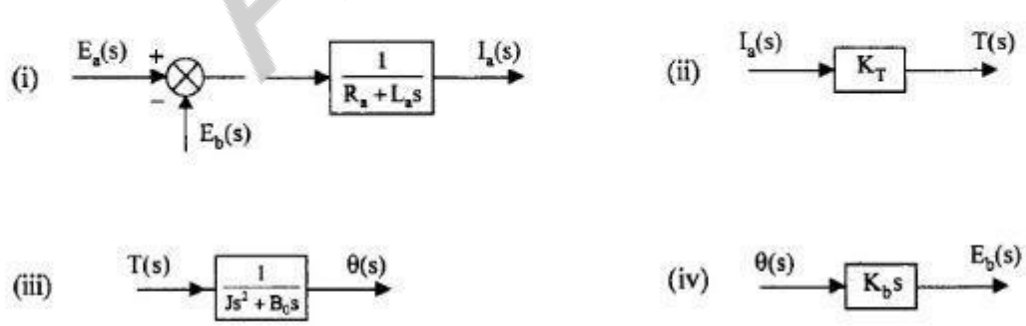
$$(Js^2 + B_0 s)\ \theta(s) = T(s)$$

$$\theta(s) = K_T\ \frac{E_a(s) - K_b s\,\theta(s)}{(R_a + sL_a)(Js^2 + B_0 s)}$$

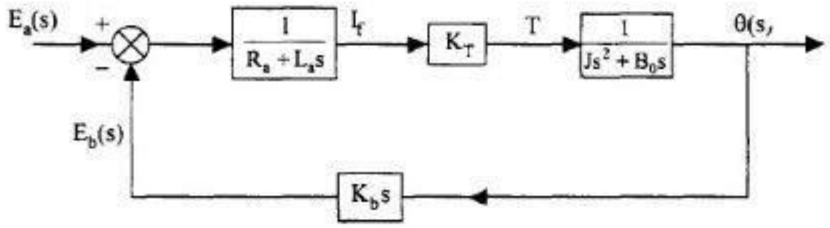
Solving for θ(s) ,we get

$$\theta(s) = \frac{K_T\ E_a(s)}{s[(R_a + sL_a)(Js + B_0) + K_T\ K_b]}$$

The block diagram representation of the armature controlled DC servo motor is developed in steps



Combining these blocks we have



Usually the inductance of the armature winding is small and hence neglected

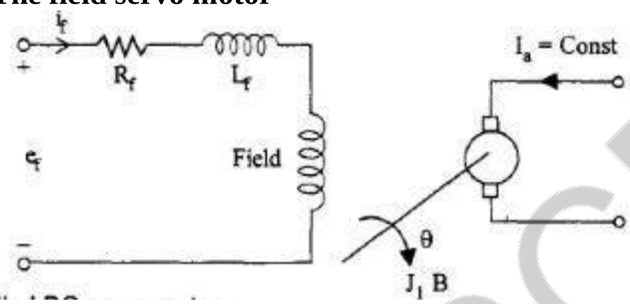
$$T(s) = \frac{\theta(s)}{E_a(s)} = \frac{K_T/R_a}{s\left[Js + B_0 + \frac{K_b K_T}{R_a}\right]}$$

$$= \frac{K_T/R_a}{s(Js + B)}$$

Where

$$B = B_0 + \frac{K_b K_T}{R_a}$$

Field Controlled Dc Servo Motor
The field servo motor



The electrical circuit is modeled as

$$I_f(s) = \frac{E_f(s)}{R_f + L_f s}$$

$$T(s) = K_T\, I_f(s)$$

$$(Js^2 + B_0)\,\theta(s) = T(s)$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K_T}{s(Js + B_0)(R_f + L_f s)}$$

$$= \frac{K_T/R_f B_0}{s\left(\frac{J}{B_0}s + 1\right)\left(\frac{L_f}{R_f}s + 1\right)}$$

$$= \frac{K_m}{s(\tau_m s + 1)(\tau_f s + 1)}$$

Where Motor gain constant

$K_m = K_T/R_f B_0$

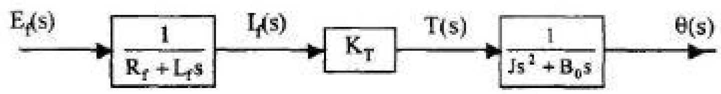
Motor time constant

$\tau_m = J/B_0$

Field time constant

$\tau_f = L_f/R_f$

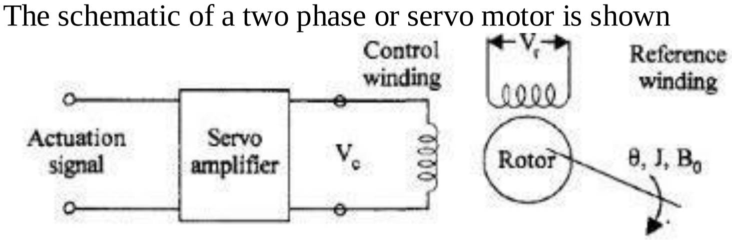
The block diagram is as shown as



2. Write a short note on AC Servo Motors

An AC servo motor is essentially a two phase induction motor with modified constructional features to suit servo applications.

The schematic of a two phase or servo motor is shown



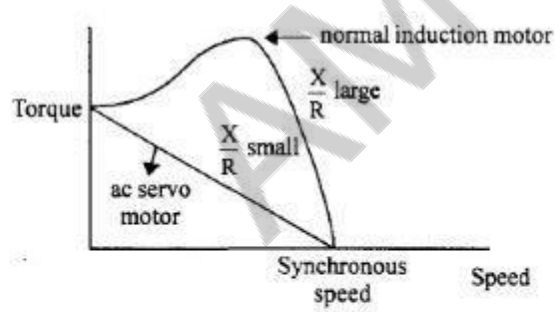
It has two windings displaced by 90 degree on the stator One winding, called as reference winding, is supplied with a constant sinusoidal voltage.

The second winding, called control winding, is supplied with a variable control voltage which is displaced by 90 ^{o} out of phase from the reference voltage.

The major differences between the normal induction motor and an AC servo motor are

The rotor winding of an ac servo motor has high resistance (R) compared to its inductive reactance (X) so that its ratio $\frac{X}{R}$ is very low.

For a normal induction motor, $\frac{X}{R}$ ratio is high so that the maximum torque is obtained in normal operating region which is around 5% of slip.



The torque speed characteristics of a normal induction motor and an ac servo motor are shown in fig

The Torque speed characteristic of a normal induction motor is highly nonlinear and has a positive slope for some portion of the curve.

This is not desirable for control applications as the positive slope makes the systems unstable. The torque speed characteristic of an ac servo motor is fairly linear and has 45 negative slope throughout.

The rotor construction is usually squirrel cage or drag cup type for an ac servo motor. The diameter is small compared to the length of the rotor which reduces inertia of the moving parts.

Thus it has good accelerating characteristic and good dynamic response.

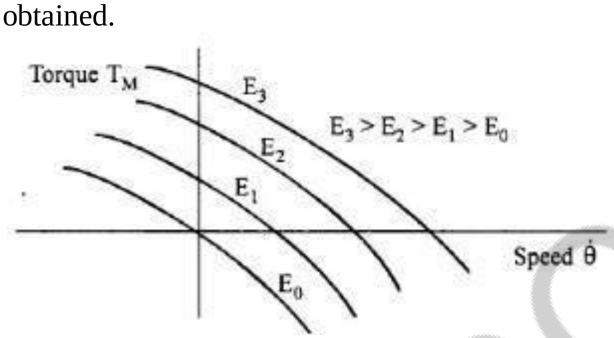
The supplies to the two windings of ac servo motor are not balanced as in the case of a normal induction motor.

The control voltage varies both in magnitude and phase with respect to the constant reference vulture applied to the reference winding.

The direction of rotation of the motor depends on the phase (± 90°) of the control voltage with respect to the reference voltage. For different rms values of control voltage the torque speed characteristics are shown in Fig.

The torque varies approximately linearly with respect to speed and also controls voltage.

The torque speed characteristics can be linearised at the operating point and the transfer function of the motor can be obtained.



3. Write a short note on Synchros

A commonly used error detector of mechanical positions of rotating shafts in AC control systems is the Synchro.
It consists of two electro mechanical devices.
Synchro transmitter

Synchro receiver or control transformer.

The principle of operation of these two devices is same but they differ slightly in their construction.
The construction of a Synchro transmitter is similar to a phase alternator.

The stator consists of a balanced three phase winding and is star connected.

The rotor is of dumbbell type construction and is wound with a coil to produce a magnetic field.

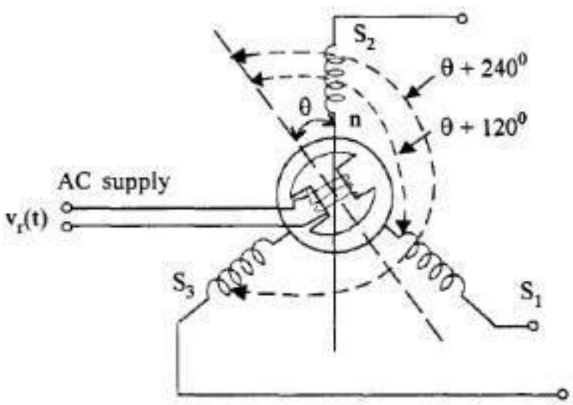
When a no voltage is applied to the winding of the rotor, a magnetic field is produced.

The coils in the stator link with this sinusoidal distributed magnetic flux and voltages are induced in the three coils due to transformer action.

Than the three voltages are in time phase with each other and the rotor voltage.

The magnitudes of the voltages are proportional to the cosine of the angle between the rotor position and the respective coil axis.

The position of the rotor and the coils are shown in Fig.



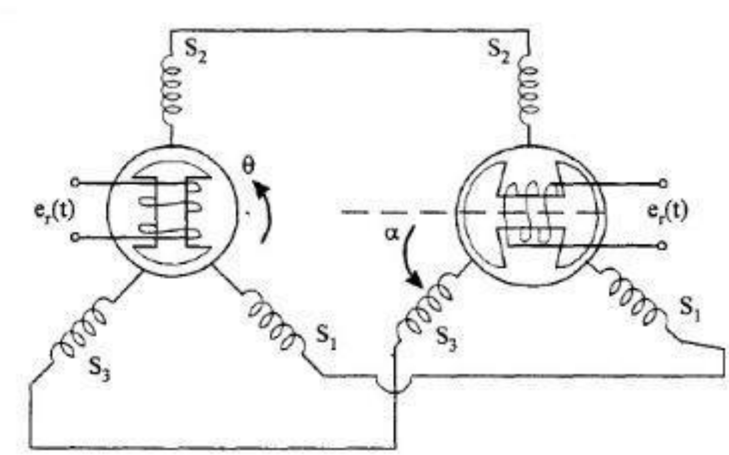
$$v_R(t) = v_r \text{ Sin } \omega_r t$$
$$v_{s_{1n}} = KV_r \text{ Sin } \omega_r t \cos(\theta + 120)$$
$$v_{s_{2n}} = KV_r \text{ Sin } \omega_r t \cos\theta$$
$$v_{s_{3n}} = KV_r \text{ Sin } \omega_r t \cos(\theta + 240)$$
$$v_{s_1 s_2} = v_{s_1 n} - v_{s_2 n} = \sqrt{3}\ KV_r \text{ Sin }(\theta + 240) \text{ Sin } \omega_r t$$
$$v_{s_2 s_3} = v_{s_2 n} - v_{s_3 n} = \sqrt{3}\ KV_r \text{ Sin }(\theta + 120) \text{ Sin } \omega_r t$$
$$v_{s_3 s_1} = v_{s_3 n} - v_{s_1 n} = \sqrt{3}\ KV_r \text{ Sin } \theta \text{ Sin } \omega_r t$$

When θ=90 the axis of the magnetic field coincides with the axis of coil S_2 and maximum voltage is induced in it as seen.

For this position of the rotor, the voltage c, is zero, this position of the rotor is known as the 'Electrical Zero' of die transmitter and is taken as reference for specifying the rotor position.

In summary, it can be seen that the input to the transmitter is the angular position of the rotor and the set of three single phase voltages is the output.

The magnitudes of these voltages depend on the angular position of the rotor as given



Hence

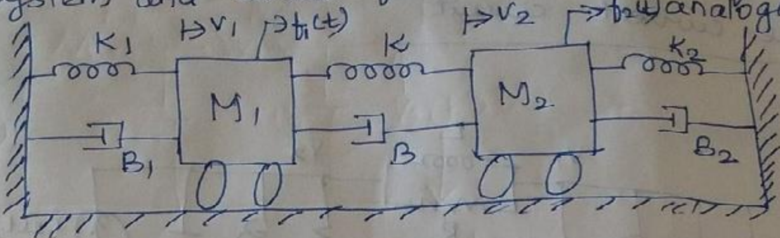
$$e_r(t) = K_1 V_r \cos\phi \sin \omega_r t$$

Now consider these three voltages to he applied to the stator of a similar device called control transformer or synchro receiver. The construction of a control transformer is similar to that of the transmitter except that the rotor is made cylindrical in shape whereas the rotor of transmitter is dumbell in shape. Since the rotor is cylindrical, the air gap is uniform and the reluctance of the magnetic path is constant. This makes the output impedance of rotor to be a constant.

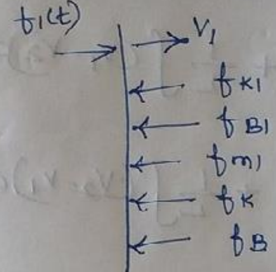
Usually the rotor winding of control transformer is connected teas amplifier which requires signal with constant impedance for better performance. A synchro transmitter is usually required to supply several control transformers and hence the stator winding of control transformer is wound with higher impedance per phase. Since the some currents flow through the stators of the synchro transmitter and receiver, the same pattern of flux distribution will be produced in the air gap of the control transformer.

The control transformer flux axis is in the same position as that of the synchro transmitter. Thus the voltage induced in the rotor coil of control transformer is proportional to the cosine of the angle between the two rotors.

11a) Write the differential equations governing the system and draw force-current & force voltage analogous circuit.



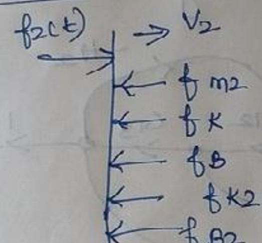
at node1



$$f_1(t) = f_{K1} + f_{B1} + f_{m1} + f_K + f_B$$

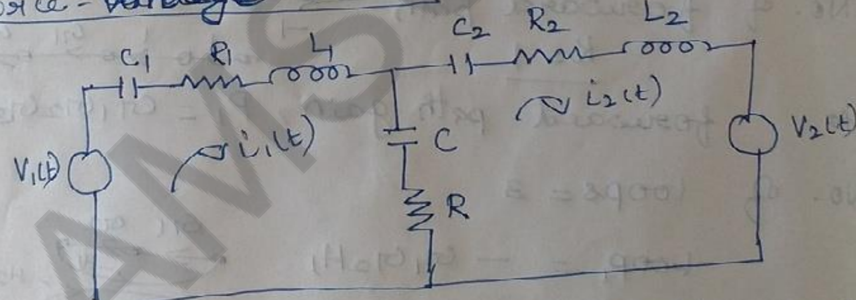
$$f_1(t) = K_1\int V_1\,dt + B_1V_1 + M_1\frac{dV_1}{dt} + K\int(V_1 - V_2)dt + B[V_1 - V_2]$$

at node2



$$f_2(t) = K_2\int V_2\,dt + B_2V_2 + M_2\frac{dV_2}{dt} + K\int(V_2 - V_1)dt + B[V_2 - V_1]$$

Force - ~~Voltage~~ circuit



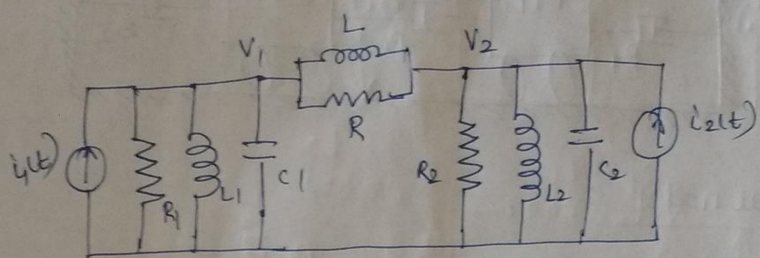
at loop1

$$V_1(t) = \frac{1}{C_1}\int i_1\,dt + R_1 i(t) + L_1\frac{di_1}{dt} + \frac{1}{C}\int(i_1 - i_2)dt + R(i_1(t) - i_2(t))$$

at loop2

$$V_2(t) = \frac{1}{C_2}\int i_2\,dt + R_2 i_2(t) + L_2\frac{di_2}{dt} + \frac{1}{C}\int(i_2 - i_1)dt + R(i_2(t) - i_1(t))$$

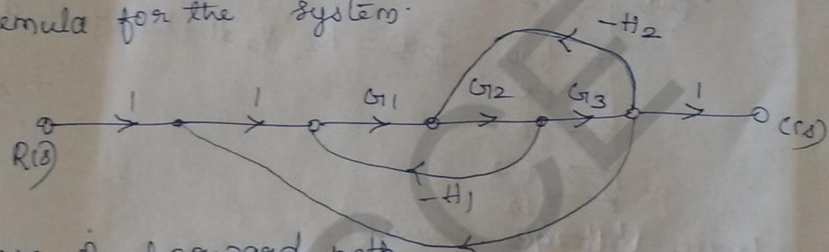
Force-current circuit



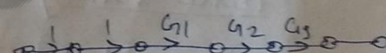
$$i_1(t) = \frac{V_1}{R_1} + \frac{1}{L_1}\int V_1\,dt + C_1\frac{dV_1}{dt} + \frac{1}{L}\int (V_1-V_2)\,dt + \frac{V_1-V_2}{R}$$

$$i_2(t) = \frac{V_2}{R_2} + \frac{1}{L_2}\int V_2\,dt + C_2\frac{dV_2}{dt} + \frac{1}{L}\int (V_2-V_1)\,dt + \frac{V_2-V_1}{R}$$

2. Obtain the transfer function using Mason's Gain formula for the system.



No. of forward path
$\underline{K = 1}$



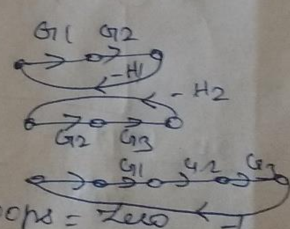
forward path gain, $P_1 = G_1G_2G_3$, $\Delta_k = 1$

No. of loops = 3

$Loop_1 = -G_1G_2H_1$

$Loop_2 = -G_2G_3H_2$

$Loop_3 = -G_1G_2G_3$



No. of two non-touching loops = Zero.

Transfer function, $T = \sum_{K=1} \frac{P_K \Delta_K}{\Delta}$

$$T = \frac{P_1 \Delta_1}{\Delta}$$

$$\Delta = 1 - [-G_1 G_2 H_1 - G_2 G_3 H_2 - G_1 G_2 G_3]$$

$$T = \frac{G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}.$$