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MA8452/ STATISTICS AND NUMERICAL METHODS
QUESTION BANK
UNIT 1
TESTING OF HYPOTHESIS
PART A

1. Define chi- square for goodness of fit.
Solution:

A test for resting the significance of discrepancy between experimental values and the theoretical values obtained under some theory or hypothesis is known as ψ^2 test for goodness of fit.

$\psi^2 = \sum \frac{(0-E)^2}{E}$ where 0 -observed frequency ,E - expected frequency and

ψ^2 is used to test whether difference between observed and expected frequency are significant

2. What are parameters and statistics is sampling ?
Solution:

The statistical constant of the population namely mean μ variance σ^2 which are usually referred to as parameters.

The statistical measured computed form sample observation alone ex: mean $\bar{x}$, variance s^2 etc., are usually referred to as statistic.

3.Write any two application of ψ^2 test ?
Solution:

ψ^2 is used to test whether difference between observed and expected frequencies are significant.

4. Define Null hypothesis and alternative hypothesis ?
Solution:
Null hypothesis:

For applying the test of significant, we first set up a hypothesis a definite statement about the population parameters. Such as hypothesis is usually a hypothesis of no difference and it is called Null hypothesis. It is denoted by H_0.

Alternative hypothesis:

Any hypothesis which is complementary to the null hypothesis is called alternative hypothesis usually denoted by H_1

5. Define level of significance.
Solution:

The probability 'α' that a random values ,of the statistic belongs to the critical region is known as the level of significance. In other words, the level if significance is the size of the Type I error .

6. State the application of F- distribution.
Solution:

1.To test if the 2 sample have come from the same population, we use F - test

2. To test , if the variance of 2 samples come from the population we use F - Test

7. What is small sample? What are test used for small sample?
Solution:

When the size of the sample (n) is less than 30, then that sample is called a small sample.

Test used for small samples

(i) Student's 't' Test
(ii) F- test
(iii) ψ^2 test

8. What is large sample? What are test used for large sample

Solution:

When the size of the sample(n) is greater than 30, then that sample is called a large sample

Test used for large sample

(i) Test of significance of single proportion
(ii) Test of significance for difference of proportion
(iii) Test of significance of single means
(iv) Test of significance for difference of means

9. What are the assumption of t-test ? (AUC/M/J 2011)

Solution:

The assumption of t-test are

(i) The parent population form which sample is drawn is noraml
(ii) The sample observation are independent
(iii) The population standard duration σ is unknown
(iv) Sample size n<30.

10. State the application of Chi - square test ? (AUC/M/J 2012)

Solution:

1.To test the goodness of fit
2. To test the independent of attributes
3. To test the homogeneous of independent estimations

11. What are the application of t-distributions?

Solution:

For small sample

(1) To test the significance of the mean of a random sample and the mean of the population.
(2) To test the significance of the difference between two sample means

12. A coin is tossed 400 times and its turns up head 216 times. Discuss whether the coin may unbiased one at 5% level if significance.

Solution:

Given n= 400, P = $\frac{1}{2}$

$\Rightarrow$Q = 1-P

= 1-1/2 $\Rightarrow$1/2

Q = 1/2

X = number of success

X = 216

Null hypothesis H_0: The coin is unbiased

Alternative hypothesis H_1 : The coin is biased

$\alpha = 5\% = 0.05$

Test Statistic

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{x - np}{\sqrt{npQ}}$$

$$= \frac{216 - (400)(1/2)}{\sqrt{(400)\,(1/2)(1/2)}}$$

Z_{cal} = 1.6

At $\alpha = 5\%$ Z_{tab} = 1.96

$\therefore$ $Z_{cal} < Z_{tab}$ (ie) 1.6 <1.96

Hence we accept null hypothesis H_o (ie) The coin is unbiased

13. Mention the various steps involved in testing of hypothesis

Solution:

The various steps involved in testing of hypothesis are

(1) Step up the null hypothesis H_o

(2) Step up the Alternative hypothesis H_1

(3) Select the appropriate level of significance

(4) Select the test statistic depends on the sample

14. The heights of a college students in Chennai are normally distributed with SD 6cm and sample of 100 students had their mean height 158cm. Test the hypothesis that the mean height of college students in Chennai is 160cm at 1% level of significance

Solution:

Null hypothesis H_o : $\mu = 160$

Alternative hypothesis H_1 : $\mu \neq 160$

Test statistic

$$Z = \frac{\bar{x} - \mu}{\sigma / \nu n}$$

$$= \frac{150 - 160}{6 / \sqrt{100}}$$

$$Z = -3.33$$

$$|Z| = 3.33$$

At α = 1% $\quad |Z| = 2.58$

$$\therefore \ |Z|_{cal} > |Z|_{tab}$$

$\Rightarrow$ Ho is rejected

PART B

1. Test mode on the breaking strength of 10 pieces of a metal gave the following results 578,572,570,568,572,570,572,596 and 584kg. Test if the breaking strength of the using can be assumed as 577kg

Solution:

Here S.D and mean of the sample is not given directly . we have determine these S.D. and mean as follows:

x	$x-\bar{x}$ x- (575.2)	$(x-\bar{x})^2$
578	2.8	7.84
572	-3.2	10.24
570	-5.2	27.04
568	-7.2	51.84
572	-3.2	10.24
570	-5.2	27.04
570	-5.2	27.04
572	-3.2	10.24
596	20.8	432.64
584	8.8	77.44

Σ= 5752		681.16

mean $\bar{x} = \frac{\sum x}{n} = \frac{5752}{10} = 575.2$

We know that, $S^2 = \frac{1}{n-1} \sum\left(x-\bar{x}\right)^2$

$= \frac{1}{10-1}(681.16)$

$S^2 = 75.7333$

Standard deviration S.D = $S = \sqrt{75.7333}$

s = 8.7025

Null Hypothesis (H_0): The data support the assumption of a population mean breaking strength μ=577kg

Alternative hypothesis (H_1): $\mu \neq 577$

Test Statistic : $t = \frac{\bar{x}-\mu}{\left(s/\sqrt{n}\right)}$

$t = \frac{575.2 - 577}{\left(8.7025/\sqrt{10}\right)}$

t = -0.6541

| t | = 0.6541

Calculated value of t = 0.6541

Tabulated value of t = 2.26 for (10-1)=9 d.f at 5% level of significance.

Since $t_{cal} < t_{tab}$, we accept the null hypothesis Ho. (ie) The data support the assumption of mean 577kg

2. A group of 10 rats fed on diet A and another group of 8 rats fed on diet B recorded the following increase in weight

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8	-	-

Show that the estimates of the population variance from the samples are not significantly different

Solution:

Null hypothesis H_0 : There is no significant difference between the variance increase in weight due to diets A & B (i.e) $S_1^2 = S_2^2$

Alternative hypothesis H_1 : $S_1^2 \neq S_2^2$

Test Statistic :

$F = \frac{S_1^2}{S_2^2}$ or $F = \frac{S_2^2}{S_1^2}$

To calculate sample means and variance:

x	$x - \bar{x}$ (x -6.4)	$(x - \bar{x})^2$	y	y- $\bar{y}$ (y-5)	(y- $\bar{y}$)^2
5	-1.4	1.96	2	-3	9
6	0.4	0.16	3	-2	4
8	1.6	2.56	6	1	1
1	-5.4	29.16	8	3	9
12	5.6	31.36	10	5	25

4	-2.4	5.76	1	-4	16
3	-3.4	11.56	2	-3	9
9	2.6	6.76	8	3	9
6	-0.4	0.16	-	-	-
10	3.6	12.96	-	-	-
Σ= 64		102.40	40		82

Mean of diet A = $\bar{x} = \frac{\sum x}{n_1}$

Here $n_1 = 10$, $\bar{x} = \frac{64}{10}$

$\bar{x} = 6.4$

Mean of diet B $\bar{y} = \frac{\sum y}{n_2}$

Here $n_2 = 8$, $\bar{y} = \frac{40}{8}$

$\bar{y} = 5$

$$S_2^2 = \frac{\sum (y-\bar{y})^2}{n_2 - 1}$$

$$S_2^2 = \frac{82}{8-1}$$

$S_2^2 = 11.7143$

$$S_1^2 = \frac{\sum (x-\bar{x})^2}{n_1 - 1}$$

$$\frac{102.40}{10-1}$$

$S^2_1 = 11.3778$

$$\therefore F = \frac{S_2^2}{S_1^2} \quad \left(\therefore S_2^2 > S_2^2\right)$$

$$= \frac{11.7143}{11.3778}$$

$F_{cal} = 1.0296$ with degrees of freedom $\upsilon = (n_2-1, n_1-1)$

$\upsilon = (7,9)$

Tabulated value of $F_{(7,9)}=3.12$

Since $F_{cal} < F_{tab}$, H_0 is a accepted.

(ie) there is no significant difference in population variance from the samples.

3. The mean value of a random sample of 60 items was found to be 145, with a standard deviation of 40. Find the 95% confidence limits for the population mean. What size of the sample is required to estimate the population mean within 5 of its actual value with 95% or more confidence, using the sample mean?

Solution:

Given n = 60

$\bar{x} = 145$ S = 40

Population mean 'μ' is not given

95% confidence limit for the population mean are

$\bar{x} \pm 1.96 \dfrac{s}{\sqrt{n}}$

$= 145 \pm 1.96\left(\dfrac{40}{\sqrt{60}}\right)$

$= 145 \pm 10.1214$

$= 155.1214$ and 134.8786

4.Test if the variances are significantly different for

X_1	24	27	26	21	25	
X_2	27	30	32	36	28	23

(NOV/DEC'2015)

Solution:

To test the variance are significantly different , we use F - test

Given $n_1 = 5, n_2 = 6$

Calculation for means and S.D of the samples

x	$x - \bar{x}$ (x -24.6)	$(x - \bar{x})^2$	y	y- $\bar{y}$ (y-29.33)	(y- $\bar{y}$)^2
24	-0.6	0.36	27	-2.33	5.4289
27	2.4	5.76	30	0.67	0.4489
26	1.4	1.96	32	2.67	7.1289
21	-3.6	12.96	36	6.67	44.4889
25	0.4	0.16	28	-1.33	1.7689
-	-	-	23	-6.33	40.0689
Σ=123		21.20	176		99.3334

$\bar{x} = \frac{\sum x}{n_1} = \frac{123}{5} = 24.6$

$\sum(x - \bar{x})^2 = 21.2$

$S_1^2 = \frac{\sum(x-\bar{x})^2}{n_1 - 1}$

$= \frac{21.20}{5-1}$

$S_1^2 = 5.3$

$\bar{y} = \frac{\sum y}{n_2} = \frac{176}{6} = 29.33$

$\sum(y - \bar{y})^2 = 99.3334$

$S_2^2 = \frac{\sum(y-\bar{y})^2}{n_2 - 1}$

$= \frac{99.3334}{6-1}$

$S_2^2 = 19.8667$

Null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$

Test statistic $F = \dfrac{S_2^2}{S_1^2} = \dfrac{19.8667}{5.3}$

$F_{cal} = 3.7484$ with degrees of freedom $\upsilon = (n_2-1, n_1-1)$

$\upsilon = (5,4)$

Tabulated value of F for (5,4) d.f at 5% level of significance is 6.26,

Since $F_{cal} < F_{tab}$,we accept H_o (ie) the variances are equal.

5. The number of accident in a certain locality was 12,8,20,2,4,10,15,6,9,4,. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 week period.

Solution:

Expected frequency of a accidents each week = $\dfrac{100}{10} = 10$

Null hypothesis H_o : The accident conditions were the same during the 10 week period

Observed frequency (0)	Expected frequency (E)	(0-E)	$\frac{(0-E)^2}{E}$
12	10	2	0.4
8	10	-2	0.4
20	10	10	10.0
2	10	-8	6.4
14	10	4	1.6
10	10	0	0
15	10	5	2.5
6	10	-4	1.6
9	10	-1	0.1
4	10	-6	3.6
Σ=100	100		26.6

Now $\Psi^2 = \Sigma\left[\frac{(0-E)^2}{E}\right]$

$\Psi^2 = 26.6$

(ie) calculated value $\Psi^2 = 26.6$

Degrees of freedom d.f= υ = n-1 = 10-1

υ = 9

Since $\Psi^2{}_{cal} > \Psi^2{}_{tab,}$ we reject the null hypothesis (ie) The accident conditions were not the same during the 10 week period.

6. A certain pesticide is packed into bags by a machine. A random sample of 10 bags is chosen and the contents of the bags is found to have the following weight (in kg) 50,49,52,44,45,48,46,45,49,and 45. Test if the average quantity packed be taken as 50 kg.

Solution:

Calculation for sample mean and S.D:

x	x- $\bar{x}$ =(x-47.3)	(x-$\bar{x}$)^2
50	2.7	7.29
49	1.7	2.89
52	4.7	22.09
44	-3.3	10.89
45	-2.3	5.29
48	0.7	0.49
46	-1.3	1.69
45	-2.3	5.29

49	1.7	2.89
45	-2.3	5.59
Σ=473		64.10

Mean = $\bar{x} = \frac{\sum x}{n} = \frac{473}{10} = 47.3$

We know that $S^2 = \frac{1}{n-1} \sum (x-\bar{x})^2$

$= \frac{64.10}{10-1}$

$S^2 = 7.12$

S = 2.67

Null hypothesis Ho : The average pack is 50kg (ie) $\mu = 50$

Alternative hypothesis H_1 : $\mu \neq 50$

Test statistic is $t = \frac{\bar{x}-\mu}{S/Vn} \rightarrow t = \frac{47.3-50}{\left(2.67/\sqrt{10}\right)}$

|t| =3.19

Calculated value of t for (n-1)= 9 d.f is 3.19

Tabulated value of 't' for 9 d.f is 2.262

$\rightarrow t_{cal} > t_{tab}$

Hence,we reject the null hypothesis H_0 (i.e)The average packing is not 50 Kgs.

7. Given $\overline{X_1}$ = 72, $\overline{X_2}$ =74, S_1 = 8, S_2 = 6, n_1 = 32, n_2 = 36. Test if the means are significant

Solution:

With the given data, it is determine that this test is large sample test to perform difference between sample means.

Null hypothesis H_o : There is no significant difference between sample means (ie) $\bar{X}_1 = \bar{X}_2$

Alternative hypothesis H_1 = $\bar{X}_1 \neq \bar{X}_2$

Test statistic $Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$

$$Z = \frac{72-74}{\sqrt{\frac{8}{32} + \frac{6}{36}}}$$

Z = -3.0984

|Z|=3.0984

At 5% level of significance |Z|= 1.96

$\rightarrow |Z|_{cal} > |Z|_{tab}$

(ie) 3.0984 > 96

We reject null hypothesis H_o

(ie) there is no significant difference between $\bar{x}_1$ and $\bar{x}_2$

8. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with S.D of 6 and the boys made an average grade of 82 with S.D of 2 . Test whether there is any difference between the performance of boys and girls.

Solution:

Given $n_1 = 50$ $\quad$ $n_2 = 75$

$\bar{x}_1 = 76$ $\quad$ $\bar{x}_2 = 82$

$\sigma_1 = 6$ $\quad$ $\sigma_2 = 2$

Null Hypothesis H_o : there is no significant difference between sample means (ie) $\bar{x}_1 = \bar{x}_2$

Alternative Hypothesis H_1 : $\bar{x}_1 \neq \bar{x}_2$

Test Statistic $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$|Z| = \dfrac{76-82}{\sqrt{\dfrac{36}{50} + \dfrac{4}{75}}}$

$|Z| = 6.8229$

At 5% level of significance, $|Z| = 1.96$

$\rightarrow |Z|_{cal} > |Z|_{tab}$

$\rightarrow H_o$ is rejected

(ie) there is significant difference between performance of girls and boys

9. Theory predicts the proportion of beans in the group A,B,C,D as 9:3:3:1. In an experiment among beans the numbers in the groups were 882,313,287 and 118. Does the experiment support theory?

Solution:

Null hypothesis Ho : The experimental result support the theory.

If we divide 1600 in the ratios 9:3:3:1, we get the expected frequencies as 900,300,300,100.

Observed frequency (0)	Expected frequency (E)	(0-E)	$\dfrac{(0-E)^2}{E}$
882	900	-18	0.360
313	300	13	0.563
287	300	-13	0.563
118	100	18	0.324
1600			4.726

$\therefore \; y^2 = \Sigma \dfrac{(0-E)^2}{E}$

$y^2 = 4.726$

(ie) Calculated value of $\chi^2 = 4.726$ for 3 d.f. At 5% level, $\chi^2_{tab} = 7.81$ for 3 d.f

$\rightarrow \chi^2_{cal} = \chi^2_{tab}$

$\rightarrow$ we accept null hypothesis

(i.e) The experimental results support the theory.

10. 400 men and 600 women were asked whether they would like to have a flyover near their residence 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same.

Solution:

Given sample sizes $n_1 = 400$, $n_2 = 600$

proportion of men $P_1 = \frac{200}{400} = 0.5$

proportion of women $P_2 = \frac{325}{600} = 0.541$

Null hypothesis H_o : Assume that there is no significant difference between the option of men and woman as for as proposal of flyover is concerned (ie) $P_1 = P_2$

Alternative hypothesis H1 : $P_1 \neq P_2$

The test statistic $Z = \frac{P_1 - P_2}{\sqrt{Pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

Where $P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$$= \frac{400\left(\frac{200}{400}\right) + 600\left(\frac{325}{600}\right)}{400 + 600} = \frac{525}{1000}$$

P = 0.525 ,Q = 1-P= 1-0.525

$$\therefore Z = \frac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400}+\frac{1}{600}\right)}}$$

$$= \frac{-0.041}{0.032} = -1.34$$

$|Z| = 1.34$

since $|Z| = 1.96$, at 5% level of significant, (ie) $|Z|_{cal} < |Z|_{tab}$,we accept the null hypothesis H_o

(ie) There is no difference of opinion between men and women as far as proposal of flyover is concerned.

11. The IQ's of 10 girls are respectively 120, 110,70,88,101,100,83,98,95,107. Test whether the population mean IQ is 100.

Solution:

Here S.D and mean of the sample is not given directly, we have to determine S.D and mean as follows.

x	$\left(x-\bar{x}\right)$ = (x-97.2)	$\left(x-\bar{x}\right)^2$
120	-27.2	739.84
110	22.8	519.84
70	12.8	163.84
88	3.8	14.44
101	-9.2	84.64
100	-14.2	201.64
83	-2.2	4.84

98	0.8	0.64
95	9.8	96.04
107	2.8	7.84
$\sum = 972$		1833.60

mean $\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$

We know that, $S^2 = \frac{1}{n-1} \sum\left(x - \bar{x}\right)^2$

$= \frac{1833.60}{9} = 203.73$

S=14.27

Null Hypothesis Ho : The data support the assumption of a population mean I.Q of 100 in the population.

Alternative hypothesis H_1: $\mu \neq 100$

Test statistic $t = \frac{\bar{x} - \mu}{S / vn}$

$= \frac{97.2 - 100}{14.27 / \sqrt{10}}$

t = -0.62

Calculated value of |t| = 2.26 for (10-1) = 9.d.f at 5% level of significance

Since $t_{cal} < t_{tab}$,we accept the null hypothesis H_o (ie) the data support the assumption of mean IQ of 100 in the population.

UNIT II

Design of Experiments

PArt-A

1. Write two advantages of completely randomised experimental design

Solution:

The advantages of a completely randomised experimental design are as follows:

(a) Easy to lay out

(b) Allows flexibility

(c) Simple statistical analysis

(d) The lots of information due to missing data is smaller than with any other design.

2. Is a 2x2 Latin square design possible ? why?

Solution:

A 2x2 Latin square design is not possible because, In a Latin square the formula for degree of freedom for residual (SSE) is d.f = (n-1) (n-2) on substituting n= 2 d.f = 0 , M.S.E = ∞

3. What do you understand by design of an experiment

Design of experiment is a systematic method to determine the relationship between factors affecting a process and the output of that process. In other words, it is used to find cause and effect relationship.

4. What are the basic principles of the design of experiments?

Solution:

The basic principles of experimental design are

(i) Randomization

(ii) Replication and

(iii) Local control

(i) Randomization is the corner stone underlying the use of statistical method in experimental design.
(ii) Replication means that repetition of the basic experiments
(iii) Local control means that experimental is unable to control extraneous sources of variation.

5. Compare one way classification model with two way classification model.

Solution:

One way classification	Two way classification
1. We cannot test two set of hypothesis	Two sets of hypothesis can be tested
2. Data are classified according to one factor	Data are classified according to two different factors

6. What is meant by Latin square?

Solution:

It may be necessary to control two sources of error or variability at the same time as the difference in rows and the difference in columns. (ie) It is desirable that each treatment should occurs once in each row and once in each column . This arrangement is called a Latin square.

7. What are the assumption involved in ANOVA?

Solution:

The assumption involved in ANOVA:

1. Each of the samples is drawn from a normal population.
2. The variation of each values around its own grand mean should to independent for each values.
3. The variances for the population from which samples have been drawn are equal.

8. Write the basic steps in ANOVA.

Solution:

1. One estimate of the population variance from the variance among the sample means
2. Determine a second estimate of the population variance from the variance within the sample
3. Compare these two estimate if they are approximately equal in values, accept the null hypothesis.

9. Define Analysis of variance.

Solution:

Analysis of variance (ANOVA) is a technique that will enable us to test for the significance of the difference among more than two sample means.

10. Define replication.

Solution:

To test the magnitude of an effect in an experiment the principle of randomisation and replication are applied. Randomisation by itself is not necessarily sufficient to yield a valid experiment. The replication or repetition of an experiment is also necessary. Randomisation must be invariably accompanied by sufficient replication so as to ensure validity in an experiment.

11. State the advantage of Latin square ones other designs.

Solution:

Advantages of the Latin square design over other design are

(a) Latin square controls more of the variation than the completely randomised block design with a two way classification

(b) The analysis is simple

(c) Even with missing data the analysis remains relatively simple.

12. Write the difference between RBD and LSD

Solution

S.No	Randomisation block design (RBD)	Latin square design (LSD)
1.	Available for a wide range of treatments	More efficient when there is diagonal trend of fertility (Variations in two directions)

2.	No restriction on the number of replication	Suitable only in the special cases where the land exhibits marked trends in fertility
3.	Flexible and easier to manage	Suitable only for 5 or 10 treatments
4.	Can be accommodated in a field of any shape	Shape of the field should be approximately square or rectangular.

13. Write down the ANOVA table for one - way classification
Solution:

Sources of variation	Sum of Square	Degrees of freedom	Mean sum of square	Variance ratio F - ratio
Between column	SSC	C-1	$MSC = \frac{SSC}{C-1}$	$F = \frac{MSC}{MSE}$ (or)
Within columns	SSE	N -C	$MSE = \frac{SSE}{N-C}$	$F = \frac{MSE}{MSC}$
Total	TSS	N-1		

14. Define Randomisation block design (RBD):
Solution:

To test the effect of 'K' fertilizing treatments on the yield of crop in an agriculture experiment, we assume that we know some information about the soil fertility of the plots. Then we divide the plots into 'K' blocks according to the soil fertility each block continuing 'K' blocks . Thus the 'K' manner such that each treatment occurs only once in any block. But the same 'K' treatments are repeated form block to block. This design is called Randomised block design.

PART B

1. The following table shows the lives in hours of four brands of electric lamps brand

A	1610	1610	1650	1680	1700	1720	1800	
B	1580	1640	1640	1700	1750	-		
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680		

Perform an analysis of variance and test the homogeneity of the mean lives of the four brands of lamps
Solution:
Null Hypothesis H_o = The lives of the 4 brands of lamps do not differ significantly.
Code the data by subtracting 1640 form the given values.

Treatment A		Treatment B		Treatment C		Treatment D	
X_1	X_1^2	X_2	X_2^2	X_3	X_3^2	X_4	X_4^2
-30	900	-60	3600	-180	32400	-130	16900
-30	900	0	0	-90	8100	-120	14400
10	100	0	0	-40	1600	-110	12100
40	1600	60	3600	-20	400	-70	4900
60	3600	110	12100	0	0	-40	1600
80	6400	-	-	20	400	40	1600
160	25600	-	-	100	10000	-	-
-	-	-	-	180	32400	-	-
Σ 290	39100	110	19300	-30	85300	-430	51500

Step 1:

Sum of all the items (T) = $\Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$

= 290+110-30-430

T = -60

Step 2:

Correction factors (C.F) = $\frac{T^2}{N} = \frac{(-60)^2}{20} = 138.46$

Step 3: TSS = total sum of square

=Sum of square of all the items - C.F

$= \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$

= 39100+19300+85300+51500-138.46

TSS = 195061.54

Step 4:

SSC = Sum of square between samples

$= \frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \frac{(\Sigma X_3)^2}{n_3} + \frac{(\Sigma X_4)^2}{n_4} - \text{C.F}$

$= \frac{(290)^2}{7} + \frac{(110)^2}{5} + \frac{(-30)^2}{8} + \frac{(-430)^2}{6} - 138.46$

SSC = 45224.99

Step 5: Mean square between sample

$= \frac{\text{Sum of square between sample}}{d.f}$

$= \frac{45224.99}{3}$

Step 6: SSE = sum of square within samples

= Total sum of square (TSS) - sum of square between samples (SSC)

= 195061.54 - 45224.99

SSE = 149836.55

Step 7: MSE = Mean square within samples

$= \frac{\text{Sum of square between sample}}{d.f}$

$= \frac{149836.55}{22} = 6810.75$

MSE = 6810.75

ANOVA table:

Source of variation	Sum of square	Degrees of freedom	Mean square	F- ratio
Between Samples	SSC= 45224.99	n-1= 4-1 = 3	$MSC = \frac{SSC}{d.f} = 15074.99$	$F_C = \frac{MSC}{MSE} = \frac{15074.99}{6810.75} = 2.2134$
Within samples	SSE = 149836.55	N-r-1 = 26-3-1 = 22	$MSE = \frac{SSE}{d.f} = 6810.75$	-

Tabulated values of F for $\upsilon_1 = 3$ and $\upsilon_2 = 22$ at 5% Level of significant is 3.05

(ie) $F_{tab} = 3.05$

$F_{calculated} = F_{cal} = 2.2134$

$\rightarrow F_{cal} < F_{tab}$

$\rightarrow$we accept the null hypothesis H_o .
The lives of the 4 brands of lamps do not differ significantly.

2. Analyse the variance in the following latin square of fields of paddy where A,B,C,D denote the different method of cultivation.

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C 122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields.
Solution:
Null hypothesis H_O : There is no significant difference between the methods of cultivation and yields.
Code the data by subtracting 120 form every value, we get

D2	A1	C3	B2
B4	C3	A2	D5
A0	B-1	D0	C1
C2	D3	B1	A2

Table I: To find TSS, SSR and SSC:

	C_1	C_2	C_3	C_4	Row Total (R_i)	$R_i^2/4$
R_1	2	1	3	2	8	16
R_2	4	3	2	5	14	49
R_3	0	-1	0	1	0	0
R_4	2	3	1	2	8	16
Column Total (Cj)	8	6	6	10	30(T)	$\Sigma \frac{R_i^2}{4} = 81$
$C_j^2/4$	16	9	9	25	$\Sigma \frac{C_j^2}{4} = 59$	

Table II: To find SST:

	1	2	3	4	Row Total (T_i)	$T_i^2/4$
A	0	1	2	2	5	6.25
B	4	-1	1	2	6	9
C	2	3	3	1	9	20.25
D	2	3	0	5	10	25
						$\Sigma \frac{T_i^2}{4} = 60.50$

Step 1:
Grand Total T = 30
Step 2:

Correction factor (C.F) $= = \frac{T^2}{N} = \frac{(30)^2}{16} = 56.25$

Step 3:
Sum of square of individual observations

$= (0)^2+(1)^2+(2)^2+ (4)^2+(-1)^2+ (1)^2+(2)^2 + (2)^2+ (3)^2 + (3)^2 + (1)^2 + (2)^2 + (3)^2 + (0)^2 + (5)^2$

= 92

Step 4:
TSS = Sum of square of individuals observation - C.F

= 92.-56.25

TSS = 35.75

Step 5:

SSR = Sum of square of rows$= \frac{\sum R_i^2}{4} - C.F$

= 81-56.25

SSR = 24.75

Step 6:

SSC = Sum of square of columns$= \frac{\sum C_j^2}{4} - C.F$

=59-56.25

=2.75

Step 7 :
SST = Sum of square of sum of treatments

$= \frac{\sum T_i^2}{4} - C.F$

= 60.50-56.25

SST = 4.25

Step 8:
SSE = Residual = RSS - (SSR+SSC+SST)

= 35.75 - (24.75 + 2.75 + 4.25)

SSE = 4

ANOVA TABLE:

Source of variation	Sum of Square	Degrees of freedom	Mean square	F -ratio	Conclusion
Rows	SSR = 24.75	4-1=3	$MSR = \frac{SSR}{D.F}$ $= \frac{24.75}{3}$ = 8.25	$F_R = \frac{MSR}{MSE}$ $= \frac{8.25}{0.67}$ = 1.231	Ho accepted
Columns	SSC = 2.75	4-1= 3	$MSC = \frac{SSC}{D.F}$ $= \frac{2.75}{3}$ = 0.917	$F_C = \frac{MSC}{MSE}$ $= \frac{0.917}{0.67}$ =1.369	Ho accepted
Treatment (or) Varieties	SST = 4.25	4-1= 3	$MST = \frac{SST}{D.F}$ $= \frac{4.25}{3}$ = 1.42	$F_T = \frac{MST}{MSE}$ $= \frac{1.42}{0.67}$ = 2.119	Ho accepted
Residual	SSE= 4	(4-1)(4-2) = 6	$MSE = \frac{SSE}{D.F}$ = 0.67	-	-

Tabulated value of F for (3,6) d.f is 4.76

(ie) $F_{tab} = 4.76$

(i) since $F_R < F_{tab}$, we accept the null hypothesis H_o .That is there is no significant difference between rows.

(ii) Since $F_C < F_{tab}$, we accept the null hypothesis $H_{o.}$ That is there is no significant difference between Columns.

(iii) Since $F_T < F_{tab}$, we accept the null hypothesis H_o .That is there is no significant difference between Treatments.

3. Given

Detergent	Engine		
	1	2	3
A	45	43	51
B	47	46	52
C	48	50	55
D	42	37	49

Perform ANOVA and test at 0.05 level of significant whether these are differences in the detergent or in the engines

Solution:

Null hypothesis H_o: (i) There is no significant difference between detergent

(ii) There is no significant difference between Engines

Code the data by subtracting 45 from each data

The coded data is

Detergent	Engine			TOTAL
	1	2	3	
A	0	-2	6	4
B	2	1	7	10
C	3	5	10	18
D	-3	-8	4	-7
Total	2	-4	27	T=25

Step 1:

Grand Total (T) = 25

Step 2:

$$\text{Correction factor (C.F} = \frac{T^2}{N} = \frac{625}{12} = 52.08$$

Step 3:

SSC = Sum of square between columns (engines)

$$= \frac{(2)^2}{4} - \frac{(-4)^2}{4} + \frac{(27)^2}{4} - C.F$$

= 4 + 4 + 182.25 - 52.08

SSC = 138.17

Step 4:

SSR = Sum of square between rows (Detergents)

$$= \frac{(4)^2}{3} + \frac{(10)^2}{3} + \frac{(18)^2}{3} + \frac{(-7)^2}{3} - C.F$$

=5.33 + 33.33 + 108 + 16.33 - 52.08

SSR = 110.91

Step 5:

Total sum of squares = sum of square of each values - correction factor

$$TSS = (0)^2 + (-2)^2 + (6)^2 + (2)^2 + (1)^2 + (7)^2 + (3)^2 + (5)^2 + (10)^2 + (-3)^2 + (-8)^2 + (4)^2 - 52.08$$

$$TSS = 264.92$$

Step 6:

SSE = Residual

= TSS - (SSC+SSR)

= 264.92 - (138.17 + 110.91)

SSE = 15.84

Source of variation	Sum of Square	Degrees of freedom	Mean square	F -ratio	F_{tab} at 0.05 level
Between Columns engines	SSC = 138.17	C-1 =-3-1 = 2	$MSR = \frac{SSC}{d.f}$ = 69.085	$F_C = \frac{MSC}{MSE}$ = 26.169	$F_{2,6} = 5.14$
Between Rows detergent	SSR = 110.91	r-1 = 4-1= 3	$MSR = \frac{SSR}{d.f}$ = 36.97	$F_R = \frac{MSR}{MSE}$ = 14.004	$F_{3,6} = 4.76$
Residual	SSE= 15.84	(c-1)(r-1) = (2) (3) = 6	$MSE = \frac{SSE}{D.F}$ = 2.64	-	-

(i) Since $F_R > T_{tab}$, we reject H_o (ie) there is difference between detergents

(ii) Since $F_C > F_{tab}$, we reject H_o (ie) there is difference between Engines

4. find out the main effects and interaction in the following 2^2 -factorial experiment and write down the ANOVA table

Block	I	a	b	Ab
	00	10	01	11
I	64	25	30	6
II	75	14	50	33
III	76	12	41	17
IV	75	33	25	10

Solution:

Null Hypothesis H_o : There is no significant difference between treatment (rows) and blocks (columns)

We re-arrange the given data in new table given below for computations of the SS due to treatment and blocks.

Treatment Combination	Blocks			
	I	II	III	IV
(1)	64	75	76	75
a	25	14	12	33
b	30	50	41	25
ab	6	33	17	10

Code the data by subtracting 37 from each data

Treatment combination	Blocks				Row Total R_i	R_i^2
	I	II	III	IV		

(1)	27	38	39	38	142 [1]	20164
a	-12	-23	-25	-4	-64[a]	4096
b	-7	+13	4	-12	-2[b]	4
ab	-31	-4	-20	-27	-82[ab]	6724
Column Total C_j	-23	24	-2	-5	-6(T)	309882 (ΣR_i^2)
C_j^2	529	576	4	25	1134 (ΣC_j^2)	

Here N = 4x4 = 16

Step 1:

Grand Total (T) = -6

Step 2:

Correction factor (C.F) = $= \frac{T^2}{N} = \frac{(-6)^2}{16} = 2.25$

Step 3:

Sum of squares of individuals observations

$= (27)^2 + (38)^2 + (39)^2 + (38)^2 + (-12)^2 + (-23)^2 + (-25)^2 + (-4)^2 + (-7)^2 + (13)^2 + (4)^2 + (-12)^2 + (-31)^2 + (-4)^2 + (-20)^2 + (-27)^2$

= 8936

Step 4:

TSS = sum of square of individual observation - C.F

= 8936 - 2.25

TSS = 8933.75

Step 5:

SSR = Sum of square of rows$= \frac{\Sigma R_i^2}{4} - C.F$

$= \frac{30988}{4} - 2.24$

SSR = 7744.75

Step 6:

SSC = Sum of Square of columns

$= \frac{\Sigma C_j^2}{4} - C.F$

$= \frac{1134}{4} - 2.24$

SSC = 281.25

Step 7:

SSE = Residual = TSS - (SSR+ SSC)

= 8933.75 - (7744.75 + 281.25)

SSE = 907.75

Step 8:

[a] = [ab] - [b] + [a] - [1]

= - 82 - (-2)+ (-64) - 142

[a] = -286

Step 9:

[b] = [ab]+ [b] - [a] - [1]

= - 82 + (-2)- (-64) - 142

[b] = -162

Step 10:

[ab] = [ab] - [b] - [a] + [1]
= - 82 - (-2)- (-64) +142
[b] = -126
Step 11:

$$S_a = \frac{[a]^2}{4r} = \frac{(-286)^2}{4X4} = 5112.25$$

Step 12:

$$S_b = \frac{[b]^2}{4r} = \frac{(-162)^2}{4X4} = 1640.25$$

Step 13:

$$S_{ab} = \frac{[ab]^2}{4r} = \frac{(126)^2}{4X4} = 992.25$$

ANOVA TABLE:

Sources of variation	Degrees of freedom	Sum of square	Mean square	F - ratio	F_{tab} (1%)	Conclusion
a	1	5112.25	MS_a = 5112.25	$F_a = \frac{MS_a}{MSE}$ = 50.69	6.99	Ho Rejected
b	1	1640.25	MS_b = 1640.25	$F_b = \frac{MS_b}{MSE}$ = 16.26	6.99	Ho Rejected
ab	1	992.25	MSa_b = 1640.25	$F_{ab} = \frac{MS_{ab}}{MSE}$ = 9.84	6.99	Ho Rejected
Residual	(4-1) (4-1) =9	907.75	$MSE = \frac{SSE}{9} = 100.86$	-	-	-

At 1% level of significance, the mean effect of a,b,ab is significance

5. Three varieties of coal were analysed by 4 chemist and the ash content is tabulated here. Perform an analysis of variance.

		Chemists			
Coal		A	B	C	D
	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

Solution:
Null hypothesis H_0 :(i) There is no significant difference between chemists
(ii)There is no significant difference between coals

		Chemists				Total
Coal		A	B	C	D	
	I	8	5	5	7	25
	II	7	6	4	4	21
	III	3	6	5	4	18
Total		18	17	14	15	64(T)

Step 1: Grand Total (T) = 64

Step 2: Correction factor = $c.f. = \frac{T^2}{N} = \frac{(64)^2}{12}$ = 341.3333

Step 3: SSC = Sum of square between columns (Chemist)

$= = \frac{1}{3}\left(18^2 + 17^2 + 14^2 + 15^2\right) - c.f$

= 344.6667-341.3333

SSC= 3.3334

Step 4: SSR = Sum of square between rows (coal

$= \frac{1}{4}\left(25^2 + 21^2 + 18^2\right) - c.f$

= 347.5 - 341.3333

SSR = 6.16667 = 6.1667

Step 5:TSS = Total sum of square

= Sum of square of each values - correction factor

= $(8)^2 + (5)^2 + (5)^2 + (7)^2 + (7)^2 + (6)^2 + (4)^2 + (4)^2 + (3)^2 + (6)^2 + (5)^2 + (4)^2$ -341.3333

TSS = 24.667

Step 6: SSE = Residual = TSS - (SSC + SSR)

= 24.6667 - (3.334+6.1667)

SSE = 15.1666

ANOVA TABLE:

Source of variation	Sum of Square	Degrees of freedom	Mean square	F -ratio	F_{tab} at 0.05 level	Conclusion
Between Columns (Chemists)	SSC = 3.3334	c-1 =4-1 = 3	$MSR = \frac{SSC}{d.f}$ = 1.1113	$\frac{MSC}{MSE}$ = 2.2746	$F_{6,3}$ = 8.94	Ho is accepted
Between Rows (Coals)	SSR = 6.1667	r-1 = 3-1= 2	$MSR = \frac{SSR}{d.f}$ = 3.0834	$\frac{MSR}{MSE}$ = 1.2198	$F_{2,6}$ = 5.14	Ho is accepted
Residual	SSE= 15.1666	(c-1)(r-1) = (3) (2) = 6	$MSE = \frac{SSE}{D.F}$ = 2.5278	-	-	-

There is no significant difference between chemists and between ash content of coal at 5% level of significance.

6. The result of RBD experiment on 3 blocks with 4 treatments A,B,C,D are tabulated here . Carry out an analysis of variance

Blocks	Treatment effects			
I	A36	D35	C21	B36
II	D32	B29	A28	C31
III	B28	C29	D29	A26

Solution:

Null Hypothesis H_0: There is no significant difference between treatments.

Blocks	Treatment effects			
	A	B	C	D
I	36	36	21	35
II	28	29	31	32
III	26	28	29	29

code the data by subtracting 30 from each to simplify

Blocks	Treatment effects				Total
	A	B	C	D	
I	6	6	-9	5	8
II	-2	-1	1	2	0
III	-4	-2	-1	-1	-8
Total	0	3	-9	6	0(T)

Step 1: Grand Total (T) = 0

Step 2: Correction factor (C.F) = $\frac{T^2}{N}$ = 0

Step 3: SSC = sum of square between columns

$$= \frac{(0)^2}{3} + \frac{(3)^2}{3} + \frac{(-9)^2}{3} + \frac{(6)^2}{3} - C.F$$

= 42-0

SSC = 42

Step 4: SSR = Sum of square between RowS

$$= \frac{(8)^2}{4} + \frac{(0)^2}{4} + \frac{(-8)^2}{4} - C.F$$

= 32-0

SSR = 32.

Step 5:TSS =Total sum of square

= Sum of square of each values - corrections factor

$=(6)^2 + (6)^2 + (-9)^2 + (5)^2 + (-2)^2 + (-1)^2 + (1)^2 + (1)^2 + (2)^2 + (-4)^2 + (-2)^2 + (-1)^2 -(-1)^2 -0$

TSS = 210

Step 6: Residual=SSE = TSS - (SSR + SSC)

= 210-42-32

SSE = 136

ANOVA Table:

Source of variation	Sum of Squares	Degrees of freedom	Mean squares	F -ratio	F_{tab} at 0.05 level	Conclusion
Between Columns (Treatments)	SSC = 42	c-1 =4-1 = 3	$MSR = \frac{SSC}{d.f}$ = 14	$F_C = \frac{MSC}{MSE}$ = 1.169	$F_{6,3}$ = 8.94	$F_{tab} > F_{cal}$ $\rightarrow H_o$ is accepted
Between Rows (Blocks)	SSR = 32	r-1 = 3-1= 2	$MSR = \frac{SSR}{d.f}$ =16	$F_R = \frac{MSE}{MSR}$ = 1.417	$F_{6,2}$ = 19.35	$F_{tab} > F_{cal}$ $\rightarrow H_o$ is accepted
Residual	SSE= 136	(c-1)(r-1) = 6	$MSE = \frac{SSE}{d.f}$ =22.67	-	-	

Conclusion:
There is no significant difference between treatment at 5% level of significance.

7. Carry out ANOVA (Analysis of variance) for the following:

Workers		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Solution:

Null Hypothesis H_o : (i) The mean productivity is the same for four different machines

(ii) The 5 men do not differ with respect to mean productivity

Code the given data by subtracting 40 from each value

The code data is,

Workers	Machine type				Total
	A	B	C	D	
1	4	-2	7	-4	5
2	6	0	12	3	21
3	-6	-4	4	-8	-14
4	3	-2	6	-7	0
5	-2	2	9	-1	8
Total	5	-6	38	-17	T = 20

Step1: Grand Total (T) = 20

Step 2 : Correction factor ($C.F$) = $\frac{T^2}{N} = \frac{(20)^2}{20} = 20$

Step 3: SSC = Sum of square between column (machine)

$$= \frac{(5)^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - C.F$$

$$= 5 + 7.2 + 288.8 + 57.8 - 20 \Rightarrow SSC = 338.8$$

Step 4: SSR = Sum of square between Rows (workers)

$$= \frac{(5)^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{(0)^2}{4} + \frac{(8)^2}{4} - C.F$$

$$= 6.25 + 110.25 + 49 + 16 - 20$$

$$\Rightarrow SSC = 161.5$$

Step 5: TSS = Total sum of squares

= Sum of square of each values - $C.F$

$=(4)^2 + (-2)^2 + (7)^2 + (-4)^2 + (6)^2 + (0)^2 + (12)^2 + (3)^2 + (-6)^2 + (-4)^2 + (4)^2 + (-8)^2 + (3)^2 + (-2)^2 + (6)^2 + (-7)^2 + (-2)^2 + (2)^2 + (9)^2 + (-1)^2 - 20$

TSS = 574

Step 6: Residual = SSE = TSS - (SSC+SSR)

= 574 - (338.8 + 161.5)

SSE = 73.7

ANOVA TABLE:

Sources of variations	Sum of squares	Degrees of freedom	Mean square	F-ratio	F-tabulated at α = 5%	Conclusion
Between Column (machines)	SSC = 338.8	c-1 = 4-1 = 3	$MSC = \frac{338.8}{3}$ = 112.93	$F_C = \frac{MSC}{MSE}$ = 18.38	$F_{3,12}$ = 3.49	$F_{cal} > F_{tab}$ $\Rightarrow H_o$ is rejected
Between Rows (workers)	SSR = 161.5	r -1 = 5-1 = 4	$MSC = \frac{161.5}{3}$ = 40.38	$F_R = \frac{MSR}{MSE}$ = 6.574	$F_{4,12}$ = 3.26	$F_{cal} > F_{tab}$ $\Rightarrow H_o$ is rejected

Residual	SSE = 73.7	(c-1) (r-1) = (3)(4) = 12	$MSE = \frac{73.7}{12} = 6.142$			

Conclusion:
Therefore from ANOVA table, H_o is rejected in both columns and rows.
(i.e) The mean productivity is not the same for the four different types of machines and
The workers differ with respect to mean productivity

8. Perform Latin square experiment for the following:

Rome	I	II	III	Three equally space concentrations of poison as extracted from the scorpion fish.
Arabic	1	2	3	Three equally spaced body weights for the animals tested
Latin	A	B	C	Three equally spaced times of storage of the poison before it is administered to the animals

	I	II	III
1	0.194 (A)	0.73 (B)	1.187 (C)
2	0.758(C)	0.311 (A)	0.589 (B)
3	0.369 (B)	0.558 (C)	0.311 (A)

Solution:
Null Hypothesis H_o : There is no significance difference between rows, between columns and treatments

A(0.194)	B(0.73)	C(1.187)
C(0.758)	A(0.311)	B(0.589)
B(0.369)	C(0.558)	A(0.311)

Table I :(To find SSC, SSR and TSS)

	C_1	C_2	C_3	Row total (Ri)	$R_i^2/3$
R_1	0.194	0.73	1.187	2.111	1.485
R_2	0.758	0.311	0.589	1.658	0.916
R_3	0.369	0.558	0.311	1.238	0.511
Column Total (C_j)	1.321	1.599	2.087	5.007(T)	$\sum R_i^2/3 = 2.912$
$C_j^2/3$	0.582	0.852	1.452	$\sum C_i^2/3 = 2.886$	

Table II :To find SST

	1	2	3	Row total(T_i)	$T_i^2/3$
A	0.194	0.311	0.311	0.816	0.2219
B	0.369	0.730	0.589	1.688	0.9498
C	0.758	0.558	1.187	2.503	2.0883
					$\sum T_i^2/3 = 3.2600$

Step 1: Grand total (T) = 5.007

Step 2: Correction factor ($C.F$) = $\frac{T^2}{N} = \frac{5.007}{9} = 2.79$

Step 3: SSR =Sum of square between Rows

$$= \frac{\sum R_i^2}{3} - C.F$$

=2.912 - 2.79

SSR = 0.122

Step 4: SSC= Sum of square between Columns

$$= \frac{\sum C_j^2}{3} - C.F$$

= 2.886-2.79

SSC = 0.096

Step 5: TSS = Sum of squares of individual observations -*C.F*

$= (0.194)^2 + (0.73)^2 + (1.187)^2 + (0.758)^2 + (0.311)^2 + (0.589)^2 + (0.369)^2 + (0.558)^2 + (0.311)^2 - 2.79$

TSS = 3.542 - 2.79

TSS = 0.752

Step 6: SST = Sum of square of treatments

$$= \frac{\sum T_i^2}{3} - C.F$$

= 3.26 - 2.79

SST = 0.47

Step 7: SSE = Residual

=TSS - (SSR + SSC + SST)

=0.752 - (0.122 + 0.096 + 0.47)

SSE = 0.064

ANOVA Table:

Sources of variations	Sum of square	Degrees of freedom	Mean square	F - ratio	F_{tab} at α = 5%	Conclusion
Rows	SSR = 0.122	r-1 = 3-1 = 2	$MSR = \frac{0.122}{2}$ = 0.061	$F = \frac{MSR}{MSE}$ F_R = 1.906	$F_{2,2}$ = 19.00	$F_R < F_{tab}$ $\Rightarrow H_o$ accepted
Columns	SSC = 0.096	c-1 = 3-1 = 2	$MSC = \frac{0.096}{2}$	$F_C = \frac{MSC}{MSE}$ = 1.5	$F_{2,2}$ = 19.00	$F_C < F_{tab}$ $\Rightarrow H_o$ accepted
Treatments	SST = 0.47	T-1= 3-1 = 2	$MST = \frac{0.47}{2}$ = 0.235	$F_T = \frac{MST}{MSE}$ = 7.34	$F_{2,2}$ = 19.00	$F_T < F_{tab}$ $\Rightarrow H_o$ accepted
Residual	SSE=0.064	(3-1)(3-2)=2	$MSE = \frac{0.064}{2}$ =0.032	-	-	-

Conclusion:

From the ANOVA table, the calculated F-ratio is lesser than the tabulated F-value ,we accept the null hypothesis for all sources of variations.

(ie) There is no significant difference between rows, between columns and between treatments as $F_R < F_{tab}$, $F_C < F_{tab}$, $F_T < F_{tab}$ at α = 5% level of significance.

9. Three varieties A,B, C of a crop are tested in a randomized block design with four replications. The plot yields in pounds are as follows:

A6	C5	A8	B9
C8	A4	B6	C9
B7	B6	C10	A6

Analyse the experimental yield and state your conclusion.

Solution:

Null hypothesis H_o :There is no significant difference between varieties (rows) and between yields(columns)

Varieties	Yields				Total
	1	2	3	4	
A	6	4	8	6	24
B	7	6	6	9	28
C	8	5	10	9	32
Total	21	15	24	24	84(T)

Step 1: Grand Total (T) = 84

Step 2: Correction factor ($C.F$) = $\frac{T^2}{N} = \frac{(84)^2}{12}$ = 588

Step 3: SSC =Sum of squares between Columns

$$= \left(\frac{C_1^2}{3} + \frac{C_2^2}{3} + \frac{C_3^2}{3} + \frac{C_4^2}{3}\right) - C.F$$

$$= \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588$$

SSC = 606 - 588

SSC = 18

Step 4: SSR =Sum of square between Rows

$$= \left(\frac{R_1^2}{3} + \frac{R_2^2}{3} + \frac{R_3^2}{3}\right) - C.F$$

$$= \frac{(24)^2}{3} + \frac{(28)^2}{3} + \frac{(32)^2}{3} - 588$$

= 596-588

SSR = 8

Step 5: TSS = Total Sum of squares

= Sum of square of individuals observation - C.F

$$= \left[(6)^2 + (7)^2 + (8)^2 + (4)^2 + (6)^2 + (5)^2 + (8)^2 + (6)^2 + (10)^2 + (6)^2 + (9)^2 + (9)^2\right] - 588$$

=624 - 588

TSS = 36

Step 6: SSE = Residual

= TSS - (SSR + SSC)

= 36 - (8+18)

SSE = 10

ANOVA Table:

Sources of variations	Sum of squareS	Degrees of freedom	Mean square	F-ratio	F-tabulated at α = 5%	Conclusion
Between Rows (varieties)	SSR = 8	r -1 = 3-1 = 2	$MSR = \frac{SSR}{d.f}$ = 4	$F_R = \frac{MSR}{MSE}$ = 2.4	$F_{2,6}$ = 5.14	$F_R < F_{tab}$ $\Rightarrow H_o$ is accepted
Between Columns (yields)	SSC = 18	c-1 = 4-1 = 3	$MSC = \frac{SSC}{d.f}$ = 6	$F_C = \frac{MSC}{MSE}$ = 3.6	$F_{3,6}$ = 4.76	$F_c < F_{tab}$ $\Rightarrow H_o$ is accepted

Residual	SSE = 10	(r-1) (c-1) = (2)(3) = 6	$MSE = \frac{SSE}{d.f}$ = 1.667	-	-	-

Conclusion:

From the ANOVA table, the calculated F - ratio is lesser than the tabulated F- Value. Hence, we accept the null hypothesis H_o.

(ie) There is no significant difference between varieties since $F_R < F_{Tab}$

and there is no significant difference between yields since $F_C < F_{Tab}$ at 5% level of significance.

10. Five varieties of paddy A,B,C,D, and E are tried. The plan ,the varieties shown in each plot and yields obtained in Kg are given in the following table:

B 95	E 85	C 139	A 117	D 97
E 90	D 89	B 75	C 146	A 87
C 116	A 95	D 92	B 89	E 74
A 85	C 130	E 90	D 81	B 77
D 87	B 65	A 99	E 89	C 93

Test whether there is a significant difference between rows and columns at 5% level of significance

Solution:

Null hypothesis H_o : There is no significant difference between rows, columns and treatment

Code the data by subtracting 100 from each value

B -5	E -15	C 39	A 17	D -3
E -10	D -11	B -25	C 46	A -13
C 16	A -5	D -8	B -11	E -26
A -15	C 30	E -10	D -19	B -23
D -13	B -35	A -1	E -11	C -7

Table I : (To find SSC, SSR, and TSS)

	C_1	C_2	C_3	C_4	C_5	Row total R_i	$R_i^2/5$
R_1	-5	-15	39	17	-3	33	217.8
R_2	-10	-11	-25	46	-13	-13	33.8
R_3	16	-5	-8	-11	-26	-34	231.2
R_4	-15	30	-10	-19	-23	-37	273.8
R_5	-13	-35	-1	-11	-7	-67	897.8
Column Total (C_j)	-27	-36	-5	22	-72	-118 (T)	$\sum R_i^2/S = 1654.4$
$C_i^2/5$	145.8	259.2	5	96.8	1036.8		$\sum \frac{C_i^2}{5} = 1543.6$

Table II: (To find SST)

	1	2	3	4	5	Row Total (T_i)	$T_i^2/5$
A	-15	-5	-1	17	-13	-17	57.8
B	-5	-35	-25	-11	-23	-99	1960.2
C	16	30	39	46	-7	124	3075.2
D	-13	-11	-8	-19	-3	-54	583.2
E	-10	-15	-10	-11	-26	-72	1036.8
							$\sum \frac{T_i^2}{5} = 5783.2$

Step 1: Grand total (T) = -118

Step 2 : Correction factor $(C.F) = \frac{T^2}{N} = \frac{(-118)^2}{25} = 556.96$

Step 3: SSR = Sum of square between Rows

$= \sum \frac{R_i^2}{5} - C.F$

=1654.4-556.96

SSR = 1097.44

Step 4: SSC = Sum of squares between columns

$= \left(\sum \frac{C_i^2}{5}\right) - C.F$

=1543.6 -556.96

SSC = 986.64

Step 5: TSS = Total sum of squares

= Sum of square of individuals observation - C.F

$= (-15)^2 + (-5)^2 + (-1)^2+(17)^2 + (-13)^2+ (-5)^2+ (-5)^2+(-25)^2+(-11)^2+(-23)^2+(-16)^2+(-30)^2+(-39)^2+(-46)^2+(-7)^2+(-13)^2+(-11)^2+(-8)^2+(-19)^2+(-3)^2+(-10)^2+(-15)^2+(-10)^2+(-11)^2+(-26)^2- 556.96$

TSS = 10022 - 556.96

TSS = 9465.04

Step 6: SST = sum of square between treatment

$= \left(\sum \frac{T_i^2}{5}\right) - C.F$

=5783.2-556.96

SST = 5226.24

Step 7: SSE = Residual

= TSS - (SSR + SSC + SST)

= 9456.04 - (1097.44 + 986.64 + 522.24)

SSE =2154.74

Source of variation	Sum of Square	Degrees of freedom	Mean square	F -ratio	F_{tab} at 0.05 level	Conclusion
Between Rows	SSR = 10944.44	n-1 =-5-1 = 4	$MSR = \frac{1097.44}{4}$ = 274.36	$F_R = \frac{MSR}{MSE}$ = 1.528	$F_{4,12} = 3.26$	$F_R < F_{tab}$ $\Rightarrow H_o$ is accepted

Between Columns	SSC = 986.64	n-1 = 5-1= 4	$MSC = \frac{986.64}{4}$ =246.66	$F_C = \frac{MSC}{MSE}$ = 1.374	$F_{4,12}$ = 3.26	$F_C < F_{tab}$ $\Rightarrow H_o$ is accepted
Treatment	SST = 5226.24	n-1 = 5-1= 4	$MST = \frac{5226.24}{4}$ = 1306.56	$F_T = \frac{MST}{MSE}$ = 7.276	$F_{4,12}$ = 3.26	$F_C < F_{tab}$ $\Rightarrow H_o$ is accepted
Residual	SSE= 2154.74	(n-1)(n-2) = (5-1) (5-2) = 12	$MSE = \frac{2154.74}{12}$ = 179.5	-	-	-

Conclusion:
From ANOVA table, since the calculated F ratio is lesser than the tabulated values of F at α =5% level of significance, we accept the null hypothesis H_o.
(ie) There is no significant difference between rows, columns and treatments at α = 5% level of significance.

UNIT III
PART A

1. Obtain the iterative formula to find $\frac{1}{N}$ using newton Raphson method .(Nov/Dec'2015)

Solution:
Let $x = \frac{1}{N}$; $N = \frac{1}{x} \Longrightarrow \frac{1}{x} - N = 0$

$f(x) = \frac{1}{x} - N$; $f^1(x) = \frac{-1}{x^2}$

The Newton's formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \frac{(1/x_n - N)}{(-1/x^2{}_n)}$$

$$= \left(x_n + \left(\frac{1}{x_n} - N \right) \right) x^2{}_n$$

$$= x_n + x_n - Nx^2{}_n$$

$$= 2x_n - Nx^2{}_n$$

$$= x_{n+1} = x_n \, (2 - Nx_n)$$

is the iterative formula to find $\frac{1}{N}$.

2. Compare Gauss elimination with Gauss – siedel.
Solution:

S.No	Gauss Elimination	Gauss-Siedel
1.	It is Direct method	It is Iterative method
2.	It gives exact values	It gives only approximate solution
3.	Simple ,take less time	Time consuming and laborious
4.	This method determine all the roots at the same time using back - substitution process	This method determine only one root at a time.

3. Perform four iteration of the newton Raphson method to find the smallest positive root of the equation $f(x)= x^3-5x +1 = 0$.

Solution:

$f(x)= x^3-5x +1 = 0$

$f'(x)= 3x^2-5$

$f(0)=1$

$f(1)=-3$

$\therefore$ A root lies between 0 and 1.

Let $x_0 = 0.5$

Newton-Raphson formula is

$$x_{n+1} = xn - \frac{f(x_n)}{f^1(x_n)}$$

$$x_{n+1} = xn - \frac{(x_n^3 - 5x_n + 1)}{(3x_n^2 - 5)}$$

$$= \frac{3x_n^2 - 5x_n - x_n^3 + 5x_n - 1}{3x_n^2 - 5}$$

$$x_{n+1} = \frac{2x_n^3 - 1}{3x_n^2 - 5}$$

$$\text{put n} = 0,\ x_1\ \frac{2x_0^3 - 1}{3x_0^2 - 5} = \frac{2(0.5)^3 - 1}{3(0.5)^2 - 5} = 0.1765$$

$$\text{put n} = 1,\ x_2\ \frac{2x_1^3 - 1}{3x_1^2 - 5} = \frac{2(0.1765)^3 - 1}{3(0.1765)^2 - 5} = 0.1883$$

$$\text{put n} = 2,\ x_3\ \frac{2x_2^3 - 1}{3x_2^2 - 5} = \frac{2(0.1883)^3 - 1}{3(0.1883)^2 - 5} = 0.2016$$

$$\text{put n} = 3,\ x_1\ \frac{2x_3^3 - 1}{3x_3^2 - 5} = \frac{2(0.2016)^3 - 1}{3(0.2016)^2 - 5} = 0.2016$$

$\therefore$ A positive root of the given equation $f(x) = x^3-5x + 1 = 0$ $x= 0.2016$

4. Solve the equation 10x-y+2z= 4: x+ 10y-z =3 ;2x +3y+20z= 7 using the guass elimination method

Solution:

The given system of eqn's are 10x-y+2z=4, x+10y-z =3, 2x+3y+20z =7

$$\text{Coefficient matrix is is A} = \begin{pmatrix} 10 & -1 & 2 \\ 1 & -10 & -1 \\ 2 & 3 & 20 \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$$

By Gauss elimination method ,

$$\text{the augmented matrix is [A,B]} = \left(\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 1 & -10 & -1 & 3 \\ 2 & 3 & 20 & 7 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 10 & -1 & 2 & 4 \\ 0 & +101 & -12 & 26 \\ 0 & +16 & +98 & 31 \end{array}\right) \quad \begin{array}{l} R_2 = R_2 - 10R_1 \\ R_3 = R_3 - 5R_1 \end{array}$$

$$= \begin{pmatrix} 10 & -1 & 2 & 4 \\ 0 + & 101 & -12 & 26 \\ 0 + & 0 + & 10090 & 2715 \end{pmatrix} \qquad R_3^1 = 101R_3^1 - 16R_2^1$$

By Back substitution,

10090z = 2715

z= 0.2691

101y - 12y = 26

y = 0.2894

10x - y+2z = 4

x = 0.3751

$\therefore$ Solution is

x= 0.3751, y = 0.2894, z= 0.2691.

5. Mention the order and condition for the convergence of Newton - Raphson Method. (May/June'2016)

solution:

The order of Newton Raphson method is Two

The condition for convergence is $|f(x) f''(x) < |f'(x)|^2$.

6. Write the procedure of Gauss Jordan method.

Solution:

To solve a system of equations we use Gauss Jordan method. The procedure are as follows:

Step1 : Write the coefficient matrix A & B of the given system AX = B

Step 2: Reduce the augmented matrix [A, B] to a diagonal matrix [D.,Y]

Step 3: Using direct substitution find the values of the corresponding variables in X

7. Using Newton - Raphson method find the iteration formula to compute $\sqrt{N}$.

Solution:

Let $x = \sqrt{N}$

$\Rightarrow x^2 = N$

$\Rightarrow x^2 - N = 0$

Let $f(x) = x^2 - N$

$f^1(x) = 2x$

Newton Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \left(\frac{x_n^2 - N}{2x_n}\right)$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{x_n^2 + N}{2x_n}$$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

is the iterative formula to find $\sqrt{N}$

8. Solve by Gauss - Jordan method the following system of equation $2x_1 + x_2 = 3$, $x_1 + 2x_2 = 3$
Solution:

The given system of equations are

$$A\,\mathrm{x} = \mathrm{B} \Rightarrow \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

The augmented matrix is

$$[\mathrm{A,B}] = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 3 \end{bmatrix} \qquad R_2^1 = 2R_2 - R_1$$

$$= \begin{bmatrix} 6 & 0 & 6 \\ 0 & 3 & 3 \end{bmatrix} \qquad R_2^{11} = 3R_2^1 \; R_1^1{}_2$$

$\therefore$ Solution is $x_1 = 1$, $x_2 = 1$

9. Define a diagonally dominant system of equations.
Solution:

A matrix is diagonally dominant if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical values of the other elements in that row.
Example:

$$\begin{pmatrix} 5 & 1 & -1 \\ 1 & 4 & 2 \\ 1 & -2 & 5 \end{pmatrix} \text{ is diagonally dominant}$$

10. Derive Newton's algorithm for finding the Pth root of a number N.
Solution:
The pth root of a positive number N is the equation

$$x = N^{1/p}$$

$$\Rightarrow x^P = N$$

$$\text{(ie)} \;\; f(x) = x^P - N = 0$$

$$f^1(x) = p_x{}^{p-1}$$

By Newton's algorithm,

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$= x_n - \left(\frac{x_n^p - N}{Px_n^{p-1}} \right)$$

$$= \frac{Px_n^{p-1} - x_n^p + N}{Px_n^{p-1}}$$

$$x_{n+1} = \frac{(P-1)\, x_n^p + N}{Px_n^{p-1}}$$

is the iterative formula to find the pth root of a positive number N

11. What is the condition for convergence of Gauss Jacobi method of iteration?
Solution:

The coefficient matrix must be diagonally dominant is the condition for convergence of Gauss- Jacobi method of iteration

12. What type of Eigen value can be obtained using power method?
Solution:

We can obtain dominant eigen value (Largest eigen Value) of a given matrix using power method

13. Find the dominant eigen value of A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method.

Solution:

Let $X_0=\begin{pmatrix}1\\1\end{pmatrix}$ be the intial eigen vector

then, $AX_0 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}3\\7\end{pmatrix} = 7\begin{pmatrix}0.4286\\1\end{pmatrix} = \lambda X_1$

$$= AX_1 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}0.4286\\1\end{pmatrix} = \begin{pmatrix}2.4286\\5.2858\end{pmatrix} = 5.2858\begin{pmatrix}0.4595\\1\end{pmatrix} = \lambda X_2$$

$$= AX_2 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}0.4595\\1\end{pmatrix} = \begin{pmatrix}2.4595\\5.3785\end{pmatrix} = 5.3785\begin{pmatrix}0.4573\\1\end{pmatrix} = \lambda X_3$$

$$= AX_3 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}0.4573\\1\end{pmatrix} = \begin{pmatrix}2.4595\\5.3719\end{pmatrix} = 5.3719\begin{pmatrix}0.4574\\1\end{pmatrix} = \lambda X_4$$

$$= AX_4 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}0.4574\\1\end{pmatrix} = \begin{pmatrix}2.4574\\5.3722\end{pmatrix} = 5.3722\begin{pmatrix}0.4574\\1\end{pmatrix} = \lambda X_5$$

$$= AX_5 = \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}0.4574\\1\end{pmatrix} = \begin{pmatrix}2.4574\\5.3722\end{pmatrix} = 5.3722\begin{pmatrix}0.4574\\1\end{pmatrix} = \lambda X_6$$

Since $AX_4 = AX_5$ & $\lambda X_5 = \lambda X_6$

(ie) $5.3722\begin{pmatrix}0.4574\\1\end{pmatrix}$, the largest eigen value of the given matrix A is λ= 5.3722 and the corresponding eigen vector is $\begin{pmatrix}0.4574\\1\end{pmatrix}$

14. Explain Gauss - siedel method to solve a system of simultaneous equations
Solution:

Let the method to solve a system of simultaneous equations

$a_1x + b_1y + C_1z = d_1$

$a_2x + b_2y + C_2z = d_2$

$a_3x + b_3y + C_3z = d_3$

Let us assume that

The coefficient matrix of the above system the diagonal dominant

Let us rearrange the given equation as,

$$x = \frac{1}{a_1}\left(d_1 - b_1y - c_1z\right)$$

$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$

$$z = \frac{1}{c_3}\left(d_3 - a_3x - b_3y\right)$$

we start with the initial values y_o and Z_0 for y and Z we get x_1 (ie)

$$x_1 = \frac{1}{a_1}(d_1 - b_1 y_0 - c_1 z_0)$$

using x_1 & z_0 , $y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - c_2 z_0)$

using x_1 ,y_1 & $z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$

This process may be continued until convergence is assured to all the solution.

15. Find the dominant eigen values of $A = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}$ using power method

solution:

Let $X_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be the initial eigen vector

Now, $Ax_0 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} = 5\begin{pmatrix} 1 \\ 0.8 \end{pmatrix} = \lambda x_1$

$$Ax_1 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 4.8 \\ 3.4 \end{pmatrix} = 4.8\begin{pmatrix} 1 \\ 0.71 \end{pmatrix} = \lambda x_2$$

$$Ax_2 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.71 \end{pmatrix} = \begin{pmatrix} 4.71 \\ 3.13 \end{pmatrix} = 4.71\begin{pmatrix} 1 \\ 0.67 \end{pmatrix} = \lambda x_3$$

$$Ax_3 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.67 \end{pmatrix} = \begin{pmatrix} 4.67 \\ 3.01 \end{pmatrix} = 4.67\begin{pmatrix} 1 \\ 0.65 \end{pmatrix} = \lambda x_4$$

$$Ax_4 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.65 \end{pmatrix} = \begin{pmatrix} 4.65 \\ 2.95 \end{pmatrix} = 4.65\begin{pmatrix} 1 \\ 0.63 \end{pmatrix} = \lambda x_5$$

$$Ax_5 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.63 \end{pmatrix} = \begin{pmatrix} 4.63 \\ 2.89 \end{pmatrix} = 4.63\begin{pmatrix} 1 \\ 0.62 \end{pmatrix} = \lambda x_6$$

$$Ax_6 = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0.62 \end{pmatrix} = \begin{pmatrix} 4.62 \\ 2.86 \end{pmatrix} = 4.62\begin{pmatrix} 1 \\ 0.62 \end{pmatrix} = \lambda x_7$$

The eigen value of A is λ = 4.62 & the corresponding Eigen vectors is $\begin{pmatrix} 1 \\ 0.62 \end{pmatrix}$

PART B

1. Find the inverse of the co - efficient matrix of the system.

$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$ by the gauss Jordan method, also solve the system.

Solution:

The coefficient matrix is $A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$

T o find inverse of A:

By Gauss Jordan method,

$$[A,I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{array}\right] \qquad \begin{array}{l} R_2^1 = R_2 - 4R_1 \\ R_3^1 = R_3 - 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right] \quad R_3^{11} = R_3 - 2R_2^1$$

$$= \left[\begin{array}{ccc|ccc} 10 & 10 & 0 & 1 & 2 & 1 \\ 0 & -2 & 0 & 3 & 0 & -1 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right] \qquad \begin{array}{l} R_1^{111} = 10R_1^{11} + R_3^{11} \\ R_2^{111} = 2R_2^{11} + R_3^{11} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 10 & 10 & 0 & 14 & 2 & -4 \\ 0 & -2 & 0 & 3 & 0 & -1 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{array}\right] \qquad R_1^{iv} = R_1^{111} + 5R_2^{111}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 14/10 & 2/10 & -4/10 \\ 0 & 1 & 0 & 3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 11/10 & -2/10 & -1/10 \end{array}\right] \qquad \begin{array}{l} R_1^v = R_1^{iv}/10 \\ R_2^v = R_2^{iv}/-2 \\ R_3^v = R_3^{iv}/-10 \end{array}$$

$=[I, A^{-1}]$

$$A^{-1} = \begin{pmatrix} 7/5 & 1/5 & -2/5 \\ -3/2 & 0 & 1/2 \\ 11/10 & -1/5 & -1/10 \end{pmatrix}$$

To solve the system of equations:

$$[A, B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2^1 = R_2 - 4R_1 \\ R_3^1 = R_3 - 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & -10 & 5 \end{array}\right] \quad R_3^{11} = R_3^1 + 2R_2^1$$

$$= \left[\begin{array}{ccc|c} 10 & 10 & 0 & 15 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & -10 & 5 \end{array}\right] \qquad \begin{array}{l} R_1^{111} = R_3^{11} + 10R_1^{11} \\ R_2^{111} = R_2^{11} - 2R_2^{11} \end{array}$$

$$= \left[\begin{array}{ccc|c} 10 & 0 & 0 & 10 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & -10 & 5 \end{array}\right] \qquad R_1^{IV} = R_1^{111} + 5R_2^{11}$$

$$= \left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 0 & +1 & 0 & 1/2 \\ 0 & 0 & 1 & -1/2 \end{array}\right] \quad \begin{array}{l} R_1^V = R_1^{IV}/10 \\ R_2^V = R_2^{IV}/-2 \\ R_3^V = R_3^V/-10 \end{array}$$

Solution:
x= 1
y = 1/2
Z = -1/2

3. Solve the equations 5x+2y +z = 12, x+4y+2z = 15 , x+2y+5z = 20 by (i) Jacobi's method and (ii) Gauss siedel method.

Solution:

Given system of equations 5x+2y +z = 12
x+4y+2z = 15
x+2y+5z = 20

This system is diagonally dominant matrix since,

$$|5| > |2| + |1|$$
$$|4| > |1| + |2|$$
$$|5| > |1| + |2|$$

(i) Jacobi's method:

$$x_{n+1} = \frac{1}{5}\left[12 - 2y_n - Z_n\right]$$
$$y_{n+1} = \frac{1}{4}\left[15 - x_n - 2Z_n\right]$$
$$z_{n+1} = \frac{1}{5}\left[20 - x_n - 2y_n\right]$$

Let $[x_0, y_0, z_0] = [0,0,0]$

Iteration	$x_{n+1} = \frac{1}{5}\left[12-2y_n - Z_n\right]$	$y_{n+1} = \frac{1}{4}\left[15 - x_n - 2Z_n\right]$	$z_{n+1} = \frac{1}{5}\left[20 - x_n - 2y_n\right]$
1	x_1 = 2.4	y_1 = 3.75	z_1 = 4
2	x_2 = 0.1	y_2 = 1.15	z_2 = 2.02
3	x_3 = 1.536	y_3 = 2.715	z_3 = 3.52
4	x_4 = 0.61	y_4 = 1.606	z_4 = 2.6068
5	x_5 = 1.2362	y_5 = 2.2941	z_5 = 3.2356
6	x_6 = 0.8352	y_6 = 1.8232	z_6 = 2.8351
7	x_7 = 1.1037	y_7 = 2.1237	z_7 = 3.1037
8	x_8 = 0.9298	y_8 = 1.9223	z_8 = 2.9298
9	x_9 = 1.0451	y_9 = 2.0527	z_9 = 3.0451
10	x_{10} = 0.9699	y_{10} = 1.9662	z_{10} = 2.5729
11	x_{11} = 0.0989	y_{11} = 2.2211	z_{11} = 3.0195
12	x_{12} = 0.9077	y_2 = 1.9655	z_{12} = 2.8918

Approximately the solution is x= 1, y = 2 , z = 3

(ii) Gauss Siedel method

The iteration formula is

Let (x_0, y_0, z_0) = (0, 0, 0)

Iteration	$x_{n+1} = \frac{1}{5}[12 - 2y_n - Z_n]$	$y_{n+1} = \frac{1}{4}[15 - x_{n+1} - 2z_n]$	$z_{n+1} = \frac{1}{5}[20 - x_{n+1} - 2y_{n+1}]$
1	x_1 = 2.4	y_1 = 3.15	z_1 = 2.26
2	x_2 = 0.6.88	y_2 = 2.448	z_2 = 2.8832
3	x_3 = 0.8442	y_3 = 2.0974	z_3 = 2.9922
4	x_4 = 0.9626	y_4 = 2.0133	z_4 = 3.0022
5	x_5 = 0.9942	y_5 = 2.0001	z_5 = 3.0011
6	x_6 = 0.9997	y_6 = 1.9995	z_6 = 23.0003
7	x_7 = 1.0001	y_7 = 1.9998	z_7 = 3.0001
8	x_8 = 1.0001	y_8 = 1.9999	z_8 = 3.0000
9	x_9 = 1	y_9 = 2	z_9 = 3
10	x_{10} = 1	y_{10} = 2	z_{10} = 3

$\therefore$ Solution is x= 1 , y = 2, Z = 3

4. Find the positive root of x^4-x-9 = 0 using Newton method.
Solution:

Let $f(x) = x^4 - x - 9 = 0$

To find the solution space:

f(0) = -9

$f(1)$ = -9 (-ve)

$f(2)$ = 5 (+ve)

$f(3)$ = 69

The solution of $f(x)$ lies between x= 1 & x = 2 by Newton method

$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

Let x_0 = 1.5

$$x_{n+1} = x_n - \frac{(x_n^4 - x_n - 9)}{4x_n^3 - 1}$$

$$= \frac{4x_n^4 - x_n - x_n^4 + x_n + 9}{4x_n^3 - 1}$$

Put n = 0, $x_1 = \frac{3(1.5)^4 + 9}{4(1.5)^3 - 1} = 1.935$

Put n= 1, $x_2 = \frac{3(1.935)^4 + 9}{4(1.935)^3 - 1} = 1.8248$

Put n = 2, $x_3 = \frac{3(1.8248)^4 + 9}{4(1.8248)^3 - 1} = 1.8135$

Put n = 3, $x_4 = \frac{3(1.8135)^4 + 9}{4(1.8135)^3 - 1} = 1.8134$

Put n = 4, $x_5 = \frac{3(1.8134)^4 + 9}{4(1.8134)^3 - 1} = 1.8134$

Since x_4 and x_5 coincides the solution of $f(x) = x^4 - x - 9 = 0$ is x = 1.8134

5. Find the largest eigen values and its corresponding eigen vector using power method for

$$A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix} \quad \text{(NOV/DEC'2015)}$$

Solution:

$$\text{Given } A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$$

Let the initial eigen vector be $X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Now by power method,

$$AX_0 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ 14 \end{pmatrix} = 14\begin{pmatrix} 0 \\ 0.5 \\ 1 \end{pmatrix} = \lambda_1 \ x_1$$

$$AX_1 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0 \\ 0.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1 \\ 6.5 \end{pmatrix} = 6.5\begin{pmatrix} 0.0769 \\ 0.1538 \\ 1 \end{pmatrix} = \lambda_2 \ x_2$$

$$AX_2 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.0769 \\ 0.1538 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.6155 \\ -0.0772 \\ 5.9228 \end{pmatrix} = 5.9228\begin{pmatrix} 0.2728 \\ 0.3130 \\ 1 \end{pmatrix} = \lambda_3 \ x_3$$

$$AX_3 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.2728 \\ 0.3130 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.3118 \\ 0.0392 \\ 6.5978 \end{pmatrix} = 6.5978\begin{pmatrix} 0.3504 \\ 0.0059 \\ 1 \end{pmatrix} = \lambda_4 \ x_4$$

$$AX_4 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.3504 \\ 0.0059 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.3327 \\ 0.4252 \\ 7.1201 \end{pmatrix} = 7.1201\begin{pmatrix} 0.3276 \\ 0.05697 \\ 1 \end{pmatrix} = \lambda_5 \ x_5$$

$$AX_5 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.3276 \\ 0.597 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.1485 \\ 0.5492 \\ 7.1447 \end{pmatrix} = 7.1447\begin{pmatrix} 0.3007 \\ 0.0769 \\ 1 \end{pmatrix} = \lambda_6 \ x_6$$

$$AX_6 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.3007 \\ 0.0769 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.0700 \\ 0.5104 \\ 7.0349 \end{pmatrix} = 7.0349\begin{pmatrix} 0.2942 \\ 0.0726 \\ 1 \end{pmatrix} = \lambda_7 \ x_7$$

$$AX_7 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.2942 \\ 0.0726 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.0764 \\ 0.4672 \\ 6.9830 \end{pmatrix} = 6.9830\begin{pmatrix} 0.2974 \\ 0.0669 \\ 1 \end{pmatrix} = \lambda_8 \ x_8$$

$$AX_8 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}\begin{pmatrix} 0.2974 \\ 0.0669 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.0967 \\ 0.4572 \\ 6.9851 \end{pmatrix} = 6.9851\begin{pmatrix} 0.3000 \\ 0.0655 \\ 1 \end{pmatrix} = \lambda_9 \ x_9$$

$$AX_9 = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix} \begin{pmatrix} 0.3000 \\ 0.0655 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.1035 \\ 0.4620 \\ 6.9965 \end{pmatrix} = 6.9965 \begin{pmatrix} 0.3007 \\ 0.660 \\ 1 \end{pmatrix} = \lambda_{10}\ x_{10}$$

$$AX_{10} = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix} \begin{pmatrix} 0.3007 \\ 0.0660 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.1027 \\ 0.4668 \\ 7.0022 \end{pmatrix} = 7.0022 \begin{pmatrix} 0.3003 \\ 0.0667 \\ 1 \end{pmatrix} = \lambda_{11}\ x_{11}$$

$\therefore$ the dominant eigen value is 7.0022 and the corresponding eigen vector is $\begin{pmatrix} 0.3003 \\ 0.0667 \\ 1 \end{pmatrix}$

6. Solve by Gauss seidel : *5x-2y+z=-4*
x+6y-2z=-1
3x+y+5z=13

Solution:

The given system of equation is , *5x-2y+z=-4*
x+6y-2z=-1
3x+y+5z=13

This system is diagonally dominant

$$|5| > |-2| + |1|$$
(ie) $$|6| > |1| + |-2|$$
$$|5| > |3| + |1|$$

Let us use Gauss siedel method to solve this system,

The iteration formula is, $x_{n+1} = \frac{1}{5}[-4 + 2y_n - z_n]$

$$y_{n+1} = \frac{1}{6}[-1 - x_{n+1} + 2z_n]$$
$$z_{n+1} = \frac{1}{5}[13 - 3x_{n+1} - y_{n+1}]$$

Let the initial solution be (x_0, y_o, z_0) = (0, 0, 0)

Iteration	$x_{n+1} = \frac{1}{5}[-4 + 2y_n - z_n]$	$y_{n+1} = \frac{1}{6}[-1 - x_{n+1} + 2z_n]$	$z_{n+1} = \frac{1}{5}[13 - 3x_{n+1} - y_{n+1}]$
1	x_1 = -0.8	y_1 = 0.0333	z_1 = 3.0867
2	x_2 = 1.4307	y_2 = 1.1007	z_2 = 3.2383
3	x_3 = 1.0074	y_3 = 1.0807	z_3 = 2.9883
4	x_4 = 0.9654	y_4 = 0.9903	z_4 = 2.9812
5	x_5 = 1.0001	y_5 = 0.9938	z_5 = 3.0013
6	x_6 = 1.0027	y_6 = 1.0009	z_6 = 3.0014
7	x_7 = 0.9999	y_7 = 1.0005	z_7 = 2.9998
8	x_8 = 0.9998	y_8 = 0.9999	z_8 = 2.9999
9	x_9 = -1	y_9 = 1	z_9 = 3
10	x_{10} = -1	y_{10} = 1	z_{10} = 3

Since the values of x, y , z are coincides in iteration 9 and 10 the solution of the given system is
x = -1, y = 1, z = 3

7. Find the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ by Gauss Jordan method

Solution:

Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

By Gauss Jordan method,

The augmented matrix is,

$$(A, I) = \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 7 & 17 & -1 & 0 & 2 \end{array}\right) \qquad \begin{array}{l} R_2^1 = 2R_2 - 3R_1 \\ R_3^1 = 2R_3 - R_1 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -4 & 20 & -14 & 2 \end{array}\right) \qquad R_2^{11} = R_3 - 7R_2^1$$

$$= \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 2 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right) \qquad R_{32}^{111} = R_3^{11} / 2$$

$$= \left(\begin{array}{ccc|ccc} 4 & 2 & 0 & 12 & -7 & 1 \\ 0 & 2 & 3 & 24 & -17 & 3 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right) \qquad \begin{array}{l} R_1^{IV} = 2R_1^{111} + R_3^{111} \\ R_2^{IV} = 2R_2^{111} + 3R_3^{111} \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 4 & 0 & 0 & -12 & 10 & -2 \\ 0 & 2 & 0 & 24 & -17 & 3 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right) \qquad R_1^V = R_1^{IV} + R_2^{IV}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 5/2 & -1/2 \\ 0 & 1 & 0 & 12 & -17/2 & 3/2 \\ 0 & 0 & 1 & 5 & 7/2 & -1/2 \end{array}\right) \qquad \begin{array}{l} R_1^{VI} = R_1^V / 4 \\ R_2^{VI} = R_2^V / 2 \\ R_3^{-VI} = R_2^V / -2 \end{array}$$

$$= \left[I \ , \ A^{-1} \right]$$

$\therefore$ The inverse of A is $A^{-1} = \begin{pmatrix} -3 & 5/2 & -1/2 \\ 12 & -17/2 & 3/2 \\ 5 & 7/2 & -1/2 \end{pmatrix}$

8. Solve the equation by Gauss elimination method:

2x + y + 4z = 12, 8x - 3y + 2z = 20 ,4x + 11y - z = 33

Solution:

The given system of equations are,

$$Ax = B \Rightarrow \begin{pmatrix} 2 & 1 & 4 \\ 8 & -1 & 2 \\ 4 & 11 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

By Gauss elimination method

The augmented matrix is,

$$[A,\ B] = \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 8 & -1 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & 28 \\ 0 & 9 & -9 & 9 \end{array}\right) \qquad \begin{array}{l} R_2^1 = R_2 - 4R_1 \\ R_3^1 = R_3 - 4R_1 \end{array}$$

$$= \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right) \qquad \begin{array}{l} R_2^{11} = R_2^1 / -7 \\ R_3^{11} = R_3^1 / -9 \end{array}$$

$$[A,\ B] = \left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -3 & -3 \end{array}\right) \qquad R_3^{111} = R_3^{11} - R_2^{11}$$

By back substitution method

$\Rightarrow \quad -3z = -3$

$z = 1$

$\Rightarrow \quad y + 2z = 4$

$y + 2 = 4$

$y = 2$

$\Rightarrow 2x + y + 4z = 12$

$2x + 2 + 4 = 12$

$x = 3$

$\therefore$ solution is x= 3, y = 2, z = 1

9. If $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$,find the A^{-1} by Gauss - Jordan method

Solution:

$$\text{Given } A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$$

By Gauss Jordan Method,

The augmented matrix is,

$$[A,\ I] = \left(\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 0 & 5 & -4 & -1 & 2 & 0 \\ 0 & -9 & 6 & -1 & 0 & 4 \end{array}\right) \qquad \begin{array}{l} R_3^1 = 4R_3 - R_1 \\ R_2 = 2R_2 - R_1 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 0 & 5 & -4 & -1 & 2 & 0 \\ 0 & 0 & -6 & -14 & 18 & 20 \end{array}\right) \quad R_3^{11} = 5R_3^1 - 9R_2^1$$

$$= \left(\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 0 & 5 & -4 & -1 & 2 & 0 \\ 0 & 0 & 3 & 7 & 9 & -10 \end{array}\right) \quad R_3^{111} = R_3^{11} / -2$$

$$= \left(\begin{array}{ccc|ccc} 12 & 3 & 0 & -11 & 18 & 20 \\ 0 & 3 & 0 & 5 & -6 & -8 \\ 0 & 0 & 3 & 7 & -9 & -10 \end{array}\right) \quad \begin{array}{l} R_3^{1V} = 3R_1^{111} - 2R_3^{111} \\ R_2^{1V} = 3R_2^{111} - 4R_3^{111} \ \& \ R_2^{1V} = R_2^{IV} / 5 \end{array}$$

$$= \left(\begin{array}{ccc|ccc} 12 & 0 & 0 & -16 & 24 & 28 \\ 0 & 3 & 0 & 5 & -6 & -8 \\ 0 & 0 & 3 & 7 & -9 & -10 \end{array}\right) \quad R_1^{1V} = R_1^{IV} - R_2^{IV}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4/3 & 2 & 7/3 \\ 0 & 1 & 0 & 5/3 & -2 & -8/3 \\ 0 & 0 & 1 & 7/3 & -3 & -10/3 \end{array}\right) \quad \begin{array}{l} R_1^{VI} = R_1^{V} / 12 \\ R_2^{VI} = R_2^{V} / 3 \\ R_3^{VI} = R_3^{V} / 3 \end{array}$$

$$= \left[I, A^{-1} \right]$$

$\therefore$ Inverse of A is $A^{-1} = \begin{pmatrix} -4/3 & 2 & 7/3 \\ 5/3 & -2 & -8/3 \\ 7/3 & -3 & -10/3 \end{pmatrix}$

10. Solve the following equation by Gauss - siedel method
x + y + 9z = 15, x + 17y -2z = 48 , 30x - 2y + 3z = 75
Solution:
The given system of equation is,

x + y 9z = 15
x + 17y -2z = 48
30x - 2y + 3z = 75

This system is not diagonally dominant since

$$|1| \ngtr |1| + |9|$$
$$|17| > |1| + |-2|$$
$$|3| \ngtr |30| + |-2|$$

$\therefore$ we interchange first and third equations

$\Rightarrow$ 30x - 2y + 3z = 75
x + 17y - 2z = 48
x + y + 9z = 15

Now this system is diagonally dominant

$$|30| > |-2| + |3|$$
(ie) $$|17| > |1| + |-2|$$
$$|9| > |1| + |1|$$

Now let us use Gauss siedel method to solve this system of equation.
The iteration formula is ,

$$x_{n+1} = \frac{1}{30}[75 + 2y_n - 3z_n]$$
$$y_{n+1} = \frac{1}{17}[48 - x_{n+1} + 2z_n]$$

$$z_{n+1} = \frac{1}{19}[15 - x_{n+1} - y_{n+1}]$$

Let the initial vector be $(x_0, y_0\ z_0) = (0, 0, 0)$

Iteration	$x_{n+1} = \frac{1}{30}[75 + 2y_n - 3z_n]$	$y_{n+1} = \frac{1}{17}[48 - x_{n+1} + 2z_n]$	$z_{n+1} = \frac{1}{19}[15 - x_{n+1} - y_{n+1}]$
1	$x_1 = 2.5$	$y_1 = 2.6765$	$z_1 = 1.0915$
2	$x_2 = 2.5693$	$y_2 = 2.8008$	$z_2 = 1.0700$
3	$x_3 = 2.5797$	$y_3 = 2.7977$	$z_3 = 1.0692$
4	$x_4 = 2.5796$	$y_4 = 2.7976$	$z_4 = 1.0692$
5	$x_5 = 2.5796$	$y_5 = 2.7976$	$z_5 = 1.0692$

Since the values of x, y, z in iteration 4 and 5 coincides, the solution of given system is ,

$x = 2.5796, y = 2.7976, z = 1.0692$

11. Using power method ,find the dominant eigen values of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$

Solution:

Let the given matrix be $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$

Let the initial vector be $X_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

By power method

$$AX_0 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 26 \\ 4 \\ 2 \end{pmatrix} = 26\begin{pmatrix} 1 \\ 0.1538 \\ 0.769 \end{pmatrix} = \lambda_1 X_1$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.1538 \\ 0.769 \end{pmatrix} = \begin{pmatrix} 25.3076 \\ 1.4614 \\ 1.6924 \end{pmatrix} = 25.3076\begin{pmatrix} 1 \\ 0.0578 \\ 0.0669 \end{pmatrix} = \lambda_2 X_2$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0578 \\ 0.0669 \end{pmatrix} = \begin{pmatrix} 25.1916 \\ 1.1734 \\ 1.7324 \end{pmatrix} = 25.1916\begin{pmatrix} 1 \\ 0.0466 \\ 0.0688 \end{pmatrix} = \lambda_3 X_3$$

$$AX_3 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0466 \\ 0.0688 \end{pmatrix} = \begin{pmatrix} 25.1842 \\ 1.1398 \\ 1.7248 \end{pmatrix} = 25.1842 = \begin{pmatrix} 1 \\ 0.0453 \\ 0.0685 \end{pmatrix} = \lambda_4 X_4$$

$$AX_4 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0453 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1843 \\ 1.1395 \\ 1.726 \end{pmatrix} = 25.1823 = \begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \lambda_5 X_5$$

$$AX_5 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.1353 \\ 1.726 \end{pmatrix} = 25.1821 = \begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \lambda_6 X_6$$

$$AX_6 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.1353 \\ 1.726 \end{pmatrix} = 25.1821 = \begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \lambda_7 X_7$$

Since the values of $\lambda_6 X_6$ & $\lambda_7 X_7$ coincides,the dominant eigen value of A is λ = 25.1821 and the corresponding eigen vector is $X = \begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$

UNIT IV

INTERPOLATIN, NUMERICAL DIFFERENTIATION AND NUMERICAL INTERGRATION

Part- A

1. Given f(2) = 5 f (2.5) = 5.5 , find the linear interpolating polynomials using Lagrange interpolation

Solution:

The given data is

x:	2	2.5
f(x):	5	5.5

Lagrange's interpolation formula is,

$$y(x) = \frac{(x-x_1)}{x_0-x_1}.y_o + \frac{x-x_0}{x_1-x_0}.y_1$$

$$= \frac{(x-2.5)}{(2-2.5)}(5) + \frac{(x-2)}{(2.5-2)}(5.5)$$

$$= \frac{-5}{0.5}(x-2.5) + \frac{5.5}{0.5}(x-2)$$

$$= -10(x-2.5) + 11(x-2)$$

$$y(x) = -10x + 25 + 11x - 22$$

$$y(x) = x+3$$

2. Construct the divided difference table for the data:

x	0.5	1.5	3.0	5.0	6.5	8.0
f(x)	1.625	5.875	31	131	282.125	521

Solution:

The divided difference table is,

x	f(x)	Δf(x)	Δ^2 f(x)	Δ^3f(x)	Δ^4f(x)	Δ^5f(x)
0.5	1.625	$\frac{5.875-1.625}{1.5-0.5} = 4.25$				

1.5	5.875		$\frac{16.75-4.25}{3.0-0.5}=5$			
		$\frac{31-5.875}{3.0-1.5}=16.75$		$\frac{9.5-5}{5.0-0.5}=1$		
3.0	31				0	
		$\frac{131-31}{5.0-3.0}=50$	$\frac{50-16.75}{5.0-1.5}=9.5$	$\frac{14.5-9.5}{6.5-1.5}=1$		0
5.0	131	$\frac{282.125-131}{6.5-5.0}=100.75$	$\frac{100.75-50}{6.5-3.0}=14.5$		0	
				$\frac{19.5-14.5}{8.0-3.0}=1$		
6.5	282.125	$\frac{521-282.125}{8.0-6.5}=159.25$	$\frac{159.25-100.75}{8.0-5.0}=19.5$			
8.0	521					

3. Give the Newton's backwards difference table for

x:	0	1	2	3
Y:	-1	-2	-1	-2

Solution:
The Backward difference table is,

x	y	Δy	Δ^2y	Δ^3y
0	-1			
		-1		
1	-2		2	
		1		0
2	-1		2	
		3		
3	2			

4. Compare Trapezoidal rule with Simpson's $\frac{1}{3}$ rule.

Solution:

S.No	Trapezoidal Rule	Simpson's 1/3 rule
1	Least accurate	Most accurate
2.	Can be divided into any number of intervals	Intervals of integration must be divided into even number of sub intervals

5. Specify the Newton's backward difference formula for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Solution:
The Newton's Backward difference formula for,

$$\frac{dy}{dx}=\frac{1}{h}\left[\nabla y_n+\frac{2v+1}{2}\nabla^2 y_n+\frac{3v^2+6v+2}{6}\nabla^3 y_n+\ldots\ldots\right]$$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\nabla^2 y_n+\frac{6v+6}{6}\nabla^3 y_n+\cdots\right] \quad \text{Where } V=\frac{x-x_n}{h}$$

6. Write down the error in Trapezoidal and Simpson's rule of numerical integration
Solution:

Error in Trapezoidal rule is $E = \frac{-(b-a)}{12} h^2 y''(\varepsilon)$

Error in Simpson's rule is $E < \frac{-h^2}{180} y''(b-a)$

7. Write down the Lagrange's interpolation formula .
Solution:
Let the given values be

x	x_0	x_1	x_2	---------	x_n
$y = f(x)$	$f(x_0)$	$f(x_1)$	$f(x_2)$	----------	$f(x_n)$
	y_o	y_1	y_2	----------	y_n

Lagrange's interpolation formula is,

$$y = f(x) = \frac{(x-x_1)(x-x_2)---(x-x_n)}{(x_0-x_1)(x_0-x_2)---(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)---(x-x_n)}{(x_1-x_0)(x_1-x_2)---(x_1-x_n)} y_1 + + \frac{(x-x_0)(x-x_1)---(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)---(x_n-x_{n-1})} y_n$$

8.Write down Newton's forward and backward difference interpolation formula for equal intervals
Solution:

Let the values of x_i's are equally spaced then the values are given by

x:	x_0	x_1	x_2	---------	x_n
Y:	y_o	y_1	y_2	----------	y_n

Where $x_{i+1}-x_i=h$
Newton's forward interpolation formula is

$$y = y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots. + \frac{u(u-1)(u-2)\ldots(u-(r-1))}{r!}\Delta^r y_0$$

where $u = \frac{x-x_0}{h}$

Newton's backward interpolation formula is,

$$y = y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots. + \frac{v(v+1)(v+2)\ldots(v+(r-1))}{r!}\nabla^r y_n$$

where $v = \frac{x-x_n}{h}$

9. Write down Newton's divided difference formula for unequal intervals.
Solution:

Let the values of x_i's are not equally spaced and the values of y are y_0 , y_1, y_2, ----- y_n corresponding to the values of x x_0, x_1, x_2, ----- x_n then,
Newton's divided difference formula is

$$y = y(x) = y_o + (x-x_0)\Delta y_o + (x-x_0)(x-x_1)\Delta^2 y_o + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_o + ----$$

10. Write down the Newton's forward difference formula for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Solution:

$$\frac{dy}{dx}=\frac{1}{h}\left[\Delta y_o+\frac{2u-1}{2!}\Delta^2 y_o+\frac{3u^2-6u+2}{3!}\Delta^3 y_o+\frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_o = ----\right]$$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\Delta^2 y_o+\frac{6u-6}{3!}\Delta^3 y_o+\frac{12u^2-36u+2}{4!}\Delta^4 y_o+----\right]$$

Where $u = \frac{x-x_0}{h}$

12. Specify the Newton's forward difference formula for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_0$

$$\therefore \left.\frac{dy}{dx}\right|_{u=0}=\frac{1}{h}\left[\Delta y_o-\frac{\Delta^2 yo}{2}+\frac{\Delta^3 yo}{3}-\frac{\Delta^4 yo}{4}+----\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{u=0}=\frac{1}{h^2}\left[\Delta^2 y_o-\frac{\Delta^3 yo}{2}+\frac{11}{12}-\Delta^4 yo----\right]$$

13. Create a formula difference table for the following table and state the degree of polynomial for the same.

Solution:

x:	0	1	2	3
y:	-1	0	3	8

Forward difference table is,

x	y	Δy	Δ^2y	Δ^3y
0	-1			
		1		
1	0		2	
		3		0
2	3		2	
		5		
3	8			

Since $\Delta^3 yo$ = 0, the degree of the polynomial will be of atleast 2 (2 = 3-1)

14. Find the divided difference for the following data

x:	2	5	10
y:	5	29	109

Solution:

Divided difference is given by,

x	y	Δy	Δ^2y
2	5		
		8	
5	29		1
		16	
10	109		

15. State any two properties of divided difference

Solution:

1. The divided difference are symmetrical in their arguments (ie) The value of any difference is independent of the order of the argument.
2. The divided difference of the sum of two function is algebraic sum of their divided differences.

16. Write down the Simpson's $\frac{1}{3}$ rule in numerical integration.

Solution:

The Simpson's $\frac{1}{3}$ rule is given by

$$\int_{x_0}^{x_n} y(x)\, dx = \frac{h}{3}\left[\left(y_o + y_n\right) + 4\left(y_1 + y_3 + ---- + y_{n-1}\right)\right] + 2\left(y_2 + y_4 + ----- + y_{2n}\right)$$

Where $h = \frac{x_n - x_o}{n}$

17. Using Simpson's rule find $\int_0^4 e^x dx$ given that $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.6$.

Solution:

Let the given values be written as,

x:	0	1	2	3	4
$y = e^x$:	1 yo	2.72 y_1	7.39 y_2	20.09 y_3	54.6 y_4

By Simpson's rule we have

$$\int_0^4 e^x dx = \frac{h}{3}\left[\left(y_o + y_4\right) + 4\left(y_1 + y_3\right) + 2\left(y_2\right)\right]$$

$$= \frac{1}{3}\left[\left(1 + 54.6\right) + 4\left(2.72 + 20.09\right) + 2\left(7.39\right)\right]$$

$$\int_0^4 e^x dx = 53.8733$$

18. Write down the expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_n$ by Newton's backward difference formula

Solution:

At $x = x_n$, $v = \frac{x - x_n}{h} = 0$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \cdots\right]$$

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

19. Evaluate $\int_{0.5}^{1} \frac{dx}{x}$ by trapezoidal rule, dividing the range into 4 equal parts

Solution:

Here $h = \frac{x_n - x_o}{n} = \frac{1 - 0.5}{4} = \frac{1}{8}$

$$y = \frac{1}{x}$$

∴ The values of y are,

x:	0.5 = 4/8	5/8	6/8	7/8	8/8= 1
y:1/x	2 y_o	8/5 y_1	8/6 y_2	8/7 y_3	1 $y_4 = y_n$

By trapezoidal rule,

$$I = \int_{0.5}^{1} \frac{dx}{x} = \frac{h}{2}\left[\left(y_0 + y_4\right) + 2\left(y_1 + y_2 + y_3\right)\right]$$

$$= \frac{.1/8}{2}\left[(2+1) + 2\left(8/5 + 8/6 + 8/7\right)\right]$$

$$\int_{0.5}^{1} \frac{dx}{x} = 0.6971$$

20. Find the area under the curve passing through the points (0,0) (1,2), (2,2.5) (3,2.3), (4,2) (5,1.7) and (6,1.5)

Solution:

Given,

x:	0	1	2	3	4	5	6
y:	0 y_0	2 y_1	2.5 y_2	2.3 y_3	2 y_4	1.7 y_5	1.5 $y_6 = y_n$

By using Trapezoidal rule,

$$\text{Area} = \int_0^6 y\,dx = \frac{h}{2}\left[\left(y_0 + y_6\right) + 2\left(y_1 + y_2 + y_3 + y_4 + y_5\right)\right]$$

$$= 1/2\left[(0+1.5) + 2\left(2 + 2.5 + 2.3 + 2 + 1.7\right)\right]$$

Area = 11.25

UNIT IV

PART B

1. Evaluate $\int_1^2 \int_1^2 \frac{dxdy}{x+y}$ by Simpson's rule and Trapezoidal rule with h=0.5and k=0.25.

Solution:

when h = 0.5 and k = 0.25, , f(x,y) = $\frac{1}{x+y}$

$x_0 = 1$, $x_m = 2$ $y_o = 1$, $y_n = 2$

The values of *f (x,y)* are,

x \ y	1	1.25	1.5	1.75	2
1	0.5	0.4444	0.4	0.3636	0.33
1.5	0.4	0.3636	0.33	0.3077	0.285
2	0.33	0.3077	0.285	0.2667	0.25

Using Trapezoidal Rule:

$$I = \frac{hK}{4}\left[\text{sum of the corner values} + 2(\text{sum of the values on the boundary}) + 4(\text{sum of the remaining values})\right]$$

$$= \frac{(0.5)(0.25)}{4}\left[(0.5+0.33+0.33+0.25)+2(0.444+0.4+0.3636+0.4+0.3077+0.285\,0.2677+0.285)+4(0.3636+0.33+0.3077)\right]$$

$$= \frac{0.125}{4}\left[1.41 + 5.5048 + 4.0052\right]$$

I = 0.3413

Using Simpson's Rule:

$$I = \frac{hK}{9}\left[f_{00} + f_{04} + f_{30} + f_{34} + 4(f_{01} + f_{03} + f_{20} + f_{24} + f_{31} + f_{33}) + 8(f_{22}) + 16(f_{21} + f_{23}) + 2(f_{02} + f_{32})\right]$$

$$= \frac{(0.5)(0.25)}{9}\left[0.5+0.333+0.333+0.25+4(0.444+0.3636+0.4+0.285+0.3077+0.2667)\right]$$

$$8(0.333) + 16(0.3636+0.3077) + 2(0.4+0.285)$$

$$= \frac{0.125}{9}\left[1.4166 + 8.2696 + 2.6664 + 10.7408 + 1.37\right]$$

I =0.3398

2. The Table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface

x:	100	150	200	250	30	350	400
y :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when x= 218 ft and 410 ft

Solution:

The forwards difference table is given by

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63						
		2.4					
150	13.03		-0.39				
		2.01		0.15			
200	15.04		-0.24		-0.07		
		1.77		0.08		-0.04	
250	16.81		0.16		-0.11		0.32
		1.61		-0.03		0.28	
300	18.42		-0.19		0.17		
		1.42		0.14			
350	19.90		-0.05				
		1.37					
400	21.27						

By Newton's forwards difference interpolation formula

$$y(x) = yo + \frac{u}{1!}\Delta yo + \frac{u(u-1)}{2!}\Delta^2 yo + \frac{u(u-1)(u-2)}{3!}\Delta^3 yo + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 yo +$$

$$\frac{u(u-1)(u-2)(u-3)(u-4)(u-5)}{5!}\Delta^4 yo + \frac{u(u-1)(u-2)(u-3)(u-4)(u-5)(u-6)}{6!}\Delta^4 yo + ----$$

Where $u=\frac{x-x_0}{n}$

$$u=\frac{218-100}{50}=2.36$$

$$y(218)=10.63+\frac{2.36}{1!}(2.4)+\frac{(2.36)(2.36-1)}{2!}(-0.39)+\frac{(2.36)(2.36-1)(2.36-2)}{3!}(0.15)+$$
$$\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)}{4!}(-0.07)+\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)(2.36-4)}{5!}(-0.04)$$
$$+\frac{(2.36)(2.36-1)(2.36-2)(2.36-3)(2.36-4)(2.36-5)}{6!}(0.32)$$

=10.63+5.664+(-0.625872)+0.02889+0.002157-(0.000404)-0.00143
=15.6974

$$\boxed{y(218)=15.6974}$$

Newton's Backward difference interpolation formula is,

$$y=y(x)=y_n+\frac{v}{1!}\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$
$$+\frac{v(v+1)(v+2)\dots(v+(r-1))}{r!}\nabla^r y_n$$

where $v=\frac{x-x_n}{h}$

To find y at x=410, v= $\frac{410-400}{50}=0.2$

$$y(410)=21.27+\frac{(0.2)}{1!}(1.37)+\frac{(0.2)(0.2+1)}{2!}(-0.05)+\frac{(0.2)(0.2+1)(0.2+2)}{3!}(0.14)+$$
$$\frac{(0.2)(0.2+1)(0.2+2)(0.2+3)}{4!}(0.17)+\frac{(0.2)(0.2+1)(0.2+2)(0.2+3)(0.2+4)}{5!}(0.28)+$$
$$\frac{(0.2)(0.2+1)(0.2+2)(0.2+3)(0.2+4)(0.2+5)}{6!}(0.32)$$

=21.27+0.274+(-0.006)+0.01232+0.011968+0.01655+0.01640

$$\boxed{y(410)=21.5952}$$

(3) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule and simpson's 1/3 rule and compare with its exact solution.

Sol:

Given I= $\int_0^6 \frac{dx}{1+x^2}$, $x_0=0, x_n=6, h=\frac{x_n-x_0}{n}$

$$\text{Let } n=6, h=\frac{6-0}{6}=1$$

The values of 'y' are,

X	0	1	2	3	4	5	6

$y=\frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

By Trapezoidal rule,

$$I=\int_{x_0}^{x_n} f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+....)\right]$$

$$\int_0^6 \frac{dx}{1+x^2}=\frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.0588+0.0385)\right]$$

$$=\frac{1}{2}[1.027+1.7946]$$

$$\boxed{I=1.4108}$$

By Simpson's 1/3 rule:

$$I=\int_{x_0}^{x_n} f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+....)+2(y_2+y_4+.....)\right]$$

$$\int_0^6 \frac{dx}{1+x^2}=\frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.0385)+2(0.2+0.0588)\right]$$

$$=\frac{1}{3}[1.027+2.554+0.5176]$$

$$\boxed{I=1.3662}$$

Exact value:

$$\int_0^6 \frac{dx}{1+x^2}=\left[\tan^{-1}(x)\right]_0^6$$

$$=\tan^{-1}(6)-\tan^{-1}(0)$$

$$\boxed{I=1.4056}$$

In Trapezoidal Rule: Error = -0.0052;In Simpson's rule: Error = 0.0394

(4) Given that:

X:	1	1.1	1.2	1.3	1.4	1.5	1.6
Y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and y^{II} at x=1.1 and x=1.6

<u>Sol:</u>

The difference table is as follows:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	7.989						
		0.414					
1.1	8.403		-0.036				
				0.006			
		0.378			-0.002		
1.2	8.781		-0.030				

				0.004		0.001	
		0.348			-0.001		
1.3	9.129		-0.026				0.002
				0.003		0.003	
		0.322			0.002		
1.4	9.451		-0.023				
				0.005			
		0.299					
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

To find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=1.1:

By Newton's forward difference formula for derivatives,

$$\frac{dy}{dx}=\frac{1}{h}\left[\Delta y_o+\frac{2u-1}{2!}\Delta^2 y_o+\frac{3u^2-6u+2}{3!}\Delta^3 y_o+\frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_o=----\right]$$ where

$$u=\frac{x-x_0}{h}$$

$u=(1.1-1.0)/0.1=1$

$$\frac{dy}{dx}=\frac{1}{0.1}\left[0.414+\frac{2(1)-1}{2!}(-0.036)+\frac{3(1)^2-6(1)+2}{3!}(0.006)+\frac{2(1^3)-9(1^2)+11(1)-3}{12}(-0.002)+....\right]$$

$$=\frac{1}{0.1}[0.414-0.018-0.001-0.00017]$$

$$\boxed{\frac{dy}{dx}=3.9483}\ \text{at}\ x=1.1$$

$$\frac{d^2y}{dx^2}=\frac{1}{1^2}\left[\Delta^2 y_0+\frac{6u-6}{3!}\Delta^3 y_0+\frac{6u^2-18u+11}{4!}\Delta^4 y_0+.....\right]$$

$$=\frac{1}{(0.1)^2}\left[-0.036+\frac{6(1)-6}{3!}(0.006)+\frac{\left(6(1^2)-18(1)+11\right)}{4!}(-0.002)+....\right]$$

$$=\frac{1}{(0.1)^2}[-0.036+0+0.00008]$$

$$\boxed{\frac{d^2y}{dx^2}=-3.592}\ \text{at}\ x=1.1$$

At x=1.6, let us use Newton's backward difference formula,

At x=1.6, $v=\frac{x-x_n}{h}$ v=(1.6-1.6)/0.1=0

$$\left.\frac{dy}{dx}\right|_{v=0} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \ldots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{(-0.018)}{2} + \frac{0.005}{3} + \frac{0.002}{4} + \frac{0.003}{5} + \frac{0.002}{6}\right]$$

$$= \frac{1}{0.1}\left[0.281 - 0.009 + 0.0017 + 0.0005 + 0.0006 + 0.00003\right]$$

$$\boxed{\left.\frac{dy}{dx}\right|_{v=0} = 2.7483} \text{ at } x = 1.6$$

$$\left.\frac{d^2y}{dx^2}\right|_{v=0} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \ldots\right]$$

$$= \frac{1}{0.1^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002)\right]$$

$$\boxed{\left.\frac{d^2y}{dx^2}\right|_{v=0} = -1.1167} \text{ at } x = 1.6$$

5) Given,

X:	0	2	3	4	7	8
Y:	4	26	58	112	466	668

Find y(10), $y^1(6)$ using Newton's divided difference formula .

Solution:

Newton's divided difference formula is,

$$y = y(x) = y_o + (x - x_0)\Delta y_o + (x - x_0)(x - x_1)\Delta^2 y_o + (x - x_0)(x - x_1)(x - x_2)\Delta^3 y_o + - - - -$$

The divided difference table is given by,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	4	$\frac{26-4}{2-0} = 11$			
2	26		$\frac{32-11}{3-0} = 7$		
		$\frac{58-26}{3-2} = 32$		$\frac{11-7}{4-0} = 1$	
3	58		$\frac{54-32}{4-2} = 11$		0
		$\frac{112-58}{4-3} = 54$		$\frac{16-11}{7-2} = 1$	
4	112		$\frac{118-54}{7-3} = 16$		0
		$\frac{466-112}{7-4} = 118$		$\frac{21-16}{8-3} = 1$	

7	466		$\frac{202-118}{8-4}=21$		
		$\frac{668-466}{8-7}=202$			
8	668				

To find y(10):

$$y(x)=4+(x-0)(11)+(x-0)(x-2)(7)+(x-0)(x-2)(x-3)(1)$$

$$=4+11x+7x^2-14x+x^3-5x^2+6x$$

$$y(x)=x^3+2x^2+3x+4$$

$$y(10)=10^3+2(10^2)+3(10)+4$$

$$\boxed{y(10)=1234}$$

Now, $y(x)=x^3+2x^2+3x+4$

$$y^1(x)=3x^2+4x+3$$

$$\Rightarrow \quad y^1(6)=3(6^2)+4(6)+3$$

$$\boxed{y^1(6)=135}$$

(6) Evaluate the integral $I=\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3 rule by taking h=1/4

Solution:

Given $I=\int_0^1 \frac{dx}{1+x^2}$, $x_0=0, x_n=1, f(x)=y=\frac{1}{1+x^2}, h=\frac{1}{4}$

$$n=\frac{x_n-x_0}{h}=\frac{1-0}{\left(\frac{1}{4}\right)}=4$$

The values of y are,

X :	0	¼	2/4	¾	1
$y=\frac{1}{1+x^2}$:	1	0.9412	0.80	0.64	0.5

By Simpson's 1/3 rule,

$$I=\int_{x_0}^{x_n} f(x)dx=\frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\ldots)+2(y_2+y_4+\ldots)\right]$$

$$\int_0^1 \frac{dx}{1+x^2}=\frac{\left(\frac{1}{4}\right)}{3}\left[(1+0.5)+4(0.9412+0.64)+2(0.8)\right]$$

$$=\frac{1}{12}\left[1.5+6.3248+1.6\right]$$

$$\boxed{I=0.7854}$$

(7) Evaluate $\int_1^2 \frac{dx}{1+x^2}$ taking h=0.2 using trapezoidal rule.

Solution:

Given $I=\int_1^2 \frac{dx}{1+x^2}, x_0=1, x_n=2, h=0.2, y=\frac{1}{1+x^2}$

$$n=\frac{x_n-x_0}{h}=\frac{2-1}{0.2}=5$$

The values of y are,

X :	1	1.2	1.4	1.6	1.8	2
$y=\frac{1}{1+x^2}$:	0.5	0.4098	0.3378	0.2809	0.2358	0.2
	y_0	y_1	y_2	y_3	y_4	$y_5=y_n$

By Trapezoidal rule,

$$I=\int_{x_0}^{x_n} f(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\ldots)\right]$$

$$\int_1^2 \frac{dx}{1+x^2}=\frac{0.2}{2}\left[(0.5+0.2)+2(0.4098+0.3378+0.2809+0.2358)\right]$$

$$=0.1\left[0.7+2.5286\right]$$

$$\boxed{I=0.3229}$$

(8) Given:

X:	140	150	160	170	180
Y:	3.685	4.854	6.302	8.076	10.225

find y(175)

Solution:

To find y (175), let us use Newton's backward difference interpolation formula.

$$y=y(x)=y_n+\frac{v}{1!}\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n+\frac{v(v+1)(v+2)\ldots(v+(r-1))}{r!}\nabla^r y_n$$

where $v=\frac{x-x_n}{h}$

The finite difference table is

X	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
140	3.685				
		1.169			
150	4.854		0.279		
		1.448		0.047	
160	6.302		0.326		0.002
		1.774		0.049	$\nabla^4 y_n$
170	8.076		0.375	$\nabla^3 y_n$	
		2.149	$\nabla^2 y_n$		
180	10.225	∇y_n			
	y_n				

<u>To find y(175):</u> $v = \frac{175-180}{10} = -0.5$

$$y(175) = 10.225 + \frac{(-0.5)}{1!}(2.149) + \frac{(-0.5)(-0.5+1)}{2!}(0.375) +$$

$$\frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.047) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(0.002)$$

$$= 10.225 - 1.0745 - 0.046875 - 0.0030625 - 0.000078125$$

$$\boxed{y(175) = 9.1004}$$

(9)Interpolate y(12), if

X:	10	15	20	25	30	35
Y:	35	33	29	27	22	14

Solution:

To find y(12), let us use Newton's forward difference formula

The difference table is,

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
10	35					
		-2				
15	33		-2			
		-4		4		
20	29		2		-9	
		-2		-5		14
25	27		-3		5	
		-5		0		
30	22		-3			
		-8				
35	14					

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + ...$$

$$\text{where}\, u = \frac{x - x_0}{h}$$

$$= \frac{12-10}{5} = 0.4$$

$$y(12) = 35 + \frac{0.4}{1!}(-2) + \frac{(0.4)(0.4-1)}{2!}(-2) + \frac{(0.4)(0.4-1)(0.4-2)}{3!}(4) +$$

$$\frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!}(-9) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)(0.4-4)}{5!}(14)$$

$$= 35 - 0.8 + 0.24 + 0.256 + 0.3744 + 0.419328$$

$$\boxed{y(12) = 35.48973} \Rightarrow \boxed{y(12) = 35.49}$$

(10)Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's (1/3) rule, dividing the range into four equal parts.

Solution:

Given I= $\int_0^1 \frac{dx}{1+x^2}$, $x_0 = 0, x_n = 1, y = \frac{1}{1+x^2}, n = 4$

$$h = \frac{x_n - x_0}{n} = \frac{1-0}{4} = \frac{1}{4}$$

The values of y are,

X :	0	¼	2/4	¾	1
$y = \frac{1}{1+x^2}$:	1	0.9412	0.80	0.64	0.5
	y_0	y_1	y_2	y_3	$y_4 = y_n$

By Simpson's 1/3 rule

$$I = \int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ...) + 2(y_2 + y_4 + ...)\right]$$

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1/4}{3}\left[(0.5+1) + 4(0.9412 + 0.64) + 2(0.8)\right]$$

$$= 0.7854$$

$$\boxed{I = 0.7854}$$

11)Find $y^{I}(1)$, if

X:	-1	0	2	3
Y:	-8	3	1	12

Sol:

Since the values of 'x' are unequal, let us use Lagrange's interpolation formula.

	x_0	x_1	x_2	x_3
X:	-1	0	2	3

Y(x):	-8	3	1	12
	y_0	y_1	y_2	y_3

Lagrange's interpolation formula is,

$$y(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1+$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$=\frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8)+\frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}(3)+$$

$$\frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)}(1)+\frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)}(12)$$

$$=(x^3-5x^2+6x)\left(\frac{2}{3}\right)+(x^3-4x^2+x+6)\left(\frac{1}{2}\right)+$$

$$(x^3-2x^2-3x)\left(\frac{-1}{6}\right)+(x^3-x^2-2x)(1)$$

$$=\frac{1}{6}\left[4x^3-20x^2+24x+3x^3-12x^2+3x+18-x^3+2x^2+3x+6x^3-6x^2-12x\right]$$

$$y(x)=\frac{1}{6}\left[12x^3-36x^2+18x+18\right]$$

$$y^{I}(x)=\frac{1}{6}(36x^2-72x+18)$$

$$\Rightarrow y^{I}(1)=\frac{1}{6}(36-72+18)$$

$$\boxed{y^{I}(1)=-3}$$

12) Using Trapezoidal rule, evaluate $\int_1^2\int_1^2 \frac{dx\,dy}{x+y}$ with h=k=0.5

Solution:The values of '*f*' are

X \ y	1	1.5	2
1	0.5	0.4	0.33
1.5	0.4	0.33	0.285
2	0.33	0.285	0.25

Using Trapezoidal rule:

$$I=\frac{hk}{4}\left[\begin{matrix}\text{Sum of the}\\ \text{four corner values}+2\begin{pmatrix}\text{Sum of values}\\ \text{in the boundary}\end{pmatrix}4(\text{sum of the remaining vaues})\end{matrix}\right]$$

$$=\frac{(0.5)(0.5)}{4}\left[0.5+0.33+0.33+0.25+2(0.4)-0.285+0.4+0.285+4(0.33)\right]$$

$$\boxed{I=0.3418}$$

UNIT-V
NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS
Part-A

1. Given $y^{I}=\frac{y-x}{y+x}$ with initial condition y=1 at x=0 find y for x=0.1 by Euler's method.

Solution:

Given $y^{I}=\frac{y-x}{y+x}=f(x),y(0)=1$

Let us take h=0.1

To find y at x=0.1 (ie) y(0.1)=y_1:

Euler's ,method is,

$y_{n+1}=y_n+hf(x_n,y_n)$ (Here $x_0=0$

Put $n=1$ $y_1=y_0+hf(x_0,y_0)$ $y_0=1)$

$=1+0.1\left(\frac{1-0}{1+0}\right)$ $h=0.1$

$$\boxed{y(0.1)=y_1=1.1}$$

2. Given the initial value problem $u^{I}=-2tu^2,u(0)=1$ estimate u(0.4) using modified Euler-Cauchy method.

Solution:

Given $u^{I}=-2tu^2$ with $u(0)=1$

Let us take h=0.4

To find u at t=0.4 (ie) u(0.4)=u,:

Modified Euler's method is,

$$y_{n+1}=y_n+h\,f\left\{x_n+\frac{h}{2},y_n+\frac{h}{2}f(x_n,y_n)\right\}$$

Put n=0, Here y=u, x=t

$$u_1 = u_0 + hf\left\{t_0 + \frac{h}{2},\ u_0 + \frac{h}{2}f(t_0, u_0)\right\}$$

$$u_1 = 1 + 0.4f\left\{0 + \frac{0.4}{2}, 1 + \frac{0.4}{2}\left(-2(0)(1^2)\right)\right\}$$

$$= 1 + 0.4f\{0.2, 1\}$$

$$= 1 + 0.4\left(-2(0.2)(1^2)\right)$$

$$\boxed{u(0.4) = u_1 = 0.84}$$

3. If $u' = -y, y(0) = 1$, then find y(0.1) by Euler method.

Solution:

Given $y' = -y,\ y(0) = 1$

Let us take h=0.1

To find y(0.1)=y(1):

Euler's method is given by,

$$y_{n+1} = y_n + h\ \ f(x_n, y_n)$$

$$\text{Put } n = 0, \quad y_1 = y_0 + h\ \ f(x_0, y_0)$$

$$= 1 + 0.1(-1)$$

(Here $x_0 = 0$
$y_0 = 1$)
$h = 0.1$

$$\boxed{y(0.1) = y_1 = 0.9}$$

4. What are single step and multi step methods? Give example.

Solution:

Single step method:

The method used to find the current value using the single previous value is called single step method.

Example: (i) Taylor's series method (ii) Runge-kutta method.

Multi step method:

The method used to find the current value using the multiple previous values is called multi-step method.

Example: Predictor-Corrector method.

5. Find y(0.1) by Euler's method if $\frac{dy}{dx} = x^2 + y^2, y(0) = 0.1$

Solution:

Given $\frac{dy}{dx} = x^2 + y^2, y(0) = 0.1$

Let us take h=0.1

Euler's method is given by,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 0.1 + (0.1)(0^2 + 0.1^2) \Rightarrow \boxed{y(0.1) = 0.101}$$

6. Give the central difference approximations for $y'(x), y''(x)$.

Solution:

The central difference approximations for,

$$y^{I}(x)=\frac{y_{i+1}-y_{i-1}}{2h},$$

$$y^{II}(x)=\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}, \text{where} h\rightarrow \text{step size}$$

7.Write down the Milne's predictor-corrector formula for solving initial value problem.

Solution:

Let $y^{I}=f(x,y), y(x_0=y_0)$ be given then, Milne's formula is given by

Predictor method,

$$y_{n+1,p}=y_{n-3}+\frac{4h}{3}\left[2y^{I}_{n-2}-y^{I}_{n-1}+2y^{I}_{n}\right]$$

Corrector method,

$$y_{n+1,c}=y_{n-1}+\frac{h}{3}\left[y^{I}_{n-1}+4y^{I}_{n}+y^{I}_{n+1,p}\right]$$

8.Using Taylor's series find y(0.1) for $\frac{dy}{dx}=1-y, y(0)=0$

Solution:

Here, $x_0=0, y_0=0, h=0.1$

$y^{I}=1-y$

$y^{II}=-y^{I}$

$y^{III}=-y^{II}$

$y^{IV}=-y^{III}$

$At(x_0,y_0)=(0,0)$

$y^{I}=1$

$y^{II}=-1$

$y^{III}=+1$

$y^{IV}=-1$

Taylor's series is given by,

$$y(0.1)=y_1=y_0+\frac{h}{1!}y^{I}_0+\frac{h^2}{2!}y^{II}_0+\frac{h^3}{3!}y^{III}_0+.......$$

$$y(0.1)=0+\frac{(0.1)}{1!}(1)+\frac{(0.1)^2}{2!}(-1)+\frac{(0.1)^3}{3!}(1)+\frac{(0.1)^4}{4!}(-1)+.....$$

$$\boxed{y(0.1)=0.0952}$$

9.Solve $y_{x+2}-4y_x=0$

Solution:

Given $y_{x+2}-4y_x=0$

$$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}-4\frac{(y_{i+1}-y_{i-1})}{2h}=0$$

$2[y_{i+1}-2y_i+y_{i-1}]-4h[y_{i+1}-y_{i-1}]=0$

$\boxed{(2-4h)y_{i+1}-4y_i+(2+4h)y_{i-1}=0}$ is the finite difference scheme for given equ.

For different values of i, we get values of y_i for a specified value of 'h'

10.Write the finite difference scheme for the differential equation $\frac{d^2y}{dx^2}-y=2$ where y(0) and y(1)=1, h=1/4.

Sol:

The given differential equation can be written as

$$y''(x)-y(x)=2$$

We have, $y''=\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$

$$\therefore y''(x)-y(x)=2 \Rightarrow \frac{y_{i+1}-2y_i+y_{i-1}}{h^2}-y_i=2$$

$$\Rightarrow y_{i+1}-2y_i+y_{i-1}-h^2y_i=2h^2$$

$$\Rightarrow y_{i+1}-(2+h^2)y_i+y_{i-1}=2h^2$$

$$\Rightarrow y_{i+1}-\left(2+\frac{1}{16}\right)y_i+y_{i-1}=2\left(\frac{1}{16}\right)$$

$$\Rightarrow \boxed{16y_{i+1}-33y_i+16y_{i-1}=2}$$ is the finite difference scheme for given differential eqn.

11.What are the special advantages of Runge-Kutta method over Taylor series method.

Sol:

(1)The use of R.K method gives quick convergence to the solutions of the differential equation than the Taylor's series.

(2)The labour involved in R.K. method is comparatively lesser

(3)In R.K. method, the derivatives of higher order are not required for calculation as in Taylor's series method.

12.State modified Euler's algorithm to solve

$$y'=f(x+y), y(x_0)=y_0 \text{ at } x=x_0+h$$

Solution:

Modified Euler's method is given by,

$$y_{n+1}=y_n+h\left[f\left\{x_n+\frac{h}{2}, y_n+\frac{h}{2}(f(x_n,y_n))\right\}\right]$$

13.By Taylor's series method, find y(1.1) given $y'=x+y, y(1)=0$.

Solution:

Given $y'=x+y,\ x_0=1, y_0=0,$

$$\text{Let } h=0.1$$

Taylor's series formula is given by,

$$y_1 = y_0 + \frac{h}{1!} y_0^{I} + \frac{h^2}{2!} y_0^{II} + \frac{h^3}{3!} y_0^{III} + \ldots\ldots \rightarrow (1)$$

Derivatives	At $(x_0, y_0) = (1,0)$
$y^{I} = x + y$	$y^{I} = 1$
$y^{II} = 1 + y^{I}$	$y^{II} = 2$
$y^{III} = y^{II}$	$y^{III} = 2$
$y^{IV} = y^{III}$	$y^{IV} = 2$

Sub. the values in (1)

$$y(1.1) = y_1 = 0 + \frac{0.1}{1!}(1) + \frac{(0.1)^2}{2!}(2) + \frac{(0.1)^3}{3!}(2) + \frac{(0.1)^4}{4!}(2) + \ldots\ldots$$

$$\boxed{y(1.1) = 0.1103}$$

14.Write the Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$

Solution:

The R.K. formula of fourth order is,

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\text{where } k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left\{x_n + \frac{h}{2}, y_n + k_{1/2}\right\}$$

$$k_3 = hf\left\{x_n + \frac{h}{2}, y_n + K_{2/2}\right\}$$

$$k_4 = hf\{x_n + h, y_n + k_3\}$$

15.Using R-K method of second order find y(0.1) when $y^{I} = -y, y(0) = 1$

Solution:

Given $y^{I} = -y, y(0) = 1$

Here $x_0 = 0, y_0 = 1, h = 0.1$

R-K method of second order is given by,

$$y_1 = y_0 + k_2$$

Where $k_1 = hf(x_0, y_0) = (0.1)[-y_0] = (0.1)(-1) = -0.1$

$$k_2 = hf\left\{x_0 + \frac{h}{2}, y_0 + k_{1/2}\right\} = (0.1)f\left\{0 + \frac{0.1}{2}, 1 + \frac{(-0.1)}{2}\right\}$$

$$= (0.1)f\{0.005, 0.95\}$$

$$= (0.1)(-0.95)$$

$$k_2 = -0.095$$

$$\therefore y_1 = y(0.1) = 1 - 0.095$$

$$\boxed{y(0.1) = 0.905}$$

16.Is Milne's predictor-corrector method is self-starting? Give reasons.

Solution:

Iteration method is self starting since we can take value which lies in the given interval [a,b] in which the root lies. But Milne's method is not self-starting, since we should know any 4 prior values to the value which we need to find.

17.Bring out the merits and demerits of Taylor's series method.

Solution:

<u>Merits of Taylor's series method:</u>

1. This method gives a straight forward adaptation classic calculus to develop the solution as an infinite series.
2. It is powerful single step method if we are able to find the successive derivatives.

<u>Demerits of Taylor's series method:</u>

If the function involves some complicated algebraic structures, then the calculation of higher derivatives becomes tedious and the method fails. This is the major drawback of this method.

18.Compute y at x=0.25 by modified Euler method given $y' = 2xy, y(0) = 1$

Solution:

Given $y' = f(x,y) = xy \text{ with } x_0 = 0, y_0 = 1 \,\&\, h = 0.25$

To find y at x=0.25 (ie) y_1:

Modified Euler method is given by,

$$y_{n+1} = y_n + hf\left\{x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right\}$$

Put n=0,

$$y_1 = y_0 + hf\left\{x_0 + \frac{h}{2}, y_0 + \frac{h}{2}f(x_0, y_0)\right\}$$

$$y_1 = 1 + (0.25)f\left\{0 + \frac{0.25}{2}, 1 + \frac{0.25}{2}f(0,1)\right\}$$

$$= 1 + (0.25)f\{0.125, 1 + 0.25(2)\}$$

$$= 1 + (0.25)f\{0.125, 1.25\}$$

$$= 1 + (0.25)2(0.125)(1.25)$$

$$= 1 + 0.078125$$

$$\boxed{y(0.25) = y_1 = 1.078125}$$

<u>Part-B</u>

(1) Apply Taylor's method to obtain the approximate value of y at x=0.2 for the differential equation $y^I = 2y + 3e^x, y(0) = 0$. Compare the numerical solution with its exact solution

<u>Sol:</u>

Given $y^I = 2y + 3e^x, y(0) = 0$

$\Rightarrow f(x,y) = 2y + 3e^x, x_0 = 0, y_0 = 0$

Let us take h=0.2

<u>To find y at x=0.2 (ie) y_1:</u>

Derivatives	At $(x_0, y_0) = (0,0)$
$y^I = 2y + 3e^x$	$y_0^I = 3$
$y^{II} = 2y^I + 3e^x$	$y_0^{II} = 2(3) + 3e^0$ $= 9$
$y^{III} = 2y^{II} + 3e^x$	$y_0^{III} = 18 + 3$ $= 21$
$y_0^{IV} = 2y^{III} + 3e^x$	$y_0^{IV} = 42 + 3$ $= 45$
$y_0^V = 2y^{IV} + 3e^x$	$y_0^V = 93$

By Taylor's series method,

$$y_{n+1} = y_n + \frac{h}{1!}y_n^1 + \frac{h^2}{2!}y_n^{II} + \frac{h^3}{3!}y_n^{III} + \frac{h^4}{4!}y_n^{IV} + ...$$

Put $n = 0$,

$$y_1 = y_0 + \frac{h}{1!}y_0^1 + \frac{h^2}{2!}y_0^{II} + \frac{h^3}{3!}y_0^{III} + \frac{h^4}{4!}y_0^{IV} + ...$$

$$= 0 + \frac{(0.2)}{1!}(3) + \frac{(0.2)^2}{2!}(9) + \frac{(0.2)^3}{3!}(21) + \frac{(0.2)^4}{4!}(45) + \frac{(0.2)^5}{5!}(93) + ...$$

$$y_1 = 0.6 + 0.18 + 0.028 + 0.003 + 0.00025$$

$$\boxed{y(0.2) = 0.8113}$$

To find the exact solution:

$$y^I = 2y + 3e^x$$

$$\frac{dy}{dx} = 2y + 3e^x \Rightarrow \frac{dy}{dx} - 2y + 3e^x$$

This is in the form of $\frac{dy}{dx} + py = Q(x)$

Its solution is $ye^{\int pdx} = \int Q e^{\int pdx} dx + c$

$$\Rightarrow ye^{\int -2dx} = \int 3e^x e^{\int -2dx} dx + c$$

$$ye^{-2x} = 3\int e^x e^{-2dx} dx + c$$

$$= 3\int e^{-x} dx + c$$

$$ye^{-2x} = -3e^{-x} + c$$

Given y(0)=0

$$(0) = -3e^0 + c$$

$$0 = -3 + c$$

$$\Rightarrow \boxed{c = 3}$$

$\therefore$ Exact solution is,

$$ye^{-2x} = -3e^{-x} + 3$$

$$y = -3e^x + 3e^{2x}$$

$$y(0.2) = -3e^{0.2} + 3e^{2(0.2)}$$

$$y(0.2) = 0.8113$$

Numerical solution is y(0.2)=0.8113

Exact solution is y(0.2)=0.8113

Error = 0

(2) Using R.K fourth order method, to find y at x=0.1, 0.2, 0.3 given that $y^I = xy + y^2, y(0) = 1$. Continue the solution at x=4 using Milne's P-C method.

Solution:

Given $y^I = xy + y^2, y(0) = 1$

$f(x,y)=xy+y^2, x_0=0, y_0=1$

R-K method is given by,

$$y_{n+1}=y_n+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$\text{where}\, k_1=hf(x_n,y_n)$$

$$k_2=hf\left(x_n+\frac{h}{2},y_n+k_{1/2}\right)$$

$$k_3=hf\left(x_n+\frac{h}{2},y_n+k_{2/2}\right)$$

$$k_4=hf(x_n+h,y_n+k_3)$$

Let us take h=0.1

To find y at x=0.1, 0.2, 0.3 :-

To find y_1 (ie) at x=0.1 :

$$k_1=hf(x_0,y_0)=(0.1)((0)(1)+1^2)=0.1$$

$$k_2=hf\left(x_0+\frac{h}{2},y_0+k_{1/2}\right)=(0.1)f(0.05,1.05)$$

$$k_2=0.1155$$

$$k_3=hf\left(x_0+\frac{h}{2},y_0+k_{2/2}\right)=(0.1)f(0.05,1.05775)$$

$$k_3=0.1172$$

$$k_4=hf(x_0+h,y_0+k_3)=(0.1)f(0.1,1.1172)$$

$$k_4=0.1360$$

$$\therefore y_1=1+\frac{1}{6}(0.1+2(0.1155)+2(0.1172)+0.1360)$$

$$\boxed{y(0.1)=y_1=1.1169}$$

To find y_2 (ie) at x=0.2 :

$$x_1=0.1, y_1=1.1169, h=0.1$$

$$k_1=hf(x_1,y_1)=(0.1)f(0.1,\ 1.1169)=0.1360$$

$$k_2=hf\left(x_1+\frac{h}{2},y_1+k_{1/2}\right)=(0.1)f(0.15,1.1849)=0.1360$$

$$k_3=hf\left(x_1+\frac{h}{2},y_1+k_{2/2}\right)=(0.1)f(0.15,1.196)=0.161$$

$$k_4=hf(x_1+h,y_1+k_3)=(0.1)f(0.2,1.2779)=0.1889$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.1169 + \frac{1}{6}(0.1360 + 2(0.1582) + 2(0.161) + 0.1889)$$

$$\boxed{y(0.2) = y_2 = 1.2775}$$

<u>To find y_3 (ie) at x=0.3 :</u>

$$x_2 = 0.2, y_2 = 1.2775, h = 0.1$$

$$k_1 = hf(x_2, y_2) = (0.1)f(0.2,\ 1.2775) = 0.1889$$

$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + k_{1/2}\right) = (0.1)f(0.3, 1.37195) = 0.2294$$

$$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + k_{2/2}\right) = (0.1)f(0.3, 1.3922) = 0.2356$$

$$k_4 = hf(x_2 + h, y_2 + k_3) = (0.1)f(0.4, 1.5131) = 0.2895$$

$$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.2775 + \frac{1}{6}(0.1889 + 2(0.2294) + 2(0.2356) + 0.2895)$$

$$\boxed{y(0.3) = y_3 = 1.5122}$$

<u>To find y at x=0.4 (ie) y(0.4) using Milne's method:</u>

$$x_0 = 0 \quad , y_0 = 1$$

$$x_1 = 0.1 \quad , y_1 = 1.1169$$

$$x_2 = 0.2 \quad , y_2 = 1.2775$$

$$x_3 = 0.3 \quad , y_3 = 1.5122$$

Milne's predictor formula is given by,

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$\text{Now}, y'_1 = (xy + y^2)_1 = (0.1)(1.1169) + 1.1169^2 = 1.359$$

$$y'_2 = (xy + y^2)_2 = (0.2)(1.2775) + 1.2775^2 = 1.8875$$

$$y'_3 = (xy + y^2)_3 = (0.3)(1.5122) + 1.5122^2 = 2.7404$$

$$y_{4,p} = 1 + \frac{4(0.1)}{3}(2(1.359) - 1.8875 + 2(2.7404))$$

$$\boxed{y_{4,p} = 1.8415}$$

Milne's corrector formula is,

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

To find y'_4:

$$y'_4 = \left(xy + y^2\right)_{4,p} = (0.4)(1.8415) + 1.8415^2$$

$$y'_4 = 4.1277$$

$$\therefore \quad y_{4,c} = 1.2775 + \frac{0.1}{3}\left[1.8875 + 4(2.7404) + 4.1277\right]$$

$$\boxed{y_{4,c} = 1.8434}$$

$$\therefore y(0.4) = 1.8434$$

By R-K method, $y(0.1) = 1.1169$

$y(0.2) = 1.2775$

$y(0.3) = 1.5122$

$y(0.4) = 1.8434$

3.Using Runge-Kutta method of fourth order, solve $y' = \frac{y^2 - x^2}{y^2 + x^2}$ given y(0)=1. Find y at x=0.2, 0.4, 0.6, 0.8

Solution:

Given $y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$

$$\Rightarrow f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}, x_0 = 0, y_0 = 1, \text{let } h = 0.2$$

<u>To find y(0.2), y(0.4), y(0.6):-</u>

By Runge-Kutta method of fourth order,

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

Where $k_1 = hf(x_n, y_n)$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + k_{1/2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + k_{2/2}\right)$$

$$k_4 = hf\left(x_n + h, y_n + k_3\right)$$

<u>To find y(0.2):</u>

$$k_1 = hf(x_0, y_0) = (0.2)f(0,1) = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + k_{1/2}\right) = (0.2)f(0.1, 1.1) = 0.1967$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + k_{2/2}\right) = (0.2)f(0.1, 1.0984) = 0.1967$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = (0.2)f(0.2, 1.1967) = 0.1891$$

$$y(0.2) = y_1 = 1 + \frac{1}{6}\left[0.2 + 2(0.1967) + 2(0.1967) + 0.1891\right]$$

$$\boxed{y(0.2) = y_1 = 1.19598}$$

<u>To find y(0.4):</u> $x_1 = 0.2,\ y_1 = 1.19598, h = 0.2$

$$k_1 = hf(x_1, y_1) = (0.2)f(0.2, 1.19598) = 0.1891$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + k_{1/2}\right) = (0.2)f(0.3, 1.29045) = 0.1794$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + k_{2/2}\right) = (0.2)f(0.3, 1.2856) = 0.1793$$

$$k_4 = hf(x_1 + h, y_1 + k_3) = (0.2)f(0.4, 1.3752) = 0.1687$$

$$y(0.4) = y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.1959 + \frac{1}{6}(0.1891 + 2(0.1794) + 2(0.1793) + 0.1687)$$

$$\boxed{y(0.4) = y_2 = 1.3751}$$

<u>To find y(0.6):</u> $x_2 = 0.4,\ y_2 = 1.3751, h = 0.2$

$$k_1 = hf(x_2, y_2) = (0.2)f(0.4, 1.3751) = 0.1687$$

$$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + k_{1/2}\right) = (0.2)f(0.5, 1.4595) = 0.158$$

$$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + k_{2/2}\right) = (0.2)f(0.5, 1.4541) = 0.1577$$

$$k_4 = hf(x_2 + h, y_2 + k_3) = (0.2)f(0.6, 1.5328) = 0.1469$$

$$y(0.6) = y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.3751 + \frac{1}{6}(0.1681 + 2(0.158) + 2(0.1577) + 0.1469)$$

$$\boxed{y(0.6) = y_3 = 1.5328}$$

<u>To find y(0.8):</u> $x_3 = 0.6, y_3 = 1.5328$

$k_1 = hf(x_3, y_3) = (0.2)f(0.6, 1.5328) = 0.1469$

$k_2 = hf\left(x_3 + \frac{h}{2}, y_3 + k_{1/2}\right) = (0.2)f(0.7, 1.6063) = 0.1362$

$k_3 = hf\left(x_3 + \frac{h}{2}, y_3 + k_{2/2}\right) = (0.2)f(0.7, 1.6009) = 0.1358$

$k_4 = hf(x_3 + h, y_3 + k_3) = (0.2)f(0.8, 1.6686) = 0.1252$

$y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.5328 + \frac{1}{6}(0.1469 + 2(0.1362) + 2(0.1358) + 0.1252)$

$\boxed{y(0.8) = y_4 = 1.6688}$

4)Compute y(0.5), y(1), y(1.5) using Taylor's series for $y^{I} = \frac{x+y}{2}$ with y(0)=2 and hence find y(2) using Milne's method.

Solution:

Given $y^{I} = \frac{x+y}{2}$,y(0)=2

Let h=0.5

Taylor's series is given by,

$$y_{n+1} = y_n + \frac{h}{1!}y_n^{I} + \frac{h^2}{2!}y_n^{II} + \frac{h^3}{3!}y_n^{III} + \frac{h^4}{4!}y_n^{IV} + ...$$

To find y(0.5)=y_1:

Derivatives	At $(x_0, y_0) = (0,2)$
$y^{I} = \frac{x+y}{2}$	$y_o^{I} = 1$
$y^{II} = \frac{1+y^{I}}{2}$	$y_0^{II} = 1$
$y^{III} = \frac{y^{II}}{2}$	$y_0^{III} = \frac{1}{2}$
$y^{IV} = \frac{y^{III}}{2}$	$y_0^{IV} = \frac{1}{4}$

$$\therefore y(0.5) = 2 + \frac{0.5}{1!}(1) + \frac{(0.5)^2}{2!}(1) + \frac{(0.5)^3}{3!}\left(\frac{1}{2}\right) + \frac{(0.5)^4}{4!}\left(\frac{1}{4}\right) + ...$$

$\boxed{y_1 = y(0.5) = 2.6361}$

To find y(1)=y_2:

Derivatives	At $(x_1, y_1) = (0.5, 2.6361)$

$y^{I}=\frac{x+y}{2}$ $y^{II}=\frac{1+y^{I}}{2}$ $y^{III}=\frac{y^{II}}{2}$ $y^{IV}=\frac{y^{III}}{2}$	$y_1^{I}=1.5681$ $y_1^{II}=1.2841$ $y_1^{III}=0.6421$ $y_1^{IV}=0.3211$

$$y_2=y(1.0)=2.6361+\frac{0.5}{1!}(1.5681)+\frac{(0.5)^2}{2!}(1.2841)+\frac{(0.5)^3}{3!}(0.6421)+\frac{(0.5)^4}{4!}(0.3211)+\ldots$$

$$\boxed{Y_2=Y(1)=3.5949}$$

To find y(1.5)=y_3:

Derivatives	At $(x_2,y_2)=(1,3.5949)$
$y^{I}=\frac{x+y}{2}$ $y^{II}=\frac{1+y^{I}}{2}$ $y^{III}=\frac{y^{II}}{2}$ $y^{IV}=\frac{y^{III}}{2}$	$y_2^{I}=2.2975$ $y_2^{II}=1.6488$ $y_2^{III}=0.8244$ $y_2^{IV}=0.4122$

$$y_3=y(1.5)=3.5949+\frac{0.5}{1!}(2.2975)+\frac{(0.5)^2}{2!}(1.6488)+\frac{(0.5)^3}{3!}(0.8244)+\frac{(0.5)^4}{4!}(0.4122)$$

$$\boxed{Y_3=Y(1.5)=4.9679}$$

<u>To find y(2) using Milne's method:</u>

$x_0=0 \qquad y_0=2$

$x_1=0.5 \qquad y_1=2.6361$

$x_2=1 \qquad y_2=3.5949$

$x_3=1.5 \qquad y_3=4.9679$

Milne's predictor formula is,

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$n = 3, y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$\text{Now}, y'_1 = \left(\frac{x+y}{2}\right)_1 = \left(\frac{0.5+2.6361}{2}\right) = 1.5681$$

$$y'_2 = \left(\frac{x+y}{2}\right)_2 = \left(\frac{1+3.5949}{2}\right) = 2.2975$$

$$y'_3 = \left(\frac{x+y}{2}\right)_3 = \left(\frac{1.5+4.9679}{2}\right) = 3.234$$

$$\therefore y_{4,p} = 2 + \frac{4(0.5)}{3}\left[2(1.5681) - 2.2975 + 2(3.234)\right]$$

$$\boxed{y(2) = y_{4,p} = 6.8711}$$

Milne's corrector formula is,

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n+1} + 4y'_n + y'_{n-1}\right]$$

$$n = 3, y_{4,c} = y_2 + \frac{h}{3}\left[y'_4 + 4y'_3 + y'_2\right]$$

$$\text{Now}, y'_4 = \left(\frac{x+y}{2}\right)_4 = \left(\frac{1.5+68711}{2}\right) = 4.1856$$

$$\therefore y_{4,c} = 3.5949 + \frac{0.5}{3}\left[4.1856 + 4(3.234) + 2.2975\right]$$

$$\boxed{y(2) = y_{4,c} = 6.8314}$$

$\therefore$solutionis,

$y(0.5) = 2.6361$

$y(1) = 3.5949$

$y(1.5) = 4.9679$

$y(2) = 6.8314$

5)If $\frac{dy}{dx} = x^2 + y^2, y(0) = 1,$ find y(0.1), y(0.2) and y(0.3) by Taylor series method. Hence find y(0.4) by Milne's predictor-corrector method.

Solution:

Given $\frac{dy}{dx} = y' = x^2 + y^2, y(0) = 1$

$$\Rightarrow x_0 = 0, y_0 = 1, \text{Let } h = 0.1$$

Taylor's series method is given by,

$$y_{n+1} = y_n + \frac{h}{1!}y'_n + \frac{h^2}{2!}y''_n + \frac{h^3}{3!}y'''_n + \frac{h^4}{4!}y^{IV}_n + \ldots..$$

To find $y(0.1)=y_1$:

Derivatives	At $(x_0, y_0)=(0,1)$
$y^I = x^2 + y^2$	$y_0^I = 1$
$y^{II} = 2x + 2yy^I$	$y_0^{II} = 2$
$y^{III} = 2 + 2yy^{II} + 2(y^I)^2$	$y_0^{III} = 8$
$y^{IV} = 2y^I y^{II} + 2yy^{III} + 4Y^I y^{II}$	$y_0^{IV} = 28$
$= 6y^I y^{II} + 2yy^{III}$	

$$y(0.1) = y_1 = 1 + \frac{0.1}{1!}(1) + \frac{(0.1)^2}{2!}(2) + \frac{(0.1)^3}{3!}(8) + \frac{(0.1)^4}{4!}(28) + \ldots..$$

$$\boxed{y(0.1) = y_1 = 1.115}$$

To find $y(0.2)=y_2$:

Derivatives	At $(x_1, y_1)=(0.1, 1.1115)$
$y^I = x^2 + y^2$	$y_1^I = 1.2454$
$y^{II} = 2x + 2yy^I$	$y_1^{II} = 2.9685$
$y^{III} = 2 + 2yy^{II} + 2(y^2)$	$y_1^{III} = 11.7010$
$y^{IV} = 6y^I y^{II} + 2yy^{III}$	$y_1^{IV} = 48.1931$

$$y(0.2) = y_2 = 1.1115 + \frac{0.1}{1!}(1.2454) + \frac{(0.1)^2}{2!}(2.9685) + \frac{(0.1)^3}{3!}(11.7010) + \frac{(0.1)^4}{4!}(48.1931) + \ldots..$$

$$\boxed{y(0.2) = y_2 = 1.253}$$

To find $y(0.3)=y_3$:

Derivatives	At $(x_2, y_2)=(0.2, 1.253)$
$y^I = x^2 + y^2$	$y_2^I = 1.61$
$y^{II} = 2x + 2yy^I$	$y_2^{II} = 4.4347$
$y^{III} = 2 + 2yy^{II} + 2(y^I)^2$	$y_2^{III} = 18.2976$
$y^{IV} = 6y^I y^{II} + 2yy^{III}$	$y_2^{IV} = 88.693$

$$y_3 = y(0.3) = 1.253 + \frac{0.1}{1!}(1.61) + \frac{(0.1)^2}{2!}(4.4347) + \frac{(0.1)^3}{3!}(18.2976) + \frac{(0.1)^4}{4!}(88.693) + \ldots..$$

$$\boxed{y(0.3) = y_3 = 1.4396}$$

<u>To find y(0.4) by Milne's predictor-corrector method:</u>

$x_0 = 0, \qquad y_0 = 1$

$x_1 = 0.1, \qquad y_1 = 1.1115$

$x_2 = 0.2, \qquad y_2 = 1.253$

$x_3 = 0.3, \qquad y_3 = 1.4396$

Milne's predictor formula is

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$\text{Put } n = 3, y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$y'_1 = \left(x^2 + y^2\right)_1 = \left(0.1^2 + 1.1115^2\right) = 1.2454$$

$$y'_2 = \left(x^2 + y^2\right)_2 = \left(0.2^2 + 1.253^2\right) = 1.610$$

$$y'_3 = \left(x^2 + y^2\right)_3 = \left(0.3^2 + 1.4396^2\right) = 2.1625$$

$$\therefore y_{4,p} = 1 + \frac{4(0.1)}{3}\left[2(1.2454) - 1.610 + 2(2.1625)\right]$$

$$\boxed{y(0.4) = y_{4,p} = 1.6941}$$

Milne's corrector formula is,

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n+1} + 4y'_n + y'_{n-1}\right]$$

$$n = 3 \Rightarrow y_{4,c} = y_2 + \frac{h}{3}\left[y'_4 + 4y'_3 + y'_2\right]$$

$$\text{Now}, y'_4 = \left(x^2 + y^2\right)_4 = \left(0.4^2 + 1.6941^2\right) = 3.03$$

$$\therefore y_{4,c} = 1.253 + \frac{0.1}{3}\left[3.03 + 4(2.1621) + 1.61\right]$$

$$\boxed{y(0.4) = y_{4,c} = 1.6969}$$

$\therefore$ Solutionis,

$y(0.1) = 1.1115$

$y(0.2) = 1.253$

$y(0.3) = 1.4396$

$y(0.4) = 1.6969$

(6) If $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$ find $y(0.2), y(0.4), y(0.6)$ by Runge-Kutta method. Hence find y(0.8) by Milne's method.
(NOV/DEC'2016)

<u>Sol:</u> Same as problem No:3, follow the procedure upto finding y(0.6),

$\therefore$ solution is $y(0.2) = 1.19598$; $y^{I} = 0.9456$

$y(0.4) = 1.3751$; $y_2^1 = 0.8439$

$y(0.6) = 1.5358$; $y_3^1 = 0.7352$

To find y(0.4) by Milne's method:-

Milne's predictor method is,

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$\Rightarrow y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$y_{4,p} = 1 + \frac{4(0.2)}{3}\left[2(0.9456) - (0.8439) + 2(0.7352)\right]$$

$$y(0.8) = y_{4,p} == 1.6712$$

Milne's corrector method is

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n+1} + 4y'_n + y'_{n-1}\right], y'_4 = 0.6272$$

$$\Rightarrow y_{4,c} = y_2 + \frac{h}{3}\left[y'_4 + 4y'_3 + y'_2\right]$$

$$y_{4,c} = 1.3751 + \frac{(0.2)}{3}\left[0.6272 + 4(0.7352) + (0.8439)\right]$$

$$\boxed{y(0.8) = y_{4,c} = 1.6692}$$

solution is $y(0.2) = 1.19598$

$y(0.4) = 1.3751$

$y(0.6) = 1.5358, y(0.8) = 1.6692$

7. Using finite differences solve the boundary value problem
   $y'' + 3y' - 2y = 2x + 3, y(0) = 2, y(1) = 1$ with $h = 0.2$

Solution:

The given differential equation can be written as

$$y'' + 3y' - 2y = 2x + 3 \rightarrow ①$$

Using the central difference approximations for $y''_{(x)}$ & $y'_{(x)}$ we have,

$$y''_{(x)} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \text{ and} \rightarrow ②$$

$$y'_{(x)} = \frac{y_{i+1} - y_{i-1}}{2h} \rightarrow ③$$

Using ② & ③ in ①

$$\left(\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}\right)+3\left(\frac{y_{i+1}-y_{i-1}}{2h}\right)-2y_i=2x_i+3$$

$$2y_{i+1}-4y_i+2y_{i-1}+3hy_{i+1}-3hy_{i-1}-4h^2y_i=4h^2x_i+6h^2$$

$$(2+3h)y_{i+1}-\left(4+4h^2\right)y_i+(2-3h)y_{i-1}=4h^2x_i+6h^2$$

$$\text{Sub}\,h=0.2=\frac{1}{5}$$

$$\frac{13}{5}y_{i+1}-\frac{104}{25}y_i+\frac{7}{5}y_{i-1}=\frac{4}{25}x_i+\frac{6}{25}$$

$$65y_{i+1}-104y_i+35y_{i-1}=4x_i+6\rightarrow ④$$

Put i=1,2,3,4 we get,

$$\left.\begin{aligned}65y_2-104y_1+35y_0&=4x_1+6\\65y_3-104y_2+35y_1&=4x_2+6\\65y_4-104y_3+35y_2&=4x_3+6\\65y_5-104y_4+35y_3&=4x_4+6\end{aligned}\right|\rightarrow A$$

Since h=1/5, we have

X:	0	1/5	2/5	3/5	4/5	1
Y:	2	Y_1	Y_2	Y_3	Y_4	1

To find y_1, y_2, y_3, y_4:

Solving the equations (A)

Sub. the known values in A. We get

$$-104y_1+65y_2=-63.2\rightarrow ⑤$$

$$35y_1-104y_2+65y_3=7.6\rightarrow ⑥$$

$$35y_2-104y_3+65y_4=8.4\rightarrow ⑦$$

$$35y_3-104y_4=-55.8\rightarrow ⑧$$

Solving ⑤& ⑥,

$$(5)\times 35\ \Rightarrow -3640y_1+2275y_2+0.y_3=-2212$$

$$(6)\times 104\Rightarrow 3640y_1+10816y_2+6760y_3=790.4$$

$$-8541y_2+6760y_3=-1421.6\rightarrow ⑨$$

Solving ⑨&⑦

$$(9)\times 35\ \Rightarrow -298935y_2+236600y_3=-49756$$

$$(7)\times 8541\Rightarrow 298935y_2-888264y_3+555165y_4=71744.4$$

$$-651664y_3+555165y_4=21988.4\rightarrow ⑩$$

Solving ⑧& ⑩

$$(8)\times 651664\ \Rightarrow 2{,}2808240y_3-67773056y_4=-36362851.2$$

$$(10)\times 35\Rightarrow\quad -2{,}2808240y_3+19430775y_4=769594$$

$$-47572687y_4=-35593257.2$$

$$\Rightarrow \boxed{y_4 = 0.7482}$$

Sub. $y_4 = 0.7482$ in ⑩ we get

$$-651664y_3 + 555165(0.7482) = 21988.4$$

$$\boxed{y_3 = 0.6037}$$

Sub. y_3 =0.6037 in (9)

$$-8541y_2 + 6760(0.6037) = -1421.6$$

$$\boxed{y_2 = 0.6443}$$

Sub. y_2 =0.6443 in (5)

$$-104y_1 + 65(0.6443) = -63.2$$

$$\boxed{y_1 = 1.0104}$$

∴ Solution is,

X:	0	1/5	2/5	3/5	4/5	1
Y:	2	1.0104	0.6443	0.6037	0.7482	1

8. Using Milne's method, obtain the solution of $\frac{dy}{dx} = x - y^2$ at x=0.8, x=1 given y(0)=0, y(0.2)=0.02, y(0.4)=0.0795, y(0.6)=0.1762.

Solution:

Given $\frac{dy}{dx} = x - y^2$ and h=0.2

$$x_0 = 0 \qquad y_0 = 0$$
$$x_1 = 0.2 \qquad y_1 = 0.02$$
$$x_2 = 0.4 \qquad y_2 = 0.0795$$
$$x_3 = 0.6 \qquad y_3 = 0.1762$$

<u>To find y(0.8)=y_4:</u>

$$y' = f(x,y) = x - y^2$$
$$y'_0 = x_0 - y_0^2 = 0$$
$$y'_1 = x_1 - y_1^2 = 0.2 - (0.02)^2 = 0.1996$$
$$y'_2 = x_2 - y_2^2 = 0.4 - (0.0795)^2 = 0.3937$$
$$y'_3 = x_3 - y_3^2 = 0.6 - (0.1762)^2 = 0.5690$$

By Milne's predictor formula, we have,

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Put n=3,

$$y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 0 + \frac{4(0.2)}{3}\left[2(0.1996) - 0.3937 + 2(0.5690)\right]$$

$$\boxed{y_{4,p} = 0.3049}$$

By Milne's corrector formula we have,

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1,p}\right]$$

Put n=3,

$$y_{4,c} = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_{4,p}\right]$$

Now,

$$y'_{4,p} = x_4 - y^2_{4,p} = 0.8 - (0.3049)^2 = 0.7070$$

$$\therefore y_{4,c} = 0.079 + \frac{(0.2)}{3}\left[0.3937 + 4(0.5690) + 0.707\right]$$

$$\boxed{y_{4,c} = 0.3046}$$

$$\therefore \boxed{y(0.8) = 0.3046}$$

<u>To find $y(1)=y_5$:</u>

We need $y'_4 = x_4 - y_4^2 = 0.8 - (0.3046)^2$

$$\boxed{y'_4 = 0.7072}$$

Put n=4 in Milne's predictor formula,

$$\boxed{y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]}$$

$$y_{5,p} = y_1 + \frac{4h}{3}\left[2y'_2 - y'_3 + 2y'_4\right]$$

$$= 0.2 + \frac{4(0.2)}{3}\left[2(0.3934) - 0.5690 + 2(0.7072)\right]$$

$$\boxed{y_{5,p} = 0.4554}$$

Milne's corrector formula is given by,

$$\boxed{y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1,p}\right]}$$

Put $n = 4$,

$$y_{5,c} = y_3 + \frac{h}{3}\left[y'_3 + 4y'_4 + y'_{5,p}\right]$$

Now, $y'_{5,p} = x_5 - y^2_{5,p} = 1 - (0.4554)^2 = 0.7926$

$y_{5,c} = 0.4556$

$\therefore \boxed{y(1) = 0.4556}$

$\therefore$ Solution:

$$\boxed{\begin{aligned} y(0.8) &= 0.3046 \\ y(1) &= 0.4556 \end{aligned}}$$

9. Use R.K method of fourth order to find y(0.2) if

$$\frac{dy}{dx} = x + y^2, y(0) = 1, h = 0.1$$

Sol:

Given $y' = x + y^2$, $x_0 = 0, y_0 = 1$

$x_1 = 0.1, x_2 = 0.2$ and $h = 0.1$

To find y(0.1) using Fourth order R-K method:-

$$\left.\begin{aligned} y_{n+1} &= y_n + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \\ \text{where } k_1 &= hf(x_n, y_n) \\ k_2 &= hf\left(x_n + \frac{h}{2}, y_n + k_{1/2}\right) \\ k_3 &= hf\left(x_n + \frac{h}{2}, y_n + k_{2/2}\right) \\ K_4 &= hf(x_n + h, y_n + k_3) \end{aligned}\right] \to (I)$$

Replacing n=0 in (I) & finding the values:

$$\begin{aligned} k_1 &= hf(x_0, y_0) \\ &= (0.1)f(0,1) \\ &= (0.1)\left[0 + 1^2\right] \end{aligned}$$

$$\boxed{k_1 = 0.14}$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + k_{1/2}\right) = 0.1f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$$

$$= 0.1f(0.05, 1.05)$$

$$= (0.1)(0.05 + 1.05^2)$$

$$\boxed{k_2 = 0.11525}$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + k_{2/2}\right) = 0.1f\left(0 + \frac{0.1}{2}, 1 + \frac{0.11525}{2}\right)$$

$$= 0.1f(0.05, 1.057625)$$

$$= (0.1)(0.05 + 1.057625^2)$$

$$\boxed{k_3 = 0.116857}$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.1f(0 + 0.1, 1 + 0.116857)$$

$$= 0.1f(0.1, 1.116857)$$

$$= (0.1)(0.17 + 1.116857^2)$$

$$\boxed{k_4 = 0.1347}$$

$$\therefore y_1 = y(0.1) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = 1 + \frac{1}{6}(0.1 + 2(0.11525) + 2(0.116857) + 0.1347)$$

$$= 1 + 0.11649$$

$$= 1.11649$$

$$\boxed{y_1 = y(0.1) = 1.1165}$$

<u>To find y(0.2) using fourth order R-K method:-</u>

Put n=1 in (I) & find the values,

$$k_1 = hf(x_1, y_1)$$

$$k_1 = 0.1f(0.1, 1.1165)$$

$$k_1 = (0.1)(0.1 + 1.1165^2)$$

$$\boxed{k_1 = 0.1347}$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + k_{1/2}\right)$$

$$= 0.1f\left(0.1 + \frac{0.1}{2}, 1.1165 + \frac{0.1347}{2}\right)$$

$$= 0.1f(0.15, 1.18385)$$

$$= (0.1)(0.15 + 1.18385^2)$$

$$\boxed{k_2 = 0.1552}$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + k_{2/2}\right)$$

$$= 0.1f\left(0.1 + \frac{0.1}{2}, 1.1165 + \frac{0.1552}{2}\right)$$

$$= 0.1f(0.15, 1.1941)$$

$$= (0.1)(0.15 + 1.1941^2)$$

$$\boxed{k_3 = 0.1576}$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= (0.1)f(0.1 + 0.1,\ 1.1165 + 0.1576)$$

$$= (0.1)f(0.2, 1.2741)$$

$$= (0.1)(0.2 + 1.2741^2)$$

$$\boxed{k_4 = 0.1823}$$

$$\therefore y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.1165 + \frac{1}{6}(0.1347 + 2(0.1552) + 2(0.1576) + 0.1823)$$

$$\boxed{y_2 = 1.2736}$$

Solution:

$$\boxed{y_2 = y(0.2) = 1.2736}$$

10. Solve by finite difference method, the equation $y'' - y = 0$, given y(0)=0, y(1)=1 taking h=.2

Solution:

The given differential equation can be written as,

$$y''(x) - y(x) = 0$$

Using the central finite difference approximation,

$$y''(x) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

We have,

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - y_i = 0$$

$$y_{i+1} - 2y_i + y_{i-1} - h^2 y_i = 0$$

$$y_{i+1} - (2 + h^2)y_i + y_{i-1} = 0 \rightarrow (I)$$

Given h=0.2=$\frac{1}{5}$

$$① \Rightarrow y_{i+1} - \left(2 + \frac{1}{25}\right)y_i + y_{i-1} = 0$$

$$\Rightarrow 25y_{i+1} - 51y_i + 25y_{i-1} = 0$$

$$\Rightarrow \boxed{25y_{i+1} - 51y_i + 25y_{i-1} = 0} \rightarrow ①$$

Now,

X:	0	1/5	2/5	3/5	4/5	1
Y:	0	Y_1	Y_2	Y_3	Y_4	1

Put i=1,2,3,4 in ① to find $y_1, y_2, y_3 y_4$

$$\left.\begin{aligned} 25y_0 - 51y_1 + 25y_2 &= 0 \\ 25y_1 - 51y_2 + 25y_3 &= 0 \\ 25y_2 - 51y_3 + 25y_4 &= 0 \\ 25y_3 - 51y_4 + 25y_5 &= 0 \end{aligned}\right] \rightarrow (2)$$

$\text{Sub}\, x_0 = 0, y_0 = 0, x_5 = 1, y_5 = 1 \text{in}(2)$

We get,

$$-51y_1 + 25y_2 = 0 \rightarrow (3)$$

$$25y_1 - 51y_2 + 25y_3 = 0 \rightarrow (4)$$

$$25y_2 - 51y_3 + 25y_4 = 0 \rightarrow (5)$$

$$25y_3 - 51y_4 = -25 \rightarrow (6)$$

Solving (3) (4)

$$(3) \times 25 \Rightarrow -1275y_1 + 625y_2 = 0$$

$$(4) \times 51 \Rightarrow \quad 1275y_1 - 2601y_2 + 1275y_3 = 0$$

$$-1976y_2 + 1275y_3 = 0 \rightarrow (7)$$

Solving (5) & (7)

$$(5) \times 1976 \Rightarrow 49400y_2 - 100776y_3 + 49400y_4 = 0$$

$$(7) \times 25 \Rightarrow \quad -49400y_2 + 31875y_3 = 0$$

$$-68901y_3 + 49400y_4 = 0 \rightarrow (8)$$

Solving (6) & (8)

$$(6) \times 68901 \Rightarrow 1722525y_3 - 3513951y_4 = -1722525$$

$$(8) \times 25 \Rightarrow \quad -1722525y_3 + 1235000y_4 = 0$$

$$-2278951y_4 = -1722525$$

$$\boxed{y_4 = 0.7558}$$

$$\text{sub.}\, y_4 = 0.7558 \text{in}(8)$$

$$-68901y_3 + 49400(0.7558) = 0$$

$$\boxed{y_3 = 0.5419}$$

$$\text{sub.}\, y_3 = 0.5419 \text{in}(7)$$

$$-1976y_2 + 1275(0.5419) = 0$$

$$\boxed{y_2 = 0.3497}$$

$sub. y_2 = 0.3497 in (3)$

$-51y_1 + 25(0.3497) = 0$

$$\boxed{y_1 = 0.1714}$$

Solution is,

X	0	1/5	2/5	3/5	4/5	1
Y	0	0.1714	0.3497	0.5419	0.7558	1