HEAT AND MASS TRANSFER

UNIT-1

CONDUCTION

PART-A

1. State Fourier's law of condition

The rate of heat conduction is propagation to the area measured normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q \ \alpha \ A\frac{dT}{dx}$$
$$Q = -KA\frac{dT}{dx}$$

Where A – area in m^2

$\frac{dT}{dx}$ – Temperature gradient in K/m

K – Temperature conductivity W/mK

2. Defined Thermal Conductivity

Thermal conductivity is defined as the ability of a substance to conduct heat.

3. Write down the equation for conduction of heat through a slab or plan wall.

$$\text{Heat transfer } Q = \frac{\Delta T_{\text{overall}}}{R}$$
$$\text{where } \Delta T = T_1 - T_2$$
$$R = \frac{1}{KA} = \text{Thermal resistance of slab}$$
$$L = \text{Thickness of slab}$$
$$K = \text{Thermal conductivity of slab}$$
$$A = \text{Area}$$

4. Write down the equation for conduction of heat through a hollow cylinder

$$\text{Heat transfer } Q = \frac{\Delta T_{\text{overall}}}{R}$$
$$\text{where } \Delta T = T_1 - T_2$$
$$R = \frac{1}{2\pi LK} \text{in} \left[\frac{r_2}{r_1}\right] \text{thermal resistance of slab}$$
$$L = \text{Length of cylinder}$$
$$K = \text{Thermal conductivity}$$
$$r_2 = \text{Outer radius}$$
$$r_1 = \text{inner radius}$$

5. State Newton's law of cooling or conservation law.

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Heat transfer by conservation given by Newton's law of cooling

$$Q = hA\left(T_s - T_\infty\right)$$

Where A – Area exposed to heat transfer in m^2

h – heat transfer co efficient in W/m^2K

T_s – Temperature of the surface in K

T_∞ - Temperature of the fluid in K

6. Write down the general equation for one dimension steady state heat transfer in slab or plane wall with and without heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\infty}\frac{\partial T}{\partial t}$$
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

7. Define overall beat transfer co efficient

The overall heat transfer by combined modes is usually expressed in terms of an overall conductance or overall heat transfer co efficient U

$$\text{Heat transfer } Q = UA\,\Delta T$$

8. Write down the equation for heat transfer through composite pipes or cylinder.

Heat transfer

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$
Where
$$\Delta T = T_a - T_b$$
$$R = \frac{1}{2\pi L}\frac{1}{h_n r_1} + \frac{\mathrm{In}\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{\mathrm{In}\left[\frac{r_2}{r_1}\right]L_2}{K_2} + \frac{1}{h_b r_3}$$

9. What is critical radius of insulation or critical thickness?

$$\text{Critical radius} = r_c$$
$$\text{Critical Thickness} = r_c - r_1$$

Additional of insulation material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumferences it actually increases the heat loose up certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius insulation and the coprresponding thickness is called critical thickness.

10. Define external surfaces

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surface used for increasing heat transfer are called extended surface used or sometimes known as fin.

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11. State applications of fins.

The main application of fins are,

1. Cooling of electronic components

2. Cooling of motor cycle engines

3. Cooling of transformers

4, Cooling of small capacity compressors.

12. Define Fin efficiency.

The efficiency of a fin is defined as the ratio of actual heat transfer by the fin to the maximum possible heat transferred by the fin

$$\eta_{min} = \frac{Q_{lim}}{Q_{max}}$$

13. Define Fin effectiveness.

Fin effectiveness is the ratio of heat transfer with fin to that without fin.

$$\text{Fin effectiveness } = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

14. List the mode of heat transfer.

1) Conduction

2)Convection

3) Radiation,

15. List down the types of boundary condition.

1) Prescribed temperature

2) Prescribed heat flux

3) Convection boundary condition

16. What is meant by lumped heat analysis?

In Newton heating or cooling process the temperature through the solid is considered to be uniform at a given time. Such an analysis is called Lumped heat capacity analysis.

17. What is meant by semi infinite solids?

In semi infinite solids, at ant instant of time, there is always a point where the effect of heating or cooling at one of its boundaries is not fell at all. At this point the temperature remains unchanged. In semi-infinite solids, the biot number value is infinite.

18. Explain the significance of Fourier number.

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It is defined as the ratio of characteristic body dimension to temperature wave penetration depth in time. It signifies the degree of penetration of heating or cooling effect of a solid

19. What are the factors affecting in thermal conductivity?

1. Moisture
2. Density of materials,
3. Pressure
4. Temperature
5. Structure of material

20.Explain the significance of thermal diffusivity?

The physics significance of thermal diffusivity is that tells us how fast heat is propagated or it diffuses through a material during changes of temperature with time.

PART - B

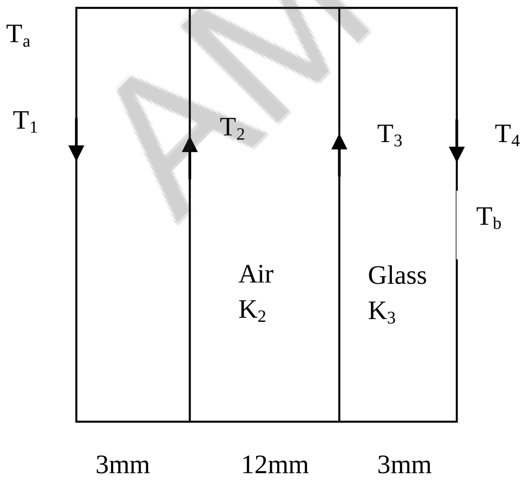
1. Consider a 1.2 m high and 2 m wide double-pane window consisting of two 3 mm thick layers of glass (k = 0.78W/mK) separate by a 12 mm wide stagnant air space (k = 0.026W/mK). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface when the room is maintained at 24°C while the temperature of the outdoors is -5°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be 10 W/m²K and 25 W/m²K respectively.

Given data

Thermal conductivity of glass

$$= K_1 = K_3 = 0.78 \text{ W/mK}$$

Thermal conductivity of air K_2 = 0.026 W/mK



Height of double pane window = 1.2 m

Width of double pane window = 2 m

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Area of double pane window = 1.2 × 2 = 2.4 m^2

Interior room temperature, T_a = 24° C

Outside air temperature, T_b = -5° c

Thickness of glass, $L_1 = L_3$ = 3mm = 0.003m

Thickness of air, L_2 = 12mm = 0.012m

Interior convective heat transfer coefficient,

h_a = 100 W/m_2K.

Outer convective heat transfer coefficient,

h_b = 25W /m^2K

To find:

(a) Rate of heat transfer, Q

(b) Inner surface temperature, T_i

Solution:

Heat flow through composite wall is given by.

$$Q = \frac{\Delta T}{R} \quad \text{where } \Delta T = T_a - T_b$$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_2}{K_3 A} + \frac{1}{h_b A}$$

$$Q = \frac{T_a - T_b}{R_{convet} + R_{12} + R_{23} + R_{conv2}} \quad \text{or}$$

$$Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_2}{K_3 A} + \frac{1}{h_b A}}$$

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$$R_{convert} = \frac{1}{h_0 A} = \frac{1}{10 \times 2.4} = 0.04167°C/W$$

$$R_{12} = \frac{L_1}{K_1 A} = \frac{0.003}{0.78 \times 2.4} = 0.0016°C/W$$

$$R_{23} = \frac{L_2}{K_2 A} = \frac{0.012}{0.026 \times 2.4} = 0.1923°C/W$$

$$R_{34} = \frac{L_3}{K_3 A} = \frac{0.003}{0.78 \times 2.4} = 0.0016°C/W$$

$$R_{conv2} = \frac{1}{h_0 A} = \frac{1}{25 \times 2.4} 0.01667°C/W$$

$$Q = \frac{24-(-5)}{0.4167 + 0.0016 + 0.1923 + 0.0016 + 0.01667}$$

$$= \frac{29}{0.25384}$$

$$Q = 114.24\,W$$

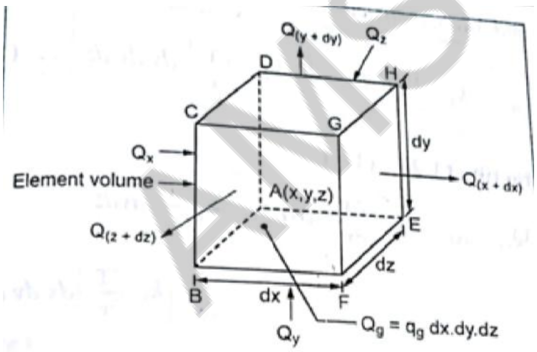
$$Q = \frac{24 - T_1}{R_{conv1}}$$

2. Derive the general 3 dimension heat conduction equation in Cartesian coordinates.

Consider a small rectangular element of sides dx, dy and dz shown in figure

The energy balance of this rectangular element is obtained from first law of thermodynamics.

$$\Rightarrow \left\{\begin{array}{l}\text{Net heat} \\ \text{conducted into} \\ \text{element from} \\ \text{all the coordinate} \\ \text{directions}\end{array}\right\} + \left\{\begin{array}{l}\text{Heat} \\ \text{generated} \\ \text{with in} \\ \text{element}\end{array}\right\} = \left\{\begin{array}{l}\text{Heat} \\ \text{Stored} \\ \text{in the} \\ \text{element}\end{array}\right\}$$



Net heat conducted into element from all the coordinates direction

Let q_x be the heat flux in a direction of face ABCD and q_{x+dx} be the heat flux in a direction of face EFGH.

The rate of heat flow into the element in x direction through the face ABCD is

$$Q_x = q_x \, dy \, dz = -k_x \frac{\partial T}{\partial x} dy \, dz$$

Where k – Thermal conductivity, W/mK

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$\frac{\partial T}{\partial x}$ – Temperature gradient

The rate of heat flow out of the element in x direction through the face EFGH is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x)dx$$

$$= -k\frac{\partial T}{\partial x}dydz + \frac{\partial}{\partial x}\left[-k_x \frac{\partial T}{\partial x}dydz\right]dx$$

$$Q_{x+dx} = -k_x \frac{\partial T}{\partial x}dydz = \frac{\partial}{\partial x}\left[k_c \frac{\partial T}{\partial x}\right]dx\,dy\,dz$$

Substiting

$$Q_x - Q_{x+dx} = k_x \frac{\partial T}{\partial x}dy\,dz = \left[-k_x \frac{\partial T}{\partial x}dydz - \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right]dx\,dy\,dz\right]$$

$$= -k_x \frac{\partial T}{\partial x}dy\,dz9 + k_x dy\,dz + \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right]dx\,dy\,dz$$

$$\Rightarrow Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right]dx\,dy\,dz$$

similarly

$$Q_y - Q_{y+dy} = \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right]dx\,dy\,dz$$

$$Q_z - Q_{z+dz} = \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]dx\,dy\,dz$$

Adding the equation

$$\text{Net heat conductance} = \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right]dx\,dy\,dz + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right]dx\,dy\,dz + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]dx\,dy\,dz$$

$$= \left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right]dx\,dy\,dz$$

Net heat conducted into element from all the coordinates direction

$$= \left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right]dx\,dy\,dz$$

Heat stored in the element

We know that

$$\begin{Bmatrix}\text{Heat stored}\\ \text{in the}\\ \text{element}\end{Bmatrix} = \begin{Bmatrix}\text{Mass of}\\ \text{the}\\ \text{element}\end{Bmatrix} \times \begin{Bmatrix}\text{Specific}\\ \text{heat of the}\\ \text{element}\end{Bmatrix} \times \begin{Bmatrix}\text{Rise in}\\ \text{temperature}\\ \text{of element}\end{Bmatrix}$$

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$$= m \times C_p \times \frac{\partial T}{\partial t}$$

$$= \rho \times dx\,dy\,dz \times C_p \times \frac{\partial T}{\partial t} \qquad [\therefore \text{Mass} = \text{Density} \times \text{Volume}]$$

$$\boxed{\left\{\begin{array}{l}\text{Heat stored in}\\ \text{the element}\end{array}\right\} = \rho C_p \frac{\partial T}{\partial t} dx\,dy\,dz}$$

Heat generated within the element

Heat generated within the element is given by

$$Q = q\,dx\,dy\,dz$$

Substituting equations in

$$= \left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right]\right] dx\,dy\,dz + \dot{q}\,dx\,dy\,dz = \rho C \frac{\partial T}{\partial t} dx\,dy\,dz$$

$$\Rightarrow \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z}\right] + q = \rho C_\rho \frac{\partial T}{\partial t}$$

Considering the material is isotropic. So,

$K_x = k_y = k_z = k$ = Constant.

$$\Rightarrow \qquad \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] k + q = \rho C_\rho \frac{\partial T}{\partial t}$$

Divided by k,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}} \qquad \text{........}(1.10)$$

It is a general three dimensional heat conduction e1uation in Cartesian co-ordinates

$$\text{Where, } \alpha = \text{Thermal diffusivity} = \frac{k}{\rho C_p} - m^2/s$$

Thermal diffusivity is nothing but how fast heat is diffused through a material during changes of temperature with time.

Case (i) : No heat sources

In the absence of internal heat generation, equation (1.10) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \qquad \text{............}(1.11)$$

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This equation is known as diffusion equation (or) Fourier's equation.

Case (ii): Steady state conditions

In steady state condition, the temperature does not change with time. So, $\frac{\partial T}{\partial t} = 0$. The heat condition equation (1.10) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0 \quad ....(1.12)$$

(or)

$$\nabla^2 T + \frac{q}{k} = 0$$

The equation is known as Poisson's equation.

In the absence of internal heat generations, equation (1.12)

Becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad ....(1.13)$$

(or)

$$\nabla^2 P = 0$$

This equation is known as Laplace equation.

Case (iii): One dimensional steady state heat conduction

If the temperature varies only in the x direction, the equation (1.10) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{q}{k} = 0 \quad ....(1.14)$$

In the heat absence of internal heat generation, equation (1.14)

Becomes:

$$\frac{\partial^2 T}{\partial x^2} = 0 \quad ....(1.15)$$

Case: (iv): Two dimensional steady state heat conduction

If the temperature varies only in the x and y directions the equation (1.10) becomes:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q}{k} = 0 \quad ....(1.16)$$

In the absence of internal heat generation, equation, (1.16) reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad ....(1.17)$$

Case (v): Unsteady state, one dimensional, without internal heat generation

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In steady state, the temperature changes with time.

i.e., $\frac{\partial T}{\partial t} \neq 0.$ So, the general conduction equation (1.10) reduces to

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{.................}(1.18)$$

3. A cylinder 1m long and 5cm in diameter is placed in an atmosphere at 45℃. It is provided with 10 longitudinal straight fins of material having k = 120 W/mk. The height of 0.76 mm thick fins is 1.27 cm from the cylinder surface. The heat transfer co-efficient between cylinder and atmosphere air is 17 W/m^{2}K. Calculate the rate of heat transfer and the temperature at the end of fins if surface temperature of cylinder is 150℃. (16)

Length of engine cylinder, L_{cy}= 1m

Diameter of the cylinder, d =-5cm = 0.05m

Atmospheric temperature, T_∞= 45℃ + 273 = 318k

Number of fins = 10

Thermal conductivity of fin, k = 120 W/mK

Thickness of the fin, t = 0.76 mm = 0.76×10^{-3}m

Length (height) of the fin, L_f= 1.27 cm = 1.27×10^{-2}m

Heat transfer co-efficient, h = 17 W/m^{2}K

Cylinder surface temperature

Or

Base temperature, T_b= 150℃ + 273 = 423K

To find:

1. Rate of heat transfer, Q

2. Temperature at the end of the fin.

Solution:

Length of the fin is 1.27 cm. So, this is short fin. Assume that the fin end is insulated.

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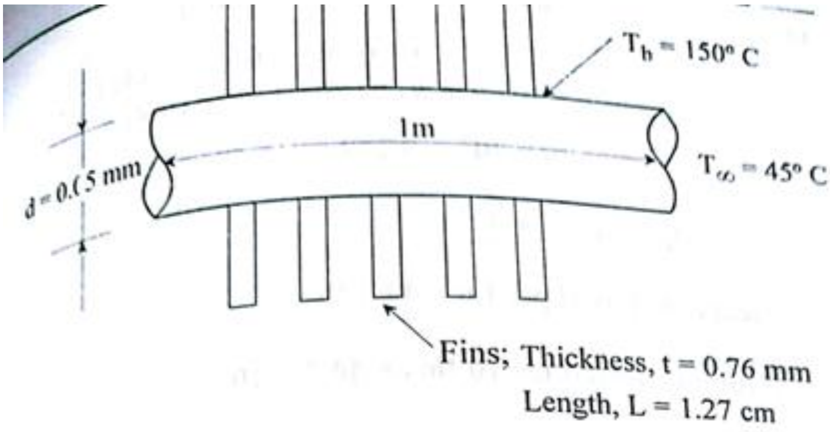

We know that,

Heat transferred, $Q_1 = (hPkA)^{1/2}(T_b - T_\infty)\tan h(mL_f)$ ....(1)

Where,

P- Parameter = 2 × Length of the cylinder

= 2 × 1

$\boxed{P = 2m}$

A- area = Length of the cylinder × thickness

= 1 × 0.76 × 10^{-3}

$\boxed{A = 0.76 \times 10^{-3} m^2}$

$$m = \sqrt{\frac{hP}{kA}}$$

$$= \sqrt{\frac{17 \times 2}{120 \times 0.76 \times 10^{-3}}}$$

$$\boxed{m = 19.30 m^{-1}}$$

(1)⟹ $Q_1 = (h\ PkA)^{1/2} (T_b - T_\infty) \tan h\ (mL_f)$

$$= \left[17 \times 120 \times 0.76 \times 10^{-3}\right]^{1/2} \times (423 - 318) \times \tan h\left(19.30 \times 1.27 \times 10^{-2}\right)$$

$$= 1.76 \times 105 \times 0.240$$

$$\boxed{Q_1 = 44.3 W}$$

Heat transferred per fin = 44.3 W

Heat transferred for 10 fins = 44.3 × 10

= 443 W

$\boxed{Q_1 = 443 W}$ ....(2)

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Heat transfer from unfinned surface due to convection is

$Q_2 = hA\,\Delta T$

$= h\,(\pi d\, L_{cy} - 10 \times t \times L_f) \times (T_b - T_\infty)$

[∵ Area of unfinned surface = Area of cylinder – Area of finn

$$= 17 \times \left[(\pi d 0.05 \times 1) - \left(10 \times 0.76 \times 10^{-3} \times 1.27 \times 10^{-2}\right) \right] \times (423 - 318)$$

$$\boxed{Q_2 = 280.21\,W}$$

So total heat transfer, $Q = Q_1 + Q_2$

$= 443 + 280.21$

$$\boxed{Q = 723.21\,W}$$

We know that,

Temperature distribution [Short fin, end insulated]

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\left[m\left(L_f - x\right)\right]}{\cos h\left(mL_f\right)}$$

We need temperature at the end of fin. So, put ∞.

$$\Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\left[m\left(L - L_f\right)\right]}{\cos h\left(mL_f\right)}$$

$$\Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cos h\left(19.30 \times 1.27 \times 10^{-2}\right)}$$

$$\frac{T - 318}{423 - 318} = \frac{1}{1.030}$$

$$\frac{T - 318}{105} = 0.970$$

$$\Rightarrow \boxed{T = 419.94\,K}$$

Result:

1. Heat transfer, Q = 723.21 W

2. Temperature at the end of the fin, T = 419.94 K

4. Explain the mechanism of heat conduction in solids and gases. (4)

Heat conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature within medium (solid, liquid or gases) or different medium in direction physical contact.

(CONTINUATION OF ABOVE) in conduction, energy exchange take place by the kinematic motion of direct impact of molecules pure conduction found only in solids

5. At a certain instant of time, the temperature distribution in a long cylindrical tube is, T = 800 + 1000-5000 r^2 where, T is in ℃ and r in m. The inner and outer radio of tube are respectively 30 cm and 50 cm. The tube material has a thermal conductivity of 58 W/mK and thermal diffusivity of 0.004 m^2/hr.

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Determine the rate heat flow at inside and outside surfaces per unit length rate of heat storage per unit length and rate of change temperature at inner and outer surfaces.

Given Data: in cylindrical tube,

$$T = 800 + 1000\ r - 5000\ r^2$$

Inner radius, $r_1 = 30$ cm $= 30 \times 10^{-2}$m

Outer radius, $r_2 = 50$ cm $= 50 \times 10^{-3}$m

Thermal conductivity, K = 58 W/mK

Thermal diffusivity, $\propto = 0.004\ m^2/hr$

$$= \frac{0.004}{3600} \times 1.11 \times 10^6\ m^2 / hr$$

To find:

(i) Rate of heat flow at inside and outside surfaces per unit length.

(ii) Rate of heat storage per unit length.

(iii) Rate of change of temperature at inner and outer surfaces.

@**Solution:**

(i) Rate of heat flow at inside surface per unit length,

$$Q_{in} = -KA_i \left(\frac{dT}{dr}\right)_{ri} = 0.3$$

$$Q_{in} = -58 \times 2\pi \times (0.3) \times 1 \times \left[\frac{d(800 + 1000r - 5000r^2)}{dr}\right]_{ri} = 0.3$$

$$Q_{in} = -109.33[-2000] = 21.86 \times 10^4\ W$$

Rate of heat flow at outside surface per unit length, Q_{out}

$$= -K\,A_0 \left(\frac{dT}{dr}\right)_{r_0} = 0.5$$

$$Q_{out} = -58 \times 3.14 \times \left[\frac{d(800 + 1000 - 5000r)}{dr}\right]_{r0} = 0.5$$

$$= -58 \times 3.14[-4000]$$

$$Q_{out} = 72.84 \times 10^4\ W$$

Rate of heat storage per unit length,

$$\therefore Q_{stored} = Q_{in} - Q_{out}$$

$$= (21.86 - 72.84) \times 10^4$$

$$\boxed{Q_{stored} = -50.98 \times 10^4\ W}$$

$T = 800 + 1000\ r - 5000\ r^2$

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$$\frac{dT}{dr} = 1000 - 1000r$$
$$\frac{d^2T}{dr^2} = -10,000$$

Rate of change of temperature at inner surfaces, at r_i=0.3m\

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{\alpha}.\frac{dT}{dt}$$
$$-1000 + \frac{1}{0.3}(1000 - 10000 \times 0.3) = \frac{1}{1.11\times10^{-6}}\left(\frac{dT}{dt}\right)_{ri=0.3}$$
$$\left(\frac{dT}{dt}\right)_{ri=0.3} = 0.01851^{\circ}C/s$$

Rate of change of temperature at outer surfaces

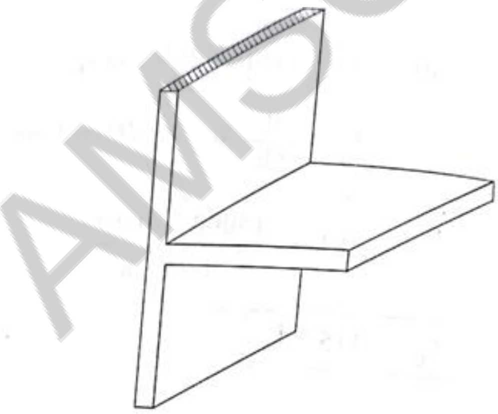
$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{\alpha}.\left(\frac{dT}{dt}\right)_{r_o=0.5} \quad -10000 + \frac{1}{0.5}(100 - 5000 \times 2 \times 0.5) = \frac{1}{1.11\times10^{-6}}\left(\frac{dT}{dt}\right)_{r_o=0.5}$$
$$\left(\frac{dT}{dt}\right)_{r_o=0.5} = \frac{-18,000}{9\times10^5} = -0.02^{\circ}C/s$$

6. With neat shetches, explain the different fin profiles. (4)

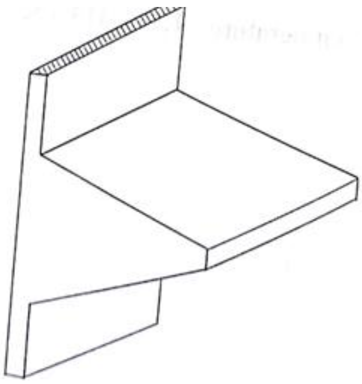
It is possible to increase the heat transfer rate by increment the surface of heat transfer. The surface used for increasing the transfer are called extended surfaces or fins.

Types of fins

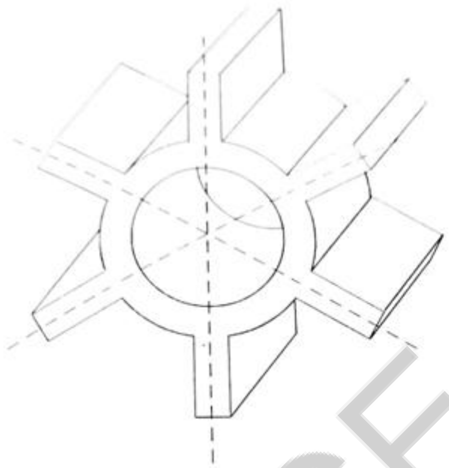
Some common types of fin configuration are shown in fig:



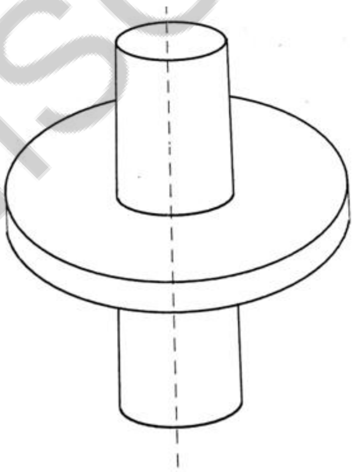
(i) Uniform Straight fin



(ii) Tappered straight fin



(iii) Splines



(iv) Annular fin

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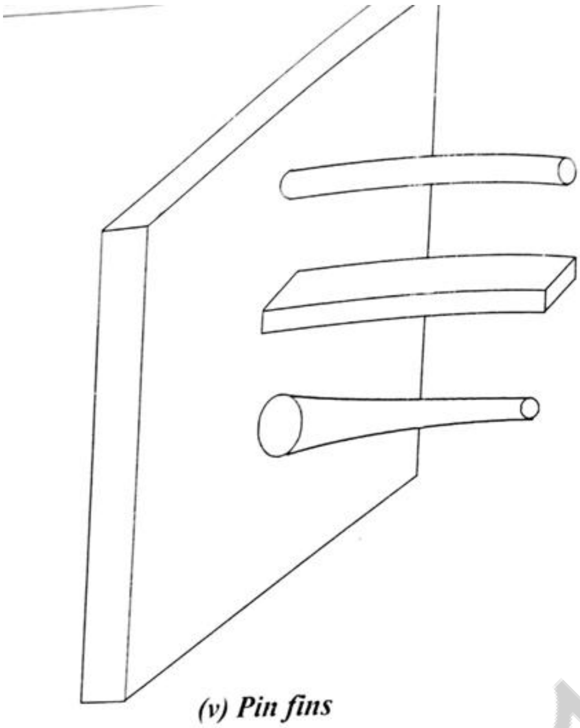


(v) Pin fins

Commonly there are three types of fin

1. Identify long fin

2. Short fin (end is insulated)

3. Short fin (end is not insulated)

7.A circulferential rectangular fins of 140 mm wide, and 5mm thick are fitted on a 200mm diameter tube. The fin base temperature is 170℃ and the ambient temperature is 25℃. Estimate fin efficiency and heat loss per fin.

Take **Thermal cionductivity, k = 200 W/mK.**

Heat transfer co-efficient, h = 140 W/m^{2}K.

Given:

Wide, L = 140 mm = 0.140 m

Thickness, t = 5 mm = 0.005 m

Diameter, d= 200 mm $\Longrightarrow$ r = 100 mm = 0.1 m

Fin base temperature, T_b = 170℃ + 273 = 443 K

Ambient temperature, T_∞= 25℃ + 273 – 298 K

Thermal conductivity, k = 220 W/mK.

Heat transfer co-efficient, h = 140 W/m^{2}K.

To find:

1. fin efficiency, η

2. Heat loss Q

Solution:

A rectangular fin is long and wide. So, heat loss is calculated by using fin efficiency curves.

[From HMT data book page no.50 (Sixth edition)]

Correlated length, $L_c = L + t/2$

$$= 0.140 + \frac{0.005}{2}$$

$$\boxed{L_c = 0.1425\,\text{m}}$$

$R_{2c} = r_1 + L_c$

$$= 0.100 + 0.1425$$

$$\boxed{r_{2c} = 0.2425\,\text{m}}$$

$A_s = 2\pi\,[r_{2c}^2 - r_1^2]$

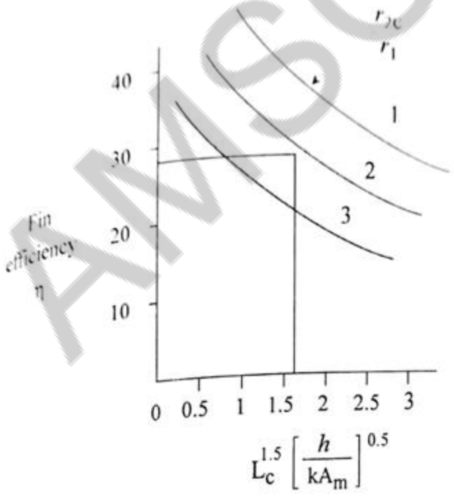
$A_s = 2\,\pi\,[(0.2425)^2 - (0.100)^2]$

$$\boxed{A_s = 0.30650\,\text{m}^2}$$

$A_m = t[r_{2c} - r_1]$

$= 0.005\,[(0.2425) - (0.100)]$

$$\boxed{A_m = 7.125 \times 10^{-4}\,\text{m}^2}$$



From the graph, we know that,

[HMT data book page no.50]

AMSCE - 1101

$$X_{axis} = L_C^{1.5}\left[\frac{h}{kA_m}\right]^{0.5}$$

$$= (0.1425)^{1.5}\left[\frac{140}{230 \times 7.125 \times 10^{-4}}\right]^{0.5}$$

$$\boxed{X_{axis} = 1.60}$$

$$\text{Curve} \longrightarrow \frac{r_{2C}}{r_1} = \frac{0.2425}{0.1} = 2.425$$

X_{axis} value is 1.60

Curve value is 2.425

By using these values, we can find fin efficiency, η from graph

$$\boxed{\text{fin efficiency}, \eta = 28\%}$$

Heat transfer, Q = η A_s h [T_b - T_∞)

[From HMT data book page no]

$$\Rightarrow \quad Q = 0.28 \times 0.30650 \times 140 \times [443 - 298]$$

$$\boxed{Q = 1742.99\,W}$$

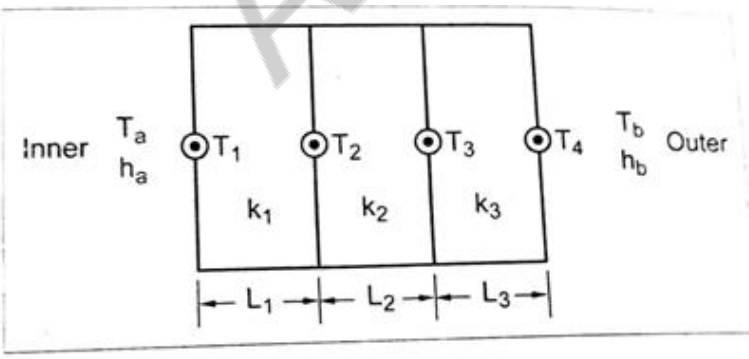
Result:

1. Fin efficiency, η = 28%

2. Heat loss, Q = 1742.99

8.**A furnace wall is mode up of three layers of thickness 25 cm. 10 cm and 15 cm with thermal conductivities of 1.65 K and 9.2 w/mK respectively. The inside is expressed to gases at 1250°Cm with a convection coefficient of 25 W/m^2°C and the inside surface is at 1100°C, the outside surface is expressed to air at 25°C with convection coefficient of 12 W/m^{2}K. Determine (i) the unknown thermal conductivity, (ii) The overall heat transfer coefficient, (iiii) All the surface temperatures. (16)**

Given:



Thickness, L_1 = 25 cm = 0.25 m

L_2 = 10 cm = 0.10 m

AMSCE - 1101

L_3 = 15 cm = 0.15 m

Thermal conductivity, k_1 = 1.65 W/mK

$K_2 = k$

K_3 = 9.2 W/mK

Inside gas temperature, $T_a = 1250°C + 273$

$= 1523$ K

Outside air temperature, $T_b = 25°C + 273$

$= 298$ K

Inside heat transfer co-efficient, $h_a = 25$ W/m^{2}K

Outside heat transfer co-efficient, $h_b = 12$ W/m^{2}K

Inner surface temperature, $T_{1=}$ $1100°C + 273$

$= 1373K$

To find: (i) Unknown thermal conductivity, k_2

(ii) Overall heat transfer co-efficient, U

(iii) Surface temperatures T_2, T_3, T_4

@Solution;

We know that,

Heat transfer Q = h_a A ($T_a - T_t$)

$$\Rightarrow \frac{Q}{A} = h_a \left(T_a - T_1\right)$$
$$= 25\left(1523 - 1373\right)$$
$$\boxed{\frac{Q}{A} = 3750\,W/m^2}$$

We know that,

Heat flow, $Q = \frac{\Delta T_{overall}}{R}$

[From HMT data book, page no. 43 & 44]

Where, $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow \qquad Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}}$$

$$\Rightarrow \qquad \frac{Q}{A} = \frac{T_a - T_b}{\frac{1}{h_a} + \frac{L_1}{k_2} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}}$$

$$3750 = \frac{1523 - 298}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.10}{k_2} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$\Rightarrow 3750 = \frac{1225}{0.2911 + \frac{0.10}{k_2}}$$

$$\Rightarrow 3750\left(0.2911 + \frac{0.1}{k_2}\right) = 1225$$

$$\Rightarrow 0.2911 + \frac{0.1}{k_2} = 0.3266$$

$$\Rightarrow \boxed{\text{Thermal conductivity, } k_2 = 2.816\,\text{W/mK}}$$

We know that,

Overall heat transfer co-efficient, $U = \frac{1}{R_{total}}$

$$\Rightarrow R_{total} = \frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b} \quad \left[\text{Take } A = 1\text{m}^2\right]$$

$$= \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.816} + \frac{0.15}{9.2} + \frac{1}{12}$$

$$\boxed{R_{load} = 0.3267\,\text{W/m}^2}$$

$$\Rightarrow U = \frac{1}{R_{total}}$$

$$= \frac{1}{0.3267}$$

$$\boxed{U = 3.06\,\text{W/m}^2\text{K}}$$

We know that,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_b}{R_b}$$

AMSCE - 1101

$(1) \Rightarrow \qquad Q = \frac{T_1 - T_2}{R_1}$ where $R_1 = \frac{L_1}{k_1 A}$

$$\Rightarrow \qquad Q = \frac{T_1 - T_2}{\frac{L_1}{k_1 A}}$$

$$\Rightarrow \qquad \frac{Q}{A} = \frac{T_1 - T_2}{\frac{L_1}{k_1}}$$

$$3750 = \frac{13730 - T_2}{\frac{0.25}{1.65}}$$

$$\Rightarrow \boxed{T_2 = 804.8\text{K}}$$

$(2) \Rightarrow \qquad Q = \frac{T_2 - T_3}{R_2}$ where $R_2 = \frac{L_2}{k_2 A}$

$$= \frac{T_2 - T_3}{\frac{L_2}{k_2 A}}$$

$$\frac{Q}{A} = \frac{T_2 - T_3}{\frac{L_2}{k_2}}$$

$$3750 = \frac{804.8 - T_3}{\frac{0.10}{2.816}}$$

$$\Rightarrow \boxed{T_3 = 671.45\text{K}}$$

$(3) \Rightarrow \qquad Q = \frac{T_3 - T_4}{R_3}$ where $R_3 = \frac{L_3}{k_3 A}$

$$= \frac{T_3 - T_4}{\frac{L_3}{k_3 A}}$$

$$\frac{Q}{A} = \frac{T_3 - T_4}{\frac{L_3}{k_3}}$$

$$3750 = \frac{671.45 - T_4}{\frac{0.15}{9.2}}$$

$$\Rightarrow \qquad \boxed{T_4 = 610.30\,\text{K}}$$

9. Pin fans are provided to increase the heat transfer rate from a hot surface. Which of the following arrangements will give higher heat transfer rate?(1) 6-fins of 10 cm length, (2) 12-fins of 5 cm length. Take k of fin material = 200 W/mK and h = 20 W/m^2°C cross sectional area of the fin = 2 cm^2; Perimeter of fin = 4cm ; Fin base temperature = 230°C; Surrounding air temperature = 30°C

Given Data:

Thermal conductivity of fin material, k = 200 W/mK

Heat transfer coefficient, h = 20 W/m^2°C = 20 W/m^{2}K

AMSCE - 1101

Cross sectional area, $A = 2\ cm^2 = 2 \times 10^{-4} m$

Perimeter of fin, $P = 0.04$ m

Fin base temperature, $T_b = 230°C + 273 = 503K$

Air temperature, $T_\infty = 30°C + 273 = 303$ K

To find: (i) Q_1, Heat transfer rate (6 fins of 10 cm length)

(ii) Q_2, Heat transfer rate (12 fins of 5 cm length)

@Solution: We know that,

$$m = \sqrt{\frac{h\,P}{k\,A}} = \sqrt{\frac{20 \times 0.04}{200 \times 2 \times 10^{-4}}}$$

$$\boxed{m = 4.4721}$$

We know that,

Heat transfer rate, $Q = n[(h\ P\ k\ A)^{1/2}\ (T_k - T_\infty)\ \tanh (mL)]$

(i) n = 6; L = 0.1m

$m\,L = 4.4721 \times 01$ [From HMT data book, page no. 49]

$$\boxed{m\,L = 0.44721}$$

$$Q_1 = n\left[\left(h\,P\,k\,A\right)^{1/2}\left(T_b - T_\infty\right)\tanh\left(0.447\right)\right]$$

$$= 6\left[\left(20 \times 0.04 \times 200 \times 2 \times 10^{-4}\right)^{1/2} \times \left(503 - 303\right) \times \tanh\left(0.447\right)\right]$$

$$\boxed{Q_1 = 90W}$$

(ii) Number of fins, n = 12, Length of fins L = 5 cm = 0.05 m

$m\,L = 4.4721 \times 0.05$

$$\boxed{m\,L = 0.2235}$$

$Q_2 = n\ [(h\ P\ k\ A)^{1/2}\ (T_b - T_\infty)\ \tanh\ (0.2235)]$

$$= 12\left[\left(20 \times 0.04 \times 200 \times 2 \times 10^{-4}\right)^{1/2} \times \left(503 - 300\right) \times \tanh\left(0.2235\right)\right]$$

$$\boxed{Q_2 = 94.34\,W}$$

Since $Q_2 > Q_1$. The second case is better.

10. A steel ball 50 mm in diameter and at 900°C is placed in still air 30°C. Calculate the initial rate of cooling of ball in °C/min. Take ρ = 7800kg/m^3; C = 2kJ/kg °C; h = 30 W/m^2 °C. Neglect the internal resistance of the ball. (8)

Given Data:

Steel ball diameter, D = 50 mm = 0.05 m

Initial temperature, $T_0 = 900°C + 273 = 1173$ K

Air temperature, $T_\infty = 30°C + 273 = 303$ K

Density, $\rho = 7800$ kg/m^3

Specific heat, $C_p = 2$ kJ/kg°C $= 2 \times 10^3$ J/kg°C

Heat transfer co-efficient, $h = 30$ W/m^2°C

Time, t = 60 seconds

To find: Initial rate of cooling of ball in °C/min.

@Solution:

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[\frac{-h\,A_s\,t}{\rho\,V\,C_p}\right]$$

$$\left[\text{From HMT data book, Page no.5}\right]$$

$$\Rightarrow \quad \frac{h\,A_s\,t}{\rho\,V\,C_p} = \frac{30 \times 4\pi R^2 \times t}{\rho \times \frac{4}{3}\pi R^3 \times C_p}$$

$$= \frac{30 \times 4\pi \times (0.025)^2 \times 60}{7800 \times \frac{4}{3}\pi (0.025)^3 \times 2 \times 10^3}$$

$$= 0.01385$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-0.01385\right]$$

$$\boxed{T = 1161\,\text{K or } 888^\circ\text{C}}$$

Rate of cooling $= T_0 - T = 12°C$/min

11.Derive the general 3- dimensional heat conduction equation cylindrical co-ordinates. Assume the material as homogeneous isotropic continues.

General Heat Conduction Equation in cylindrical Co-ordinates

The general heat conduction equation in Cartesian co-ordinates derived in the previous section is used for solid with rectangular boundaries like square, cubes, slabs, etc. But the Cartesian co-ordinates system is not applicable for the solid, like cylinders, cones, spheres etc. For cylindrical solids a cylindrical co-ordinate system is used.

Consider a small cylindrical element of sides dr, d∅ and dz shown in fig. 1.2

AMSCE - 1101

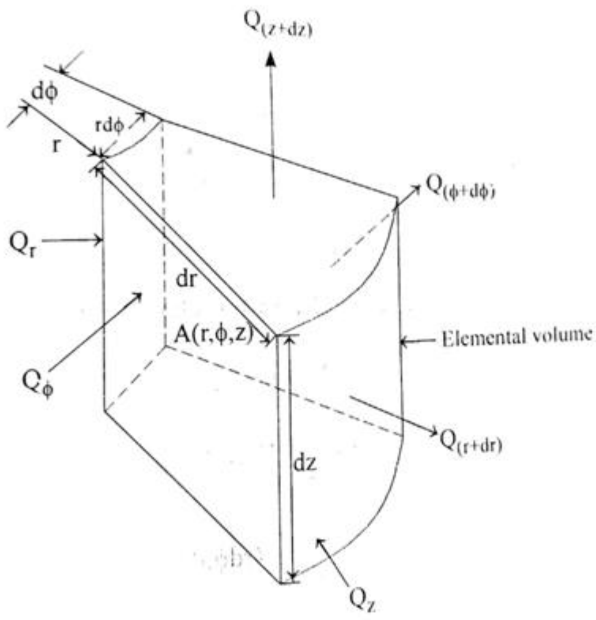


Fig. 1.2

The volume of element dv = r dØ dr dz

Let us assume that thermal conductivity k, Specific heat C_p and density ρ are constant.

The energy balance of this cylindrical element is obtained from first law of thermodynamics.

$$= \left\{\begin{array}{l}\text{Set heat}\\\text{conducted into}\\\text{element from all}\\\text{the co-ordinate}\\\text{directions}\end{array}\right\} + \left\{\begin{array}{l}\text{Heat}\\\text{generated within the}\\\text{element}\end{array}\right\} + \left\{\begin{array}{l}\text{Heat}\\\text{stored}\\\text{in the}\\\text{element}\end{array}\right\}$$

Net heat conducted into element from all the co-ordinate directions

Heat entering in the element through (r, Ø) plane in time dθ

$$Q_z = -k\left(r\,d\phi\,dr\right)\frac{\partial T}{\partial z}d\theta$$

Heat leaving from the element through (r, Ø) plane in time d

$$Q_{z-dz} = Q_z + \frac{\partial}{\partial z}\left(Q_z\right)dz$$

Net heat conducted into the element through (r, Ø) plane time dθ.

$$= Q_z - Q_{z+dz}$$

$$= -\frac{\partial}{\partial z}\left(Q_z\right)dz$$

$$= \frac{\partial}{\partial z}\left[k\left(rd\phi.dr\right).\left[\frac{\partial T}{\partial z}\right]d\theta\right]dz$$

$$= k\left[\frac{\partial^2 T}{\partial z^2}\right]\left(dr.rd\phi.dz\right)d\theta$$

$$\boxed{\text{Net heat conducted through}\left(r,\phi\right)\text{Plane} = k\left[\frac{\partial^2 T}{\partial z^2}\right]\left(dr.rd\phi.dz\right)d\theta} \qquad ....(1.20)$$

Heat entering in the element through (Ø, z) plane in time dθ.

$$Q_r = -k\left(r\,d\phi\,dz\right)\frac{\partial T}{\partial r}d\theta$$

Heat leaving from the element through (Ø, z) plane in time dθ.

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r}\left(Q_r\right)dr$$

Net heat conducted into teh element through (Ø, z) plane in time dθ.

$$= Q_r - Q_{r+dr}$$

$$= -\frac{\partial}{\partial r}\left(Q_r\right)dr$$

$$= -\frac{\partial}{\partial r}\left[-k\left(rd\phi, dz\right), \left[\frac{\partial T}{\partial r}\right]d\theta\right]^{dr}$$

$$= k\left(dr\,d\phi, dz\right), \frac{\partial}{\partial r}\left[r.\frac{\partial T}{\partial r}\right]d\theta \qquad =$$

$$= k\left(dr.rd\phi.dz\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]d\theta$$

$$\boxed{\text{Net heat conducted through}\left(\phi,z\right)\text{Plane} = k\left(dr.rd\phi.dz\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]d\theta} \quad ....(1.21)$$

Heat entering in the element through (z, r) plane in time dθ.

$$Q_\phi = -k\left(dr.dz\right)\frac{\partial T}{r\partial \phi}d\theta$$

Heat leaving from the element through (z, r) plane in time dθ.

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\partial \phi}\left(Q_\phi\right)rd\phi$$

Net heat conducted into the element through (z, r) plane in time dθ.

AMSCE - 1101

$$Q_\phi - Q_{\phi+d\phi} = \frac{\partial}{r\partial^2\phi}\left(Q_\phi\right) rd\phi$$

$$= \frac{\partial}{r\partial^2\phi}\left[-k\left(dr\,dz\right).\frac{\partial T}{r\partial\phi}d\theta\right] rd\phi$$

$$= K\frac{\partial}{\partial\phi}\left[\frac{1}{r}\frac{\partial T}{\partial\phi}\right]\left(dr\,d\phi\,dz\right)d\theta$$

$$= k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}\right]\left(dr\,rd\phi\,dz\right)d\theta$$

$$\boxed{\text{Net heat conducted through}\left(z,r\right)\text{plane} = k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}\right]\left(dr\,rd\phi\,dz\right)d\theta} \qquad ....(1.22)$$

Net heat conducted into element from all the co-ordinate directions

$$= k\frac{\partial^2 T}{\partial z^2}\left(dr\,rd\phi\,dz\right)d\theta$$

$$+$$

$$k\left(dr\,rd\phi\,dz\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]d\theta$$

$$k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}\right]\left(dr\,rd\phi\,dz\right)d\theta$$

[Adding equation 1.20, 1.21 and 1.22]

$$= k\left(dr\,rd\phi\,dz\right)d\theta\left[\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}\right]$$

$$= k\left(dr\,rd\phi\,dz\right)d\theta\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

$$\boxed{\begin{array}{l}\text{Net heat conducted int oelement from all the co} - \text{ordiante directions}\\ \qquad = k\left(dr\,rd\phi\,dz\right)d\theta\left[\dfrac{\partial^2 T}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial T}{\partial r} + \dfrac{1}{r^2} + \dfrac{\partial^2 T}{\partial\phi^2} + \dfrac{\partial^2 T}{\partial z^2}\right]\end{array}} \qquad ....(1.23)$$

Heat generated within the element

Total heat generated within the element is given by

$$Q = q\left(dr\,rd\phi\,dz\right)d\theta \qquad ....(1.24)$$

Heat stored in the element

The increase in internal energy element is equal to the net heat stored in the element.

Increase in internal energy,

=Net heat stored in the element

AMSCE - 1101

$$= \rho\left(dr\, rd\phi\, dz\right) C_p \frac{\partial T}{\partial \theta} \times d\theta \quad ....(1.25)$$

Substituting equation (1.23), (1.24) and (1.25) in (1.19)

$$(1.19) \Rightarrow k\left(dr\, rd\phi\, dz\right) d\theta \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + q\left(dr\, rd\phi\, dz\right) d\theta$$

$$= \rho\left(dr\, rd\phi\, dz\right) C_p \frac{\partial T}{\partial \theta} \times d\theta$$

Dividing by (dr rdϕ dz) dθ

$$\Rightarrow k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + q = \rho . C_p \frac{\partial T}{\partial \theta}$$

$$\Rightarrow \boxed{\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{q}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial \theta}} \quad ....(1.26)$$

It is a general three dimensional heat conduction equation in cylindrical co-ordinates,

$$\Rightarrow \quad \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \theta}$$

$$\left[\because \alpha = \frac{k}{\rho C_p}\right]$$

If the flow is steady, one dimensional and no heat generatin, equation (1.26) becomes:

$$\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \quad ....(1.27)$$

$$(OR)$$

$$\Rightarrow \boxed{\frac{1}{r}\frac{d}{dr}\left[r.\frac{dT}{dr}\right] = 0} \quad ....(1.28)$$

12. The wall of a cold room is composed of three layer. The outer layer is brick 30cm thick. The middle layer is cork 20cm thick, the inside layer is cement 15 cm thick. The temperatures of the outside air is 25°C and on the inside air is 20°C. The film co-efficient for outside air and brick is 55.4 W/m^{2}K. Film co-efficient for inside air and cement is 17 W/m^{2}K. Find heat flow rate.

Take:

K for brick = 2.5 W/mK

k for cor5k = 0.05 W/mK

k for cement = 0.28 W/mK

Given:

Thickness of brick, L_3 = 30 cm = 0.3 m

Thickness of cork, L_2 = 20 cm = 0.2 m

Thickness of cement, L_1 = 15cm = 0.15m

AMSCE - 1101

Inside air temperature, T_a = -20°C + 273 = 253 K

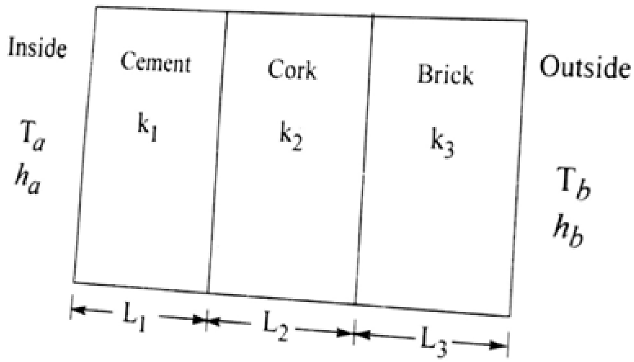
Outside air temperature, T_b = 25°C + 273. = 298 K

Film co-efficient for outside, h_b = 55.4 W/m^{2}K

$K_{brik} = k_3$ = 2.5 W/mK

$K_{cork} = k_2$ = 0.05 W/mK

$K_{cement} = k_1$ = 0.28 W/mK



To find:

Heat flow rate (Q/A)

Solution:

Heat flow through composite wall is given by

$$Q = \frac{\Delta T_{overall}}{R} \qquad \text{[From Equation no.1.42 or HMT Data book Page no.43 and 44]}$$

Where

$\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{[T_a - T_b]}{\frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{[T_a - T_b]}{\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}}$$

$$\Rightarrow Q/A = \frac{253 - 298}{\frac{1}{17} + \frac{0.15}{0.28} + \frac{0.2}{0.05} + \frac{0.3}{2.5} + \frac{1}{55.4}}$$

$$\boxed{Q/A = -9.5 W/m^2}$$

AMSCE - 1101

The negative sign indicates that the heat flows from the outside into the cold room.

Result:

Heat flow rate, $Q/A = -9.5 \text{ W/m}^2$.

13. A wall is constructed of several layers. The first layer consists of masonry brick 20 cm. thick of thermal conductivity 0.66 W/mK, the second layer consists of 3 cm thick mortar of thermal conductivity 0.6 W/mK, the third layer consists of 8 cm thick lime stone of thermal conductivity 0.58 W/mK and the outer layer consists of 1.2 cm thick plaster of thermal conductivity 0.6 W/mK. The heat transfer coefficient on the interior and exterior of the wall are 5.6 W/m²K and 11 W/m²K respectively. Interior room temperature is 22°C and outside air temperature is -5°C.

Calculate

a) **Overall heat transfer coefficient**
b) **Overall thermal resistance**
c) **The rate of heat transfer**
d) **The temperature at the junction between the mortar and the limestone.**

Given Data

Thickness of masonry L_1 = 20cm = 0.20 m

Thermal conductivity K_1 = 0.66 W/mK

Thickness of mortar L_2 = 3cm = 0.03 m

Thermal conductivity of mortar K_2 = 0.6 W/mK

Thickness of limestone L_3 = 8 cm = 0.08 m

Thermal conductivity K_3 = 0.58 W/mK

Thickness of Plaster L_4 = 1.2 cm = 0.012 m

Thermal conductivity K_4 = 0.6 W/mK

Interior heat transfer coefficient $h_a = 5.6 \text{ W/m}^2\text{K}$

Exterior heat transfer co-efficient $h_b = 11 \text{ W/m}^2\text{K}$

Interior room temperature T_a = 22°C + 273 = 295 K

Outside air temperature T_b = -5°C + 273 = 268 K.

Solution:

Heat flow through composite wall is given by

$Q = \dfrac{\Delta T_{overall}}{R}$ [From equation (13)] (or) [HMT Data book page No. 34]

AMSCE - 1101

Where, $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{295 - 268}{\frac{1}{5.6} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}}$$

$$\boxed{\text{Heat transfer per unit area } Q/A = 34.56 \text{ W/m}^2}$$

We know, Heat transfer $Q = UA (T_a - T_b)$ [From equation (14)]

Where U – overall heat transfer co-efficient

$$\Rightarrow U = \frac{Q}{A \times (T_a - T_b)}$$

$$\Rightarrow U = \frac{34.56}{295 - 268}$$

$$\boxed{\text{Overall heat transfer co-efficient } U = 1.28 \text{ W/m}^2 K}$$

We know

Overall Thermal resistance (R)

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

For unit Area

$$R = \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{L_4}{K_4} + \frac{1}{h_b}$$

$$= \frac{1}{56} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}$$

$$\boxed{R = 0.78 \; K/W}$$

Interface temperature between mortar and the limestone T_3

Interface temperatures relation

AMSCE - 1101

$$\Rightarrow Q=\frac{T_a-T_1}{R_a}=\frac{T_1-T_2}{R_1}=\frac{T_2-T_3}{R_2}=\frac{T_3-T_4}{R_3}=\frac{T_4-T_5}{R_4}=\frac{T_5-T_b}{R_b}$$

$$\Rightarrow Q=\frac{T_a-T_1}{R_a}$$

$$Q=\frac{295\text{-}T_1}{1/h_aA} \qquad \left[\because R_a=\frac{1}{h_aA}\right]$$

$$\Rightarrow Q/A=\frac{295-T_1}{1/h_a}$$

$$\Rightarrow 34.56=\frac{295-T_1}{1/5.6}$$

$$\Rightarrow \boxed{T_1=288.8\ K}$$

$$\Rightarrow Q=\frac{T_1-T_2}{R_1}$$

$$Q=\frac{288.8-T_2}{\frac{L_1}{K_1A}} \qquad \left[\because R_1=\frac{L_1}{k_1A}\right]$$

$$\Rightarrow Q/A=\frac{288.8-T_2}{\frac{L_1}{K_1}}$$

$$\Rightarrow 34.56=\frac{288.8-T_2}{\frac{0.20}{0.66}}$$

$$\Rightarrow \boxed{T_2=278.3\ \text{K}}$$

$$\Rightarrow Q=\frac{\text{T}_2-T_3}{R_2}$$

$$Q=\frac{278.3-T_3}{\frac{L_2}{K_2A}} \qquad \left[\because R_2=\frac{L_2}{K_2A}\right]$$

$$\Rightarrow Q/A=\frac{278.3-T_3}{\frac{L_2}{K_2}}$$

$$\Rightarrow 34.56=\frac{278.3-T_3}{\frac{0.03}{0.6}}$$

$$\Rightarrow \boxed{T_3=276.5\ \text{K}}$$

Temperature between Mortar and limestone (T_3 is 276.5 K)

14.A furnace wall made up of 7.5 cm of fire plate and 0.65 cm of mild steel plate. Inside surface exposed to hot gas at 650°C and outside air temperature 27°C. The convective heat transfer co-efficient for inner side is 60 W/m²K. The convective heat transfer co-efficient for outer side is 8W/m²K. Calculate the heat lost per square meter area of the furnace wall and also find outside surface temperature.

AMSCE - 1101

Given Data

Thickness of fire plate L_1 = 7.5 cm = 0.075 m

Thickness of mild steel L_2 = 0.65 cm = 0.0065 m

Inside hot gas temperature T_a = 650°C + 273 = 923 K

Outside air temperature T_b = 27°C + 273 = 300°K

Convective heat transfer co-efficient for

Inner side $h_a = 60 W/m^2K$

Convective heat transfer co-efficient for

Outer side $h_b = 8 W/m^2K$.

Solution:

(i) Heat lost per square meter area (Q/A)

Thermal conductivity for fire plate

$K_1 = 1035 \times 10^{-3}$ W/mK [From HMT data book page No.11]

Thermal conductivity for mild steel plate

K_2 = 53.6W/mK [From HMT data book page No.1]

Heat flow $Q = \dfrac{\Delta T_{overall}}{R}$, Where

$\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

[The term L_3 is not given so neglect that term]

AMSCE - 1101

$$\Rightarrow Q= \frac{T_a - T_b}{\frac{1}{h_a A}+\frac{L_1}{K_1 A}+\frac{L_2}{K_2 A}+\frac{L_3}{K_3 A}+\frac{1}{h_b A}}$$

The term L_3 is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A}+\frac{L_1}{K_1 A}+\frac{L_2}{K_2 A}+\frac{1}{h_b A}}$$

$$Q/A = \frac{923-300}{\frac{1}{60}+\frac{0.075}{1.035}+\frac{0.0065}{53.6}+\frac{1}{8}}$$

$$\boxed{Q/A = 2907.79\ W/m^2}$$

(ii) **Outside surface temperature T_3**

We know that, Interface temperatures relation

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_b} ......(A)$$

$$(A) \Rightarrow Q = \frac{T_3 - T_b}{R_b}$$

where

$$R_b = \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_3 - T_b}{\frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{T_3 - T_b}{\frac{1}{h_b}}$$

$$\Rightarrow 2907.79 = \frac{T_3 - 300}{\frac{1}{8}}$$

$$\boxed{T_3 = 663.473\ K}$$

15. A steel tube (K = 43.26 W/mK) of 5.08 cm inner diameter and 7.62 cm outer diameter is covered with 2.5 cm layer of insulation (K = 0.208 W/mK) the inside surface of the tube receivers heat from a hot gas at the temperature of 316°C with heat transfer co-efficient of 28 W/m^{2}K. While the outer surface exposed to the ambient air at 30°C with heat transfer co-efficient of 17 W/m^{2}K. Calculate heat loss for 3 m length of the tube.

Given

Steel tube thermal conductivity K_1 = 43.26 W/mK
Inner diameter of steel d_1 = 5.08 cm = 0.0508 m
Inner radius r_1 = 0.0254 m
Outer diameter of steel d_2 = 7.62 cm = 0.0762 m
Outer radius r_2 = 0.0381 m
Radius r_3 = r_2 + thickness of insulation

AMSCE - 1101

Radius r_3 = 0.0381 + 0.025 m $\qquad$ r_3 = 0.0631 m
Thermal conductivity of insulation K_2 = 0.208 W/mK
Hot gas temperature T_a = 316°C + 273 = 589 K
Ambient air temperature T_b = 30°C + 273 = 303 K
Heat transfer co-efficient at inner side h_a = 28 W/m^{2}K
Heat transfer co-efficient at outer side h_b = 17 W/m^{2}K
Length L = 3 m

Solution :

Heat flow $Q = \dfrac{\Delta T_{overall}}{R}$ [From equation No.(19) or HMT data book Page No.35]

Where $\quad \Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{K_3} In\left[\frac{r_4}{r_3}\right] + \frac{1}{h_b r_4}\right]$$

$$\Rightarrow Q = \frac{T_a - T_b}{\dfrac{1}{2\pi L}\left[\dfrac{1}{h_a r_1} + \dfrac{1}{K_1} In\left[\dfrac{r_2}{r_1}\right] + \dfrac{1}{K_2} In\left[\dfrac{r_3}{r_2}\right] + \dfrac{1}{K_3} In\left[\dfrac{r_4}{r_3}\right] + \dfrac{1}{h_b r_4}\right]}$$

[The terms K_3 and r_4 are not given, so neglect that terms]

$$\Rightarrow Q = \frac{T_a - T_b}{\dfrac{1}{2\pi L}\left[\dfrac{1}{h_a r_1} + \dfrac{1}{K_1} In\left[\dfrac{r_2}{r_1}\right] + \dfrac{1}{K_2} In\left[\dfrac{r_3}{r_2}\right] + \dfrac{1}{h_b r_3}\right]}$$

$$\Rightarrow Q = \frac{589 - 303}{\dfrac{1}{2\pi \times 3}\left[\dfrac{1}{28 \times 0.0254} + \dfrac{1}{43.26} In\left[\dfrac{0.0381}{0.0254}\right] + \dfrac{1}{0.208} In\left[\dfrac{0.0381}{0.0381}\right] + \dfrac{1}{17 \times 0.0631}\right]}$$

$$\boxed{Q = 1129.42 \text{ W}}$$

Heat loss Q = 1129.42 W.

16.**Derive an expression of Critical Radius of Insulation For A Cylinder.**

Consider a cylinder having thermal conductivity K. Let r_1 and r_0 inner and outer radii of insulation.

Heat transfer $Q = \dfrac{T_i - T_\infty}{\dfrac{In\left[\dfrac{r_0}{r_1}\right]}{2\pi KL}}$ [From equation No.(3)]

Considering h be the outside heat transfer co-efficient.

AMSCE - 1101

$$\therefore Q = \frac{T_i - T_\infty}{\frac{\ln\left[\frac{r_0}{r_1}\right]}{2\pi KL} + \frac{1}{A_0 h}}$$

Here $A_0 = 2\pi r_0 L$

$$\Rightarrow Q = \frac{T_i - T_\infty}{\frac{\ln\left[\frac{r_0}{r_1}\right]}{2\pi KL} + \frac{1}{2\pi r_0 Lh}}$$

To find the critical radius of insulation, differentiate Q with respect to r_0 and equate it to zero.

$$\Rightarrow \frac{dQ}{dr_0} = \frac{0-(T_i - T_\infty)\left[\frac{1}{2\pi KLr_0} - \frac{1}{2\pi hLr_0^{\ 2}}\right]}{\frac{1}{2\pi KL}\ln\left[\frac{r_0}{r_1}\right] + \frac{1}{2\pi hLr_0}}$$

since $(T_i - T_\infty) \neq 0$

$$\Rightarrow \frac{1}{2\pi KLr_0} - \frac{1}{2\pi hLr_0^{\ 2}} = 0$$

$$\Rightarrow \boxed{r_0 = \frac{K}{h} = r_c}$$

17. **A wire of 6 mm diameter with 2 mm thick insulation (K = 0.11 W/mK). If the convective heat transfer co-efficient between the insulating surface and air is 25 W/m²L, find the critical thickness of insulation. And also find the percentage of change in the heat transfer rate if the critical radius is used.**

Given Data

d_1= 6 mm
r_1 = 3 mm = 0.003 m
$r_2 = r_1 + 2 = 3 + 2 = 5$ mm = 0.005 m
K = 0.11 W/mK
h_b = 25 W/m^{2}K

Solution :

1. Critical radius $r_c = \frac{K}{h}$ [From equation No.(21)]

$$r_c = \frac{0.11}{25} = 4.4 \times 10^{-3} m$$

$$\boxed{r_c = 4.4 \times 10^{-3} m}$$

Critical thickness = $r_c - r_1$

$$= 4.4 \times 10^{-3} - 0.003$$
$$= 1.4 \times 10^{-3}\ m$$

$$\boxed{\text{Critical thickness } t_c = 1.4 \times 10^{-3} \text{ (or) } 1.4 \text{ mm}}$$

2. Heat transfer through an insulated wire is given by

$$Q_1 = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{\ln\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{1}{h_b r_2}\right]}$$

$$\left[\text{From HMT data book Page No.35}\right]$$

$$= \frac{2\pi L\ (T_a - T_b)}{\left[\frac{\ln\left[\frac{0.005}{0.003}\right]}{0.11} + \frac{1}{25 \times 0.005}\right]}$$

$$Q1 = \frac{2\pi L\ (T_a - T_b)}{12.64}$$

Heat flow through an insulated wire when critical radius is used is given by

$$Q_2 = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{\ln\left[\frac{r_c}{r_1}\right]}{K_1} + \frac{1}{h_b r_c}\right]} \qquad \left[r_2 \to r_c\right]$$

$$= \frac{2\pi L\ (T_a - T_b)}{\frac{\ln\left[\frac{4.4 \times 10^{-3}}{0.003}\right]}{0.11} + \frac{1}{25 \times 4.4 \times 10^{-3}}}$$

$$Q_2 = \frac{2\pi L\ (T_a - T_b)}{12.572}$$

$\therefore$ Percentage of increase in heat flow by using

Critical radius = $\frac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{\frac{1}{12.57} - \frac{1}{12.64} \times 100}{\frac{1}{12.64}}$$

$$= 0.55\%$$

AMSCE - 1101

18. **Calculate the critical radius of insulation for asbestos (k = 0.172 W/mK) surrounding a pipe and exposed to room air at 300 K with h = 2.8 W/m²K. Calculate the heat loss from a 475 K, 60 mm diameter pipe when covered with the critical radius of insulation and without insulation.**

Given:

Thermal conductivity of asbestos, k = 0.172 W/mK

Room temperature, T_2 = 300 K

Heat transfer coefficient, h = 2.8 W/m²k

Diameter of the pipe, d = 60 mm = 0.06 m

Temperature in the pipe, T_1 = 475 K

To find:

1. Critical radius of insulation of asbestos
2. Heat loss from the pipe.

Solution:

1. Critical radius of insulation $r_c = \frac{k}{h_o}$

$$r_c = \frac{0.172}{2.8} = 0.06142 \text{ m } 61.42 \text{ mm}$$

2. $$Q_{\text{with insulation}} = \frac{2\pi(T_1 - T_2)}{\frac{\ln\left(\frac{r_C}{r_1}\right)}{k} + \frac{1}{hr_C}}$$

$$Q_{\text{with insulation}} = \frac{2\pi(475-300)}{\frac{\ln\left(\frac{0.06142}{0.03}\right)}{0.172} + \frac{1}{2.8 \times 0.06142}}$$

$$Q_{\text{with insulation}} = \frac{1099}{4.162 + 5.814}$$

$$\boxed{Q_{\text{without insulation}} = 110.16 \text{ W/m}}$$

$$Q_{\text{without insulation}} = \frac{h \times 2\pi L(T_1 - T_2)}{1/h_a r} = \frac{2\pi L(T_1 - T_2)}{1/h_a r_1}$$

$$h_a\, 2\pi r_1 (T_1 - T_2) = 2.8 \times 2 \times \pi \times (.03\ (475-300))$$

$$\boxed{Q_{\text{without insulation}} = 92.31 \text{ W/m}}$$

Result:

1. Critical radius of insulation = 61.43 mm
2. Heat loss $Q_{\text{with insulation}}$ = 110.16 W/m

$Q_{\text{without insulation}}$ = 92.31 W/m.

19. **Aluminum fins of rectangular profile are attached on a plane wall with 5 mm spacing. The fins have thickness y = 1 mm and length = 10 mm and the thermal conductivity k = 200 W/mK. The wall in maintained at a temperature of 200°C and the fins dissipate heat by connection into the ambient air at 40°C with heat transfer coefficient h = 50 W/m²K. Determine the heat loss**

Given:

Fin dimension y = 1 mm; 0.001 m

l = 10 mm; 0.01 m

AMSCE - 1101

$$\text{Fin spacing} = 5 \text{ mm}$$

Thermal conductivity of fins, k = 200 W/mk

Wall temperature of fin, $T_b = 200 + 273 = 473$ K

To find:

Heat loss

Solution:

No heat loss from the tip of fin, (ie) tip is insulated. This is short fin end insulated type problem.

Heat transferred, [Short fin, end insulated]

$$Q = (hPkA)^{\frac{1}{2}} (T_b - T_\infty) \tan h(mL) \quad \text{[From HMT data book page no; 50]}$$

$$= (50 \times 1 \times 2 \times 200 \times 1 \times 0.001)^{\frac{1}{2}} (473 - 313) \tan h(m \times 0.01)$$

$$= 4.472 \times 160 \text{ Tan } h \text{ } (m \times 0.01)$$

where,

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{h \times (b + y) \times 2}{k(b \times y)}} \qquad \boxed{\text{Assuming } b >> y}$$

$$= \sqrt{\frac{h \times 2\cancel{b}}{k\cancel{b}y}} = \sqrt{\frac{2h}{ky}}$$

$$= \sqrt{\frac{2 \times 50}{200 \times 0.001}} = 22.36$$

$$\boxed{m = 22.36 \text{ m}^{-1}}$$

$$Q = 4.472 \times 160 \times \text{Tan } h \text{ } (22.36 \times 0.01)$$

$$= 715.52 \times \tan h \text{ } (0.2236)$$

$$\boxed{Q = 157.38 \text{ W/m}}$$

Result:

Heat loss, Q = 157.38 W/m.

AMSCE - 1101

20. *To reduce frosting it is desired to keep the outside surface of a glazed window at 4°C. The outside is at – 10°C and the convection coefficient is 60 W/m²K. In order to maintain the conditions a uniform heat flux is provided at the inner surface which is in contact with room air at 22°C with a convection coefficient of 12 W/m²K. The glass is 7 mm thick and has a thermal conductivity of 1.4 W/mK. Determine the heating required per m² area*

The data are shown is Fig. P. 2.9.

Solution: The heat flow through the barrier = heat convected on the outside

$= h\,(T - T_{\infty 1})$

$= 60\,(4 - (-10)) = 840\ \text{W/m}^2$

The heat flow through the barrier is the same

$$840 = \frac{\Delta T}{R} = \frac{T_1 - 4}{0.007/1.4} \quad \therefore \quad T_1 = 8.2°\text{C}$$

The heat flux + heat received by convection from room = heat flow through barrier

heat flux = heat flow through glass barrier – heat convected from inside

$= 840 - 12\,(22 - 8.2) = 840 - 165.6 = 674.4$ W

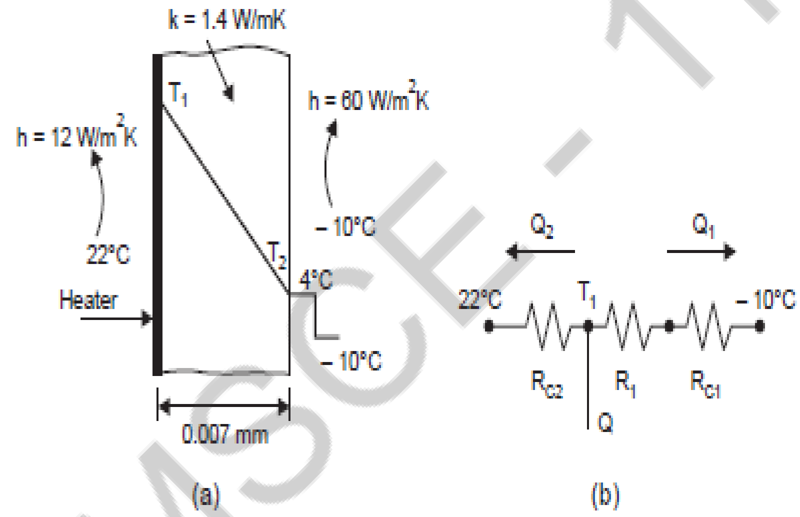


Fig. P. 2.9. Problem model.

If it is desired that the inside well temperature and room temperatures should be equal for comfort, determine the heat flux. In this case $T_1 = 22°$C and T_2 is not known

But heat conducted = heat convected

$$\frac{22 - T_2}{0.007/1.4} = 60\,(T_2 - (-10))$$

solving $\quad 22 - T = 0.3\,T + 3, T_2 = 14.62°\text{C}$

$$Q = \frac{22 - 14.62}{0.007/1.4} = 1477\ \text{W}$$

Check $\quad Q = h(T - T_\infty) = 60 \times 24.62 = 1477.2$ W

AMSCE - 1101

21. *A long cylinder of radius 15 cm initially at 30°C is exposed over the surface to gases at 600°C with a convective heat transfer coefficient of 65 W/m²K. Using the following property values determine the temperatures at the centre, mid radius and outside surface after 20 minutes. Density = 3550 kg/m³, sp. heat = 586J/kg K, conductivity = 19.5 W/mK. Also calculate the heat flow.*

Solution: The procedure is described in articles 6.1.2 and 6.1.3.

$$Bi = \frac{hR}{k} = \frac{65 \times 0.15}{19.5} = 0.5,\ \text{Fo} = \frac{\alpha\tau}{R^2} = \frac{19.5}{3550 \times 586} \times \frac{20 \times 60}{0.15 \times 0.15} = 0.5$$

Entering the chart for centre temperature as schematically shown in Fig. 6.14 (*a*) the temperature ratio is read as 0.72.

∴ Centre temperature is found using

$$\frac{T_{o,\tau} - 600}{30 - 600} = 0.72$$

∴ $$T_{o,\tau} = 189.6°\text{C}$$

To calculate the temperatures at the surface and mid radius, the location chart as schematically shown in Fig. 6.14 (*b*) is entered at $Bi = 0.5$ and values are read at $r/R = 1$ and 0.5, as 0.78 and 0.92.

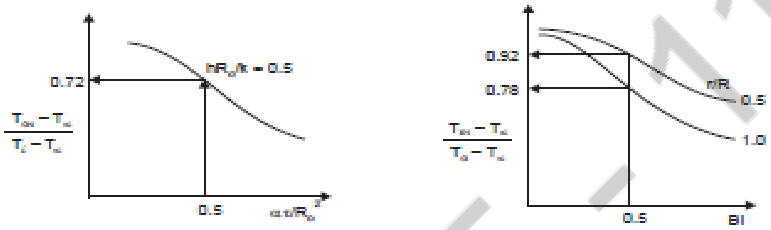


Fig. 6.14 (a, b)

∴ **Surface temperature** is obtained using

$$\frac{T_{R,\tau} - T_\infty}{T_i - T_\infty} = \frac{T_{R,\tau} - 600}{30 - 600} = 0.72 \times 0.78$$

∴ $$\mathbf{T_{R,\tau} = 279.9°C}$$

Mid radius temperature is obtained using

$$\frac{T_{r,\tau} - T_\infty}{T_i - T_\infty} = 0.72 \times 0.92$$

∴ $$\mathbf{T_{r,\tau} = 222.4°C}$$

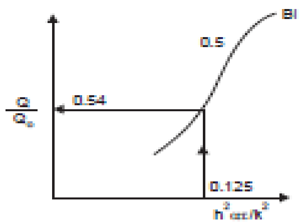


Fig. 6.14 (c)

Heat flow: Calculating the value of $h^2\alpha\tau/k^2$, as schematically shown the heat flow chart is entered at this value 0.125 and the meeting point with $Bi = 0.5$ is read of as Q/Q_o is equal to 0.54 (Fig. 6.14 (*c*))

$$h^2\,\alpha\,\tau/k^2 = \frac{65 \times 65 \times 19.5}{3350 \times 586} \times \frac{30 \times 60}{19.5 \times 19.5} = 0.125$$

∴ $$\mathbf{Q} = \pi \times 0.15^2 \times 1 \times 3550 \times 586 \times (600 - 30) \times 0.54$$
$$= \mathbf{45.26 \times 10^6\ J/m\ length}$$

Note that the cylinder of equal dimension gets heated up quicker due to larger surface area for a given volume.

22. **A body of an electric motor in 360 mm in diameter and 240 mm long. It dissipates 360 W of heat and its surface temperature should not exceed 55°C. Longitudinal fins of 15 mm thickness and 40 mm height are prepared. The heat transfer coefficient in 40 W/m^2K when the ambient temperature 30°C. Determine the number of fins required, if k of the fin material in 40 W/mK.**

Given data:

Diameter, $D = 360 \times 10^{-3}$ m

Length, $L = 240 \times 10^{-3}$ m

Base temperature, $T_b = 55°C$

Q, heat dissipated = 360 W

Longitudinal fin

$t_{fin} = 15 \times 10^{-3}$ m

h_f or length of fin $L_f = 40 \times 10^{-3}$ m

AMSCE - 1101

Heat transfer coefficient, $h = 40\ \text{W/m}^2\text{K}$

Thermal conductivity, $k = 40\ \text{W/mK}$

Ambient temperature, $T_\infty = 30^\circ\text{C}$

To find:

No. of fins required (N)

Solution:

Here length (or) height of fin is given. It is short fin (end not insulated)

[From HMT data book, Page No. 150]

$$Q = \sqrt{hpkA}\,(T_b - T_\infty)\,\frac{\tanh(mL) + \frac{h}{km}}{1 + \frac{h}{m}\tanh(mL)}$$

$$m = \sqrt{\frac{hP}{kA}}$$

$$= \sqrt{\frac{40 \times 0.51}{40 \times 3.6 \times 10^{-3}}} = 11.9\ \text{m}^{-1}$$

$$\boxed{m = 11.9\ \text{m}^{-1}}$$

$$Q = \sqrt{hpkA}\,(T_b - T_\infty)\left[\frac{\tanh(mL) + \frac{h}{km}}{1 + \frac{h}{m}\tanh(mL)}\right]$$

$$= 1.713 \times 25 \times \left[\frac{0.443 + 0.089}{1 + 0.084 \times 0.443}\right]$$

$$= 1.713 \times 25 \times \frac{0.532}{1.0372} = 21.96\ \text{W}$$

$$\text{Number of fins} = \frac{Q}{Q_{fin}} = \frac{360}{21.96} = 16.39$$

$$\boxed{= 16\ \text{fins}}$$

Result

Number of fins = 16 fins

AMSCE - 1101

Unit –2

CONVECTION

Part – A

1. Define convection

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperature.

2. Define Reynolds number (Re).

Reynolds number is defined as the ratio of inertia force viscous force

$$R_e = \frac{\text{Inertia force}}{\text{Viscouse force}}$$

3. Define Nusselt number (Nu)

It is defined as the ratio of the heat flow buy convection process under an unit temperature gradient to the heat flow rate by conduction under an unit temperature gradient through a stationary thickness(L) of meter.

$$\text{Nusselt number (Nu)} = \frac{Q_{conv}}{Q_{cond}}$$

4. Define Grashof number (Gr)

It is defined as the ratio of production of inertia force and Buoyancy force to the square of viscous force

$$G_r = \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{Viscous force})^2}$$

5. What is meant by non-Newtonian fluids?

The fluid which obey the Newton’s Law of viscosity are called Newtonian and those which do not obey are called non-Newtonian thinks.

6. What is meant by Stanton number(St)

Stanton number is the ratio of Nusselt number to the product of Reynolds number and parandtl number

$$St = \frac{Nu}{Re \times Pr}$$

7. What is meant by free or natural convection?

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection

8. Define boundary layer thickness

The thickness of boundary layer has been defined as the distance from the surface at which the velocity or temperature reaches 99% of the external velocity or temperature.

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9. What is the from of equation used to calculate heat transfer for flow through cylindrical pipes?

$$Nu = 0.023(Re)^{0.8}(Pr)^{n}$$
$$n = 0.4 \text{ for heating of fluids}$$
$$n = 0.3 \text{ for cooling of fluids}$$

10. What is dimensional analysis?

Dimensional analysis is mathematical method which makes us the study of dimension for solving several engineering problems. This method can be applied to all types of fluid resistance, heat flow problems in fluid mechanism and thermodynamics.

11. What are all advantage of dimensional analysis?

1. It expressed the functional relationship between the variable in dimensional terms
2. It enables getting up a theoretical solution in a simplified dimensionless form
3. The result of one series of tests can be applied to a large number of other similar problems with the help of dimensional analysis.

12. What is hydrodynamic boundary layer?

In hydrodynamic boundary layer, velocity of the fluid less than 99% of free steam velocity.

13. What is thermal boundary layer?

In thermal boundary layer, temperature of the fluid is less than 99% of the free stream temperature.

14. What are the dimensionless parameters used in forced convection?

1. Reynolds number (Re)

2. Nusselt number (Nu)

3. Prandtl number (Pr)

15.Indicate the concept or significance of boundary layer.

1. A thin region teh body called the boundary layer where the velocity and the temperature gradients are large.

2. The region outside the boundary layer where the velocity and the temperature gradients are very nearly equal to their free stream values.

16. Define displacement thickness.

The displacement thickness is the distance, measured perpendicular to the boundary, by which the free stream is displaced an account of formation of the boundary layer.

17. Define momentum thickness.

The momentum thickness is defined as the distance through which the total loss of momentum per second be equal to if it were passing a stationary plate.

18. Define energy thickness.

AMSCE - 1101

It is defined as the distance, measured perpendicular to the boundary of the solid, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the following fluid on account of boundary layer.

19. Define Prandtl number (P_r).

Prandtl number is the ratio of the momentum diffusivity of the thermal diffusivity.

$$P_r = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

20. Define Stanton number (S_t).

Stanton number is the ratio of nusselt number to the product of Reynolds number and prandtl number.

$$S_t = \frac{Nu}{Re \times Pr}$$

21. What is meant by forced convection.

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of heat transfer is known as forced convection.

21 What is meant by displacement thickness?

It is the distance measured perpendicular to the boundary, by which the main free stream is displaced on account of formation of boundary layer. The displacement thickness is denoted by δ^*.

22 State the characteristics of a boundary layer.

The characteristics of a boundary layer.

1. The thickness of boundary layer increases as distance from leading edge increases.
2. Thickness of boundary layer decreases as U increases.
3. Thickness of boundary layer increase as kinematic viscosity increases.

AMSCE - 1101

1. A long 10 cm diameter steam pipe whose external surface temperature is 110℃ passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit length when the air is 1 atm and 10℃ and the wind is blowing across the pipe at a velocity of 8 m/s.(8)

Given data:

Diameter of steam pipe, d = 10 cm = 0.1 m

Surface temperature, T_w = 110℃

(Air) fluid temperature, T_∞ = 10℃

Velocity, u = 8 m/s

To find: rate of heat loss from the pipe per unit length

$$Q = h\,A\left(T_s - T_\infty\right)$$

Solution:

Film temperature,

$$T_f = \frac{T_w + T_\infty}{2} = \frac{110+10}{2} = \frac{120}{2} = 60^\circ C$$

From HMT data book, page No.34 (Seventh edition) Properties of air at 60°C.

Density, $\rho = 1.060\ kg/m^3$

Kinematic viscosity, $v = 18.97 \times 10^{-6}\ m^2/s$

Prandtl number, $Pr = 0.696$

Thermal conductivity, $K = 0.02896\ W/mK$

Reynolds number,

$$Re = \frac{u \times D}{v} = \frac{8 \times 0.1}{18.96 \times 10^{-6}} = 0.421 \times 10^5$$
$$= 42194 < 5 \times 10^5$$

Nusselt number, $Nu = C\,(Re)^m\,(Pr)^{0.333}$

From HMT data book, page No. 116 (Seventh edition)

Re value is 42,194, corresponding C and values are 0.0266 and 0.805respectively.

$$Nu = 0.0266\,(42194)^{0.805}\,(0.696)^{0.333}$$

$$Nu = 124.67$$

We know,

$$Nu = \frac{hD}{K}$$
$$h = \frac{Nu.K}{D} = \frac{124.67 \times 0.02896}{0.1}$$

Heat transfer co-efficient, $h = 35.91\ W/m^2K$

$(\therefore A = \pi DL)$

Heat transfer, $Q = h A (T_s - T_\infty)$

$$= 35.91 \times (\pi \times 0.1 \times 1)\,(110 - 10)$$

$$Q = 1128.14\ W$$

Result:

Rate of heat loss from the pipe per unit length

$$Q = 1128.14\ W$$

2.Air at 0°C flow over a flat plate at a speed of 90 m/s and heated to 100°C. The plate is 60 cm long and 75cm wide. Assuming the transition of boundary layer take place at Re = 5×10^5. Calculate the following:

AMSCE - 1101

1. Average friction transfer co efficient.

2. Average heat transfer co-efficient

3. Rate of energy dissipation.

Given: Fluid temperature, $T_\infty = 0°C$

Speed, U = 90m/s

Surface temperature, $T_w = 100°C$

Length, L = 60 cm = 0.60 m

Wide , W = 75 cm = 0.75 m

To Find:

1. Average friction co-efficient
2. Average heat transfer co-efficient
3. Rate of energy dissipation.

Solution:

Film temperature,
$$T_f = \frac{T_w + T_\infty}{2} = \frac{100+0}{2}$$
$$\boxed{T_f = 50^\circ C}$$

Properties of air at 50°C:

[From HMT data book, page no. 33 (sixth edition)

$P = 1.093\ kg/m^3$

$V = 17.95 \times 10^{-6} m^2/s$

$Pr = 0.698$

$K = 0.02826\ W/mK$

We know,

Reynolds Number,
$$Re = \frac{UL}{v} = \frac{90 \times 0.60}{17.95 \times 10^{-6}}$$
$$\boxed{Re = 3.0 \times 10^6 > 5 \times 10^5}$$

Since $Re > 5 \times 10^5$, flow is turbulent.

[Note: Transition occurs means flow is combination of laminar and turbulent flow. i.e. the flow is said to be laminar upto Re value is 5×10^5. After that flow is turbulent]

AMSCE - 1101

For flat plate, Laminar- turbulent flow

[From HMT data book, page No. 114 (sixth edition)]

$$\left.\begin{array}{l}\text{Average friction}\\ \text{Co efficient}\end{array}\right\} C_{fL} = 0.074(Re)^{-0.2} - 1742(Re)^{-1.0}$$

$$\Rightarrow \quad C_{fL} = 0.074\left[3.0\times10^{6}\right]^{-0.2} - 1742\left[3.0\times10^{6}\right]^{-1.0}$$

$$C_{fL} = 3.16\times10^{-3}$$

$$\boxed{\left.\begin{array}{r}\text{Average friction}\\ \text{co}-\text{efficient}\end{array}\right\} C_{fL} = 3.16\times10^{-3}}$$

$$\left.\begin{array}{r}\text{Average Nusselt}\\ \text{number}\end{array}\right\} Nu = (Pr)^{0.333}\left[0.037(Re)^{0.8} - 871\right]$$

[From HMT data book, page no.114 (sixth edition)]

$$= (Pr)^{0.333}\left[0.037\left(3\times10^{6}\right)^{0.8} - 871\right]$$

$$= (0.698)^{0.333}\left[0.037\left(3\times10^{6}\right)^{0.8} - 87\right]$$

$$\boxed{Nu = 4215}$$

We know,

$$Nu = \frac{hL}{k}$$

Average Nusselt number, $h = 198.5\,W/m^2K$

$$\boxed{\left.\begin{array}{r}\text{Average heat transfer}\\ \text{co}-\text{efficient}\end{array}\right\} h = 198.5\,W/m^2K}$$

Rate of energy dissipation,

$$Q = hA(T_w - T_\infty)$$

$$= h\times L\times W(T_w - T_\infty)$$

$$= 198.5\times0.60\times0.75(100-0)$$

$$\boxed{Q = 8932.5\,K}$$

Result:

1. $C_{fL} = 3.16 \times 10^{-3}$

2. h = 198.5 W/m^2K

3.Q = 8932.5 W

3.A 6 m long section of an 8 cm diameter horizontal hot water pipe passes through a large room whose temperature is 20℃. If the outer surface temperature and emissivity of the pipe are 70℃ and 0.8 respectively, pipe by

(1) Natural Convection

(2) Radiation. **(10)**

Given data:

AMSCE - 1101

In horizontal hot water pipe (cylinder and internal flow)

Length of hot water pipe = 6m

Diameter of hot water pipe = 8 cm = 0.08 m

Emissivity of the pipe, $\varepsilon = 0.8$

Outer surface temperature, $T_w = 70°C$

Hot (fluid) water temperature, $T_\infty = 20°C$

To find:

Rate of heat loss from the pipe,

(1) Natural convection, Q_{conv}

(2) Radiation (Q_{rad})

Solution:

Film temperature, $T_f = \dfrac{T_w + T_\infty}{2} = \dfrac{70+20}{2} = 45^\circ C$

$$10^{-5} < G_{rD}.\ Pr < 10^{12}$$

$$= \left\{ 0.60 + 0.387 \left[\frac{1.868 \times 10^6}{\left\{ 1 + \left(\frac{0.559}{0.7241} \right)^{0.5625} \right\}^{0.296}} \right]^{0.167} \right\}^2$$

$$Nu_D = 22.89$$

We know,

$$Nu = \frac{h\,D}{K}$$

$$h = \frac{K.Nu}{D} = \frac{0.02699 \times 22.89}{0.08} = 7.7225\,W/m^2K$$

Rate of heat loss from the pipe

$$Q_{conv} = h\,A_s\left(T_s - T_\infty\right)$$
$$= 7.7225 \times 1.508 \times (70 - 20)$$
$$Q_{conv} = 582.46\,W$$

Area of the surface,

$$A_s = \pi\,DL$$

$$= \pi \times 0.08 \times 6$$

$$A_s = 1.508\ m^2$$

AMSCE - 1101

$Q_{rad} = \varepsilon\, A_s\, \sigma\, (T_s^4 - T^4_{\infty})$

$= 0.8 \times 1.508 \times 5.67 \times 10^{-8} \times (343^4 - 293^4)$

$Q_{rad} = 442.65$ W

Result:

Rate of heat loss from the pipe by,

(i) Natural convection, Q_{conv}= 582.46 W

(ii) Radiation, Q_{rad}= 442.65 W

4. (i) Explain the concept of hydrodynamic and thermal boundary layers.

(1) Velocity distribution in hydrodynamic boundary layer.

(2) Temperature distribution in thermal boundary layer.

(3) Variation of local heat transfer co-efficient along the flow. (8)

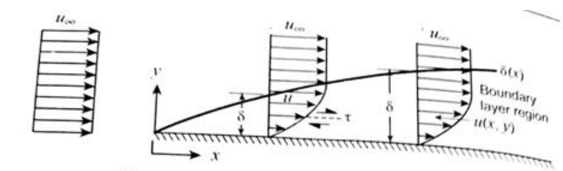


Fig. 1. Velocity boundary layer on a flat plate

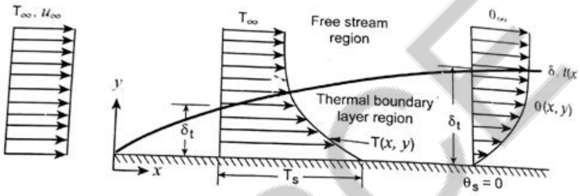


Fig. 2. Thermal boundary layer on a flat plate

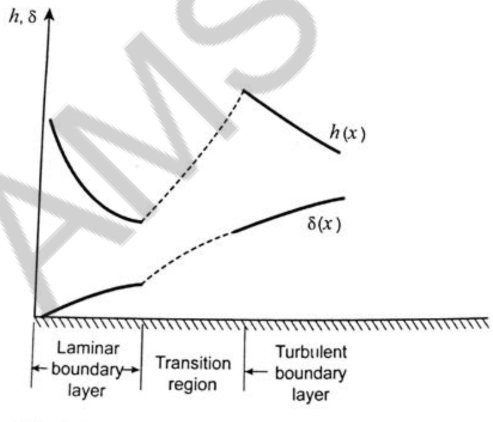


Fig. 3. Boundary layer thickness and heat transfer coefficient for a flat plate

5. Discuss briefly the development of velocity boundary layer for flow through a pipe.

The velocity at any cross section of a pipe varies from zero at wall to a maximum at the centre, and that there is no well defined free stream. There is a need to define and work in terms of a mean velocity, u_m. It is

AMSCE - 1101

defined as that velocity which is multiplied by the fluid density and the cross-sectional area of the tube gives the rate of mass flow through the tube.

Thus,

$$m = \rho u_m \frac{\pi}{4}.D^2 \backslash$$

The velocity distribution for fully developed, stead laminar flow can be determined by considering the force equilibrium of a cylindrical fluid element in fig. 1.

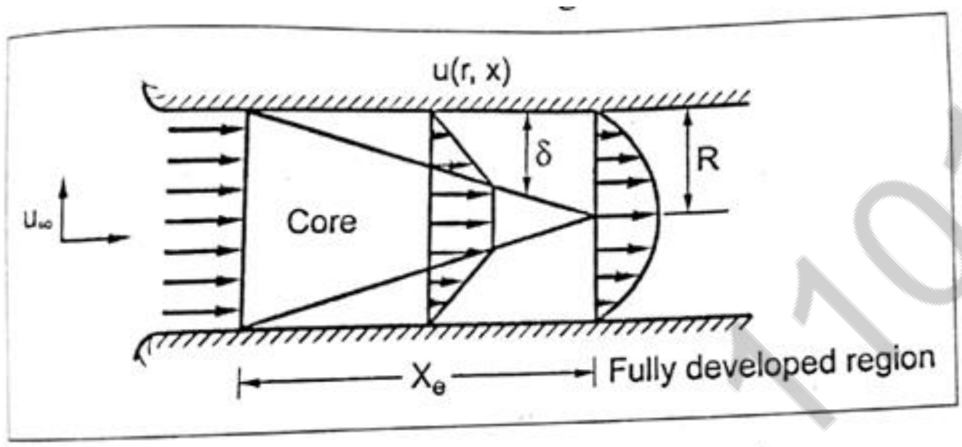


Fig. 1. Flow regions in a circular tube

The various forces are,

(i) Shear on the cylindrical surface.

(ii) Normal force due to pressure on the ends.

Since for a fully developed flow,

$$V_r = 0 \text{ and} \left(\frac{\partial u}{\partial x}\right) = 0$$

The axial velocity u,is only a function of r.

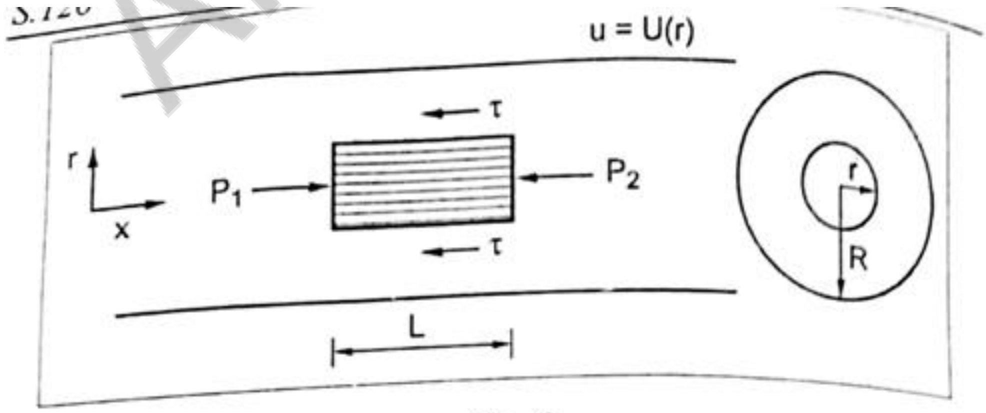


Fig. 2.

Force balance on a differential element in laminar development flow and the net momentum flow is zero everywhere.

6. Water at 60℃ and a velocity of 2 cm/s flows over a 5 m long flat plate which is maintained at a temperature of 20℃. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

Given Data: T_∞ = 60℃ (fluid temperature)

Velocity, u = 2 cm/s = 0.02 cm/s

X= 5m, T_s = 20℃, L = 5m

To find:

(i) Drag force, F_D

(ii) Rate of heat transfer per unit width of the entire plate.

@ Solution: We know that,

Film temperature, $T_f = \frac{T_w + T_\infty}{2} = \frac{60^\circ + 20^\circ}{2} = 40^\circ C$

Properties of water @ 40℃

[From HMT data book, page no. 33, 6th edition]

P = 995 kg/m^3

V = 0.657 × 10^{-6}m^2/s

Pr = 4.34

K = 0.628 W/mK

Reynolds's number, R_{e1}= $\frac{uL}{v} = \frac{0.02 \times 5}{0.657 \times 10^6}$

$= \frac{0.1 \times 10^6}{0.657}$

$\boxed{Re_L = 1.522 \times 10^5} < 5 \times 10^5$

Since, Re < 5 × 10^5, ∴ The flow is laminar.

$\bar{C}_{fL} = 1.328 \times Re_L^{-0.5}$

Average friction co-efficient, $= 1.328 \times \left(1.522 \times 10^5\right)^{-0.5}$

$= \frac{1.328}{390.128} = 3.4 \times 10^{-3}$

Drag force, F_D = Area × Average shear stress

$$= 1 \times 5 \times \bar{C}_{fL} \frac{\rho u^2}{2}$$

$$= 5 \times \frac{995(0.02)^2}{2} \times 3.4 \times 10^{-3}$$

$$= 3.4 \times 10^{-3} \times \frac{1.99}{2}$$

$$= 0.995 \times 3.43 \times 10^{-3}$$

$$\boxed{F_D = 3.41 \times 10^{-3}\,N}$$

Local heat transfer co-efficient, h_x:

Local Nusselt number, $Nu_x = 0.332\ (Re)^{0.5}\ \ (Pr)^{0.333}$

$$= 0.332\left(1.522 \times 10^5\right)^{0.5}\left(4.34\right)^{0.333}$$

$$= 0.332 \times 390.128 \times 1.6303$$

$$Nu_x = 211.168$$

$$Nu_x = \frac{h_x L}{k} \Rightarrow \frac{211.16 \times 0.628}{5} = h_x$$

$$h_x = 26.52\,W/m^2K$$

$$h = 2 \times h_x = 2 \times 26.52 = 53.04\,W/m^2K$$

(ii) Rate of heat transfer

$$Q = h\,A\left(T_s - T_\infty\right)$$

$$= 53.04 \times 5 \times (333 - 293)$$

$$\boxed{Q = 10608\,W}$$

7. Considering a heated vertical plate in a quiescent fluid, draw the velocity and temperature profiles.

AMSCE - 1101

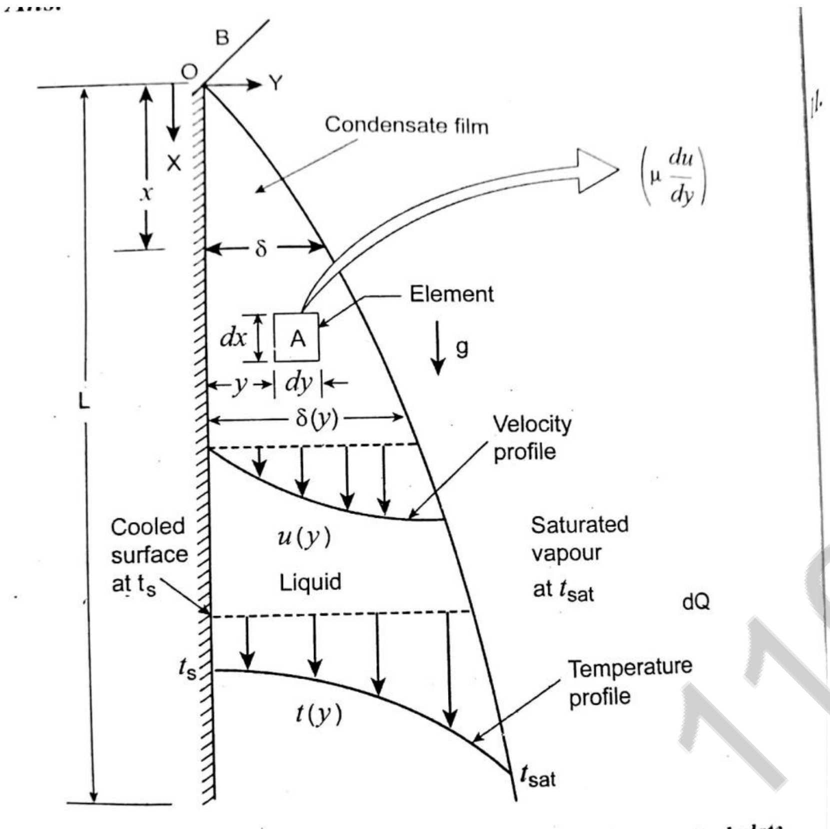


Fig. Film growth, velocity and temperature profiles of vertical plate

AMSCE - 1101

8. A horizontal pipe of 6m length and 8 cm diameter passes through a large room in which the air and walls are at 18℃. The pipe outer surface is at 70℃. Find the rate of heat loss from the pipe by natural convection.

Given Data:

Length of the pipe, L = 6m

Diameter of the pipe, d = 8 cm = 0.08 m

Surface temperature, T_s = 70℃ + 273 = 343 K

Fluid temperature, T_∞ = 18℃ + 273 = 291 K

To find: Q/L

@ Solution:

Film temperature,
$$T_f = \frac{T_s + T_\infty}{2}$$
$$= \frac{70+18}{2} = 44^\circ C \,(\text{or})\, 317K$$

The properties of air @ 44℃ ≈ 45℃

$P = 1.11 \text{ kg/m}^3$

$V = 17.45 \times 10^{-6} \text{ m}^2/\text{s}$

$\propto = 25.014 \times 10^{-6}\ m^2/s$

K = 0.02791 W/mK

Pr = 0.6985

$$\beta = \frac{1}{T_f} = \frac{1}{317} = 0.00315\,K^{-1}$$

$$Gr = \frac{g\beta\Delta T d^3}{v^2}$$

$$= \frac{9.81 \times 0.00315 \times (343-291)(0.08)^3}{\left(17.455\times10^{-6}\right)^2}$$

$$= \frac{8.229\times10^{-4}\times10^{12}}{304.677} = 2.7\times10^6$$

$$Gr.Pr = 2.7\times10^6\times0.6985$$

$$\boxed{Gr_D\,Pr = 1.88\times10^6}$$

For horizontal cylindrical,

$$Nu_D = \left\{0.6 + 0.387\left[\frac{Gr_D.Pr}{\left\{1+\left(\frac{0.559}{Pr}\right)^{0.5625}\right\}^{0.296}}\right]^{0.167}\right\}^2$$

$$10^{-5} < Gr_D\,Pr < 10^{12}$$

$$= \left\{0.6 + 0.387\left[\frac{1.88\times10^6}{\left\{1+\left(\frac{0.559}{0.6985}\right)^{0.5625}\right\}^{0.296}}\right]^{0.167}\right\}^2$$

$$= \left\{0.6 + 0.387\left[\frac{1.88\times10^6}{1.2058}\right]^{0.167}\right\}^2$$

$$= \left\{0.6 + 0.387(1559130.86)^{0.167}\right\}^2$$

$$= \left\{0.6 + 0.387(10.819)\right\}^2$$

$$= \left\{0.6 + 4.186953\right\}^2$$

$$\boxed{Nu_D = 22.916} \quad \Rightarrow Nu_D = \frac{hd}{k}$$

$$h = \frac{22.916\times0.02791}{0.08} = 7.99$$

$$Q = hA\left(T_s - T_\infty\right)$$

$$= 7.99\times\pi\times0.08\times6\times(343-291)$$

$$\boxed{Q = 626.529\,W}$$

AMSCE - 1101

9.Castor oil at 30℃ flows over a flat plate at a velocity of 1.5 m/s. The length of the plate is 4m. The plate is heated uniformly and maintained at 90℃. Calculate the following.

1. Hydrodynamic boundary layer thickness,

2. Thermal boundary layer thickness,

3. Total track force per unit width on one side of the plate,

4. Heat transfer rate.

At the mean film temperature $T_f = \frac{90+30}{2} = 60^\circ C.$

Properties are taken as follows:

P = 956.8 kg/m^3; v = 0.65 × 10^4 m^2/s;

K = 0.213 W/mK; ∝ = 7.2 × 10^{-8} m^2/s.

Given: Fluid temperature, T_∞= 30℃

Velocity, U = 1.5 m/s

Length, L = 4m

Plate surface temperature, T_w = 90℃

At T_f = 60℃, ρ = 956.8 kg/m^2

k = 0.213 W/mK

v = 0.65 × 10^{-4} m^2/s

∝ = 7.2× 10^{-8} m^2/s

To find:

1. Hydrodynamic boundary layer thickness,
2. Thermal boundary layer thickness,
3. Total drag force per unit width on one side of the place
4. Heat transfer rate.

Solution: We know that,

$$Re = \frac{UL}{v}$$

Reynold's Number, $= \frac{1.5 \times 4}{0.65 \times 10^{-4}}$

$$\boxed{Re = 9.23 \times 10^4 < 5 \times 10^5}$$

Since Re < 5 × 10^5, flow is laminar.

For flat plate, laminar flow:

AMSCE - 1101

[Refer HMT data book, page No. 112 (Sixth edition)

1. Hydrodynamic boundary layer thickness:

$$\delta_{hx} = 5 \times x \times (Re)h - 0.5$$
$$= 5 \times 4 \times \left(9.23 \times 10^4\right)^{-0.5}$$
$$[\because x = L = 4m]$$
$$\boxed{\delta_{hx} = 0.065\,m}$$

2. Thermal boundary layer thickness:

$$\delta_{Tx} = \delta_{hx} \times (Pr)^{-0.333}$$
$$= 0.065 \times (902.77)^{-0.333}$$
$$\left[Pr = \frac{v}{\alpha} = \frac{0.65 \times 10^{-4}}{7.2 \times 10^{-8}} = 902.77\right]$$
$$\boxed{\delta_{Tx} = 6.74 \times 10^{-3}\,m}$$

3. Total drag force on one side of the plate:

Average skin friction co-efficient,

$$\bar{C}_{fL} = 1.328(Re)^{-0.5}$$
$$\bar{C}_{fL} = 1.328 \times \left(9.23 \times 10^4\right)^{-0.5}$$
$$\boxed{\bar{C}_{fL} = 4.37 \times 10^{-3}}$$

$$\bar{C}_{fL} = \frac{\tau}{\frac{\rho U^2}{2}}$$

We know $\Rightarrow$ $4.37 \times 10^{-3} = \dfrac{\tau}{\dfrac{956.8 \times (1.5)^2}{2}}$

$\Rightarrow$ $\tau = 4.70\,N/m^2$

Average shear stress $\tau = 4.70 N/m^2$

Drag force, F_D = Area × Average shear stress

$= (L \times W) \times 4.70$

$= (4 \times 1) \times 4.70$ $[\because W = 1m]$

$$\boxed{\text{Drag force, } F_D = 18.8N}$$

4. Heat transfer rate:

We know that,

Local Nusselt Number

$$Nu_x = 0.332 \times (Re)^{0.5} (Pr)^{0.333}$$

AMSCE - 1101

$$=0.332 \times (9.23 \times 10^4)^{0.5} \times (902.77)^{0.333}$$

$$\boxed{Nu_x = 972.6}$$

We know,

$$Nu_x = \frac{h_x L}{k}$$

Local Nusselt Number $972.6 = \frac{h_x \times 4}{0.213}$

$$\Rightarrow \quad h_x = 51.7 W/m^2K$$

$$\boxed{\text{local heat transfer co-efficient}, h_x = 51.7\, W/m^2K}$$

Average heat transfer co-efficient

$$h = 2 \times h_x$$

$$h = 2 \times 51.7$$

$$\boxed{h = 103.58\, W/m^2K}$$

Heat transfer, Q=h A (T_w - T_∞)

$$= h \times L \times W\ (T_w - T_\infty)$$

$$= 103.58 \times 4 \times 1\ (90-30)$$

$$\boxed{Q = 24.859\, kW}$$

Result:

1. δ_{hx} = 0.065 m,

2. $\delta_{rx} = 6.74 \times 10^{-3}$m,

3. Drag force, F_D = 18.8 N,

4. Heat transfer, Q = 24.859 kW.

Example 2: A vertical plate of 0.7m wide and 1.2 m length maintained at a temperature of 90°C in a room at 30°C, calculate the convective heat loss.

Given: Wide, W = 0.7m

Height (or) Length, L = 1.2m

Wall temperature, T_w = 90°C

Room temperature, T_∞= 30°C

To find: Convective heat loss (Q),

Solution: Velocity (U) is not given, So, this is natural convection type problem.

We know that,

AMSCE - 1101

$$\text{Film temperature, } T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{90+30}{2}$$

$$\boxed{T_f = 60^\circ C}$$

Properties of air at 60°C:

[From HMT data book, page No. 33 (Sixth Edition)]

$\rho = 1.060 \text{ kg/m}^3$

$v = 18.97 \times 10^{-6} \text{ m}^2/\text{s}$

$Pr = 0.696$

$k = 0.02896 \text{ W/mK}$

we know,

$$\left.\begin{array}{r}\text{Co-efficient of} \\ \text{thermal expansion}\end{array}\right\} \beta = \frac{1}{T_f \text{ in K}}$$

$$\Rightarrow \quad \beta = \frac{1}{60+273} = 3 \times 10^{-3} K^{-1}$$

$$\boxed{\beta = 3 \times 10^{-3} K^{-1}}$$

$$\text{Grashof Number, } Gr = \frac{g \times \beta \times L^3 \times \Delta T}{v^2}$$

[From HMT data book, page No. 134 (Sixth Edition)]

$$= \frac{9.81 \times 3 \times 10^{-3} \times (1.2)^3 \times (90-30)}{(18.97 \times 10^{-6})^2}$$

$$\boxed{Gr = 8.4 \times 10^9}$$

$$Gr = Pr = 8.4 \times 10^9 \times 0.696$$

$$\boxed{Gr\,Pr = 5.9 \times 10^9}$$

Since Gr Pr > 10^9, flow is turbulent.

For turbulent flow,

Nusselt Number, $Nu = 0.10\ (Gr\ Pr)^{0.333}$

[From HMT data book, Page no.135 (Sixth edition)]

$$Nu = 0.10\left[5.9 \times 10^9\right]^{0.333}$$

$$\boxed{Nu = 179.3}$$

We know that,

AMSCE - 1101

$$Nu = \frac{h\ L}{k}$$

Nusselt number, $179.3 = \frac{h \times 1.2}{0.02896}$

$$\boxed{\left.\begin{array}{l}\text{Convective heat}\\ \text{transfer co-efficient}\end{array}\right\} h = 4.32\ W/m^2K}$$

Heat loss, $Q = h\ A\ (\Delta T)$

$= h \times W \times L \times (T_w - T_\infty)$

$= 4.32 \times 0.7 \times 1.2 \times (90 - 30)$

$$\boxed{Q = 218.16\ W}$$

Result: Convective heat loss, Q = 218.16 W.

10. Calculate the heat transfer from a 60 W incandescent bulb at 115°C to ambient air at 25°C. Assume the bulb as a sphere of 50 mm diameter. Also find the % of power lost by free convection.

Given Data:

Assume bulb as a sphere, D = 50mm = 0.050 m

Surface temperature, T_w = 115°C + 273 = 383 K

Ambient air temperature, T_∞= 25°C + 273 = 298 K

To find: Film temperature, $T_f = \frac{T_w + T_\infty}{2}$

$$= \frac{115 + 25}{2} = 70^\circ C$$

To properties of air at 70°C,

k = 0.02966W/mK

v= $20.02 \times 10^{-6}\ m^2/s$

Pr = 0.694

$$\therefore \beta = \frac{1}{T_f \text{ in K}} = \frac{1}{70 + 273}$$

$$= \frac{1}{343} = 2.915 \times 10^{-3} K^{-1}$$

$$Gr = \frac{g D^3 \beta (T_w - T_\infty)}{v^2}$$

Grashof number, $= \frac{9.81 \times (0.050)^3 \times 2.915 \times 10^{-3} \times (383 - 298)}{(20.02 \times 10^{-6})^2}$

$Gr = 7.58 \times 10^5$

$$\text{Gr Pr} = 7.58 \times 10^5 \times 0.694$$

$$= 5.26 \times 10^5$$

[Refer HMT data book , page no. 137]

Nusselt number, $Nu = 2 + 0.50\ (Gr\ Pr)^{0.25}$

$$\boxed{Nu = 15.46}$$

$$\Rightarrow \frac{h\,D}{k} = 15.46$$

$$\boxed{h = 9.15 W/m^2K}$$

$$Q = \bar{h}\,A\left(T_s - T_\infty\right)$$

Heat transfer, $Q = 9.15 \times 4\pi r^2 \left(383 - 298\right)$

$$\boxed{Q = 6.10W}$$

Percentage of power lost by free convection

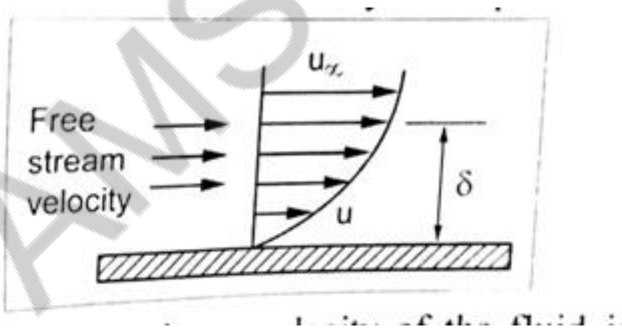
$$= \frac{Q}{60} \times 100$$

$$= \frac{6.10}{60} \times 100 = 10.18\%$$

11. Define teh velocity boundary layer and thermal boundary layer and thermal boundary layer thickness for flow over a flat plate.

Velocity boundary layer and Thermal boundary layer

The thickness of the boundary layer has been defined as the distance from the surface at which the velocity or temperature reaches 99% of the external velocity or temperature.



In velocity boundary layer, velocity of the fluid is less than 99% of free stream velocity and in thermal boundary layer, temperature of the fluid is less than 99% of the free stream.

12.Air at 30℃, at a pressure of 1 bar is flowing over a flat plate at a velocity of 4 m/s. If the plate is maintained at a uniform temperature of 130℃, calculate the average heat transfer co-efficient over the 1.5 m length of the plate and the air per 1m width of the plate.

Given: Fluid temperature, T_∞=30℃

Velocity, U = 4 m/s

Plate temperature, T_w = 130℃

AMSCE - 1101

Length, L = 1.5m

Width, W = 1m

To find: 1. Average heat transfer co-efficient, h

2. Heat transfer, Q.

Solution: We know that,

$$T_f = \frac{T_w + T_\infty}{2}$$

Film temperature, $= \frac{130 + 130}{2}$

$$\boxed{T_f = 80^\circ C}$$

Properties of air at 80℃:

[From HMT data book, Page No. 33 (Sixth Edition)]

ρ= 1 kg/m^3

v= 21.09 × 10^{-6} m^2/s

Pr = 0.692

k= 0.03047 W/mK.

$$Re = \frac{UL}{v}$$

Reynolds number, $= \frac{4 \times 1.5}{21.09 \times 10^{-6}}$

$$\boxed{Re = 2.84 \times 10^5} < 5 \times 10^5$$

Since Re < 5 × 10^5, flow is laminar flow,

For flat plate, laminar flow,

$$\left.\begin{array}{l}\text{Local nusselt} \\ \text{Number, } Nu_x\end{array}\right\} = 0.332 (Re)^{0.5} (Pr)^{0.333}$$

[From HMT data book, page No. 112 (Sixth edition)]

$$\Rightarrow \quad Nu_x = 0.332 \left(2.84 \times 10^5\right)^{0.5} \times (0.692)^{0.333}$$

$$\boxed{Nu_x = 156.51}$$

We know that,

Local Nusselt Number, $Nu_x = \frac{h_x L}{k}$

AMSCE - 1101

$$156.51 = \frac{h_x \times 1.5}{0.03047}$$

$$\Rightarrow \quad h_x = 3.179\,W/m^2K$$

$$\boxed{\left.\begin{array}{r}\text{Local heat transfer}\\ \text{coefficient, } h_x\end{array}\right\} = 3.179\,W/m^2K}$$

We know that,

$$\left.\begin{array}{r}\text{Average heat transfer}\\ \text{coefficient, } h\end{array}\right\} = 2 \times h_v$$

$$= 2 \times 3.179$$

$$\boxed{h = 6.358}$$

We know that,

Heat transfer, $Q = h\,A\,(T_w - T_\infty)$

$= h \times W \times L\,(T_w - T_\infty)$

$= 6.358 \times 1 \times 1.5 \times (130 - 30)$

$$\boxed{Q = 953.7\,W}$$

Result: 1. $h = 6.358\ W/m^2K$

2. $Q = 953.7\ W$

13. A steam pipe 80 mm in diameter is convect with 30mm thick layer of insulation which has a surface emissivity of 0.94. The insulation surface temperature is 85℃ and the pipe is placed in atmosphere air at 15℃. If the heat lost both by radiation and free convection, find the following:

1. The heat loss from 5m length of the pipe.

2. The overall heat transfer co-efficient.

3. Heat transfer co-efficient due to radiation.

Given: Diameter of pipe =-80 mm

= 0.080 m

Insulation thickness = 30 mm = 0.030 m

$$\left.\begin{array}{r}\text{Actual diameter of}\\ \text{the pipe.D}\end{array}\right\} = 0.080 + 2 \times 0.030$$

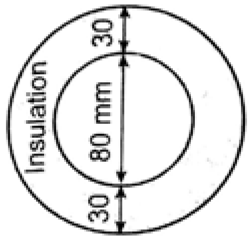
$$= 0.14\,m$$

Emissivity, $\varepsilon = 0.94$

Air surface temperature, $T_u = 85℃$

Air temperature, $T_\infty = 15℃$

AMSCE - 1101



To find: 1. Heat transfer from 5m length of pipe, Q

2. Overall heat transfer co-efficient, h_r.

3. Heat transfer co-efficient due to radiation, h_r.

Solution: We know that,

$$\text{Film temperature, } T_f = \frac{T_w + T_\infty}{2}$$
$$= \frac{85+15}{2}$$
$$\boxed{T_f = 50^\circ C}$$

Properties of air at 50°C:

[From HMT data book, Page no. 33 (Sixth edition)]

$\rho = 1.093 \text{ kg/m}^3$

$v = 17.95 \times 10^{-6} \text{ m}^2/\text{s}$

$Pr = 0.698$

$k = 0.02826 \text{ W/mK}$

$$\left.\begin{array}{r}\text{Co} - \text{efficent of thermal} \\ \text{expansion}, \beta\end{array}\right\} = \frac{1}{T_f \text{ in K}}$$
$$= \frac{1}{50+273}$$
$$\boxed{\beta = 3.095 \times 10^{-3} K^{-1}}$$

We know that,

$$\text{Grashof number, } Gr = \frac{g \times \beta \times D^3 \times \Delta T}{v^2}$$

[From HMT data book, Page No.134 (Sixth edition)]

$$= \frac{9.81 \times 3.095 \times 10^{-3} \times (0.14)^3 \times (85-15)}{(17.95 \times 10^{-6})^2}$$
$$\boxed{Gr = 18.10 \times 10^6}$$
$$Gr\,Pr = 18.10 \times 10^6 \times 0.698$$
$$\boxed{Gr\,Pr = 1.263 \times 10^7}$$

AMSCE - 1101

For horizontal cylinder,

Nusselt number, $Nu = C\ [Gr\ Pr]^m$

[From HMT data book, Page No. 137 (Sixth edition0]

$C = 1.263 \times 10^7$,

Corresponding $C = 0.125$, and $m = -0.333$

$$\Rightarrow \quad Nu = 0.125\left[1.263 \times 10^7\right]^{0.333}$$

$$\boxed{Nu = 28.952}$$

We know that, $\quad Nu = \dfrac{h\,D}{k}$

$$\Rightarrow \quad 28.952 = \frac{h \times 0.14}{0.02826}$$

$$\Rightarrow \quad h = 5.84\,W/m^2K$$

$$\boxed{\text{Convective heat transfer coefficient, } h_c = 5.84\,W/m^2K}$$

Heat lost by convection,

$$\begin{aligned} Q_{conv} &= h\,A\left(\Delta T\right) \\ &= h \times \pi D \times L \times \left(T_w - T_\infty\right) \\ &= 5.84 \times \pi \times 0.14 \times 5 \times \left(85 - 15\right) \end{aligned}$$

$$\boxed{Q_{conv} = 898.88\,W}$$

Heat lost by radiation,

$Q_{rad} = \varepsilon\ \sigma\ A\ [T^4{}_w - T^4{}_\infty]$

Where, ε = Emissivity

A= Area – m^2

σ = Stelen Boltzman Constant

$= 5.67 \times 10^{-8}\ W/m^2\ K^4$

$T_{u=}$ = Surface temperature, K.

T_∞= Fluid temperature, K.

$T_w = 85 + 273 \qquad T_\infty = 15 + 273$

$\boxed{T_w = 358K} \qquad \boxed{T_\infty = 288K}$

$$\begin{aligned} \Rightarrow \quad Q_{rad} &= \varepsilon \times \sigma \times \pi DL \times \left[T^4{}_w - T^4{}_\infty\right] \\ &= 0.94 \times 5.67 \times 10^{-8} \times \pi \times 0.14 \times 5 \times \left[358^4 - 288^4\right] \end{aligned}$$

$$\boxed{Q_{rad} = 1118.90\,W}$$

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Total heat loss, $Q_t = Q_{conv} + Q_{rad}$

$= 898.99 + 118.90$

$\boxed{Q_t = 2017.89 W}$

Total heat transfer, $Q_t = h_t \; A \; \Delta \; T$

$= h_t \times \pi \; DL \times (T_w - T_\infty)$

$2017.89 = h_t \times \pi \times 0.14 \times 5 \times (85 - 15)$

$\Rightarrow \quad h_t = 13.108 \, W/m^2K$

$\boxed{\text{Overall heat coefficient, } h_t = 13.108 \, W/m^2K}$

Radiative heat transfer coefficient,

$H_r = h_t - h_c$

$= 13.108 - 5.84$

$\boxed{h_r = 7.268 \, W/m^2K}$

Result:

1. Heat loss from 5m length of pipe

(i) By convection, Q_c = 898.99 W

(ii) By radiation, $Q_{r=}$= 118.90 W

2. Overall heat transfer coefficient, h_t= 13.108W/m^2K

3. Radiative heat transfer co-efficient, h_r= 7.268 W/m^2K.

14. **Air at 40°C flows over a flat plate, 0.8 m long at a velocity of 50 m/s. The plate surface is maintained at 300°C. Determine the heat transferred from the entire plate length to air taking into consideration both laminar and turbulent portion of the boundary layer. Also calculate the percentage error if the boundary layer is assumed to be turbulent nature from the very leading edge of the plate.**

Given : Fluid temperature T_∞ = 40°C, Length L = 0.8 m, Velocity U = 50 m/s , Plate surface temperature T_w = 300°C

To find :

1. Heat transferred for:

   i. Entire plate is considered as combination of both laminar and turbulent flow.
   ii. Entire plate is considered as turbulent flow.

2. Percentage error.

Solution: We know Film temperature $T_f = \frac{T_w - T_\infty}{2}$ T

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$$= \frac{300+40}{2} = 443 \text{ K}$$

$T_f = 170°C$

Properties of air at 170°C:

ρ= 0.790 Kg/m^3

$\nu = 31.10 \times 10^{-6}$ m^2/s

$Pr = 0.6815$

$K = 37 \times 10^{-3}$ W/mK

We know

$$\text{Reynolds number Re} = \frac{UL}{v}$$

$$= \frac{50 \times 0.8}{31.10 \times 10^{-6}} = 1.26 \times 10^6$$

$Re = 1.26 \times 10^6 > 5 \times 10^5$

$Re > 5 \times 10^5$, so this is turbulent flow

Case (i): Laminar – turbulent combined. [It means, flow is laminar upto Reynolds number value is 5×10^5, after that flow is turbulent]

Average nusselt number = Nu = $(Pr)^{0.333}$ $(Re)^{0.8}$ – 871

Nu = $(0.6815)^{0.333}$ [0.037 $(1.26 \times 10^6)^{0.8}$ – 871

Average nusselt number Nu = 1705.3

$$\text{We know Nu} = \frac{hL}{K}$$

$$1705.3 = \frac{h \times 0.8}{37 \times 10^{-3}}$$

$h = 78.8$ W/m^{2}K

Average heat transfer coefficient

h=78.8 W/m^{2}K

Head transfer $Q_1 = h \times A \times (T_w + T_\infty)$

$= h \times L \times W \times (T_w + T_\infty)$

$= 78.8 \times 0.8 \times 1 \times (300 - 40)$

$Q_1 = 16390.4$ W

Case (ii) : Entire plate is turbulent flow:

Local nusselt number} Nux = 0.0296 × $(Re)^{0.8}$ × $(Pr)^{0.333}$

$NU_x = 0.0296 \times (1.26 \times 10^6)^{0.8} \times (0.6815)^{0.333}$

AMSCE - 1101

$NU_x = 1977.57$

We know $NU_x = \frac{h_x \times L}{K}$

$$1977.57 = \frac{h_x \times 0.8}{37 \times 10^{-3}}$$

$$h_x = 91.46 \text{ W/m}^2\text{K}$$

Local heat transfer coefficient $h_x = 91.46$ W/m^{2}K

Average heat transfer coefficient (for turbulent flow)

$h = 1.24 \times h_x$

$= 1.24 \times 91.46$

Average heat transfer coefficient} $h = 113.41$ W/m^{2}K

We know Heat transfer $Q_2 = h \times A \times (T_w + T_\infty)$

$= h \times L \times W \times (T_w + T_\infty)$

$= 113.41 \times 0.8 \times 1\,(300 - 40)$

$Q_2 = 23589.2$ W

2. Percentage error = $\frac{Q_2 - Q_1}{Q_1}$

$$= \frac{23589.2 - 16390.4}{16390.4} \times 100$$

$= 43.9\%$

15. **250 Kg/hr of air are cooled from 100°C to 30°C by flowing through a 3.5 cm inner diameter pipe coil bent in to a helix of 0.6 m diameter. Calculate the value of air side heat transfer coefficient if the properties of air at 65°C are**

K = 0.0298 W/mK

μ = 0.003 Kg/hr – m

Pr = 0.7

ρ = 1.044 Kg/m^3

Given : Mass flow rate in = 205 kg/hr

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$$= \frac{205}{3600} \text{Kg/s in} = 0.056 \text{ Kg/s}$$

Inlet temperature of air T_{mi} = 100°C

Outlet temperature of air T_{mo} = 30°C

Diameter D = 3.5 cm = 0.035 m

Mean temperature $T_m = \frac{T_{mi} + T_{mo}}{2} = 65°C$

To find: Heat transfer coefficient (h)

Solution:

Reynolds Number Re = $\frac{UD}{\nu}$

Kinematic viscosity $\nu = \frac{\mu}{\rho}$

$$\frac{\frac{0.003}{3600} \text{Kg/s} - \text{m}}{1.044 \text{ Kg/m}^3}$$

$$v = 7.98 \times 10^{-7} \text{ m}^2/\text{s}$$

Mass flow rate in = ρ A U

$$0.056 = 1.044 \times \frac{\pi}{4} \times D^2 \times U$$

$$0.056 = 1.044 \times \frac{\pi}{4} \times (0.035)^2 \times U$$

$\Rightarrow$ U = 55.7 m/s

(1) $\Rightarrow$ Re = $\frac{UD}{\nu}$

$$= \frac{55.7 \times 0.035}{7.98 \times 10^{-7}}$$

Re = 2.44×10^6

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Since Re > 2300, flow is turbulent

For turbulent flow, general equation is (Re > 10000)

$$Nu = 0.023 \times (Re)^{0.8} \times (Pr)^{0.3}$$

This is cooling process, so n = 0.3

$$\Rightarrow \; Nu = 0.023 \times (2.44 \times 10^{6})^{0.8} \times (0.7)^{0.3}$$

$$Nu = 2661.7$$

We know that, $Nu = \dfrac{hD}{K}$

$$2661.7 = \frac{h \times 0.035}{0.0298}$$

Heat transfer coefficient h = 2266.2 W/m^{2}K

16. **In a long annulus (3.125 cm ID and 5 cm OD) the air is heated by maintaining the temperature of the outer surface of inner tube at 50°C. The air enters at 16°C and leaves at 32°C. Its flow rate is 30 m/s. Estimate the heat transfer coefficient between air and the inner tube.**

Given : Inner diameter D_i = 3.125 cm = 0.03125 m

Outer diameter D_o = 5 cm = 0.05 m

Tube wall temperature T_w = 50°C

Inner temperature of air T_{mi} = 16°C

Outer temperature of air t_{mo} = 32°C

Flow rate U = 30 m/s

To find: Heat transfer coefficient (h)

Solution:

Mean temperature $T_m = \dfrac{T_{mi} + T_{mo}}{2}$

AMSCE - 1101

$$= \frac{16+32}{2}$$

$$T_m = 24°C$$

Properties of air at 24°C:

ρ = 1.614 Kg/m^3

ν = $15.9 \times 10^{-6}\ m^2/s$

Pr = 0.707

K = $26.3 \times 10^{-3} W/mK$

We know,

Hydraulic or equivalent diameter

$$D_h = \frac{4A}{P} = \frac{4 \times \frac{\pi}{4}\left[D^2 - D_i^2\right]}{\pi\left[D_o + D_i\right]}$$

$$= \frac{\left(D_o + -D_i\right)\ \left(D_o - D_i\right)}{(D_o + D_i)}$$

$$= D_o - D_i$$

= 0.05 – 0.03125

D_h = 0.01875 m

Reynolds number Re = $\frac{UD_h}{\nu}$

$$= \frac{30 \times 0.01875}{15.9 \times 10^{6}}$$

Re = 35.3×10^{-6}

Since Re > 2300, flow is turbulent

For turbulent flow, general equation is (Re > 10000)

Nu = 0.023 (Re)$^{0.8}$ (Pr)^n

This is heating process. So n = 0.4

$\Rightarrow$ Nu = $0.023 \times (35.3 \times 10^3)^{0.8} (0.707)^{0.4}$

Nu $= 87.19$

AMSCE - 1101

We know $Nu = \dfrac{hD_h}{K}$

$\Rightarrow\ 87.19 = \dfrac{h \times 0.01875}{26.3 \times 10_{-3}}$

$\Rightarrow\ h = 122.3\ W/m^2K$

17. **Engine oil flows through a 50 mm diameter tube at an average temperature of 147°C. The flow velocity is 80 cm/s. Calculate the average heat transfer coefficient if the tube wall is maintained at a temperature of 200°C and it is 2 m long.**

Given : Diameter D = 50 mm = 0.050 m

Average temperature T_m = 147°C

Velocity U = 80 cm/s = 0.80 m/s

Tube wall temperature T_w = 200°C

Length L = 2m

To find: Average heat transfer coefficient (h)

Solution : Properties of engine oil at 147°C

$\rho = 816\ Kg/m^3$

$\nu = 7 \times 10^{-6}\ m^2/s$

$Pr = 116$

$K = 133.8 \times 10^{-3}\ W/mK$

We know

Reynolds number $Re = \dfrac{UD}{\nu}$

$= \dfrac{0.8 \times 0.05}{7 \times 10^{-6}}$

$Re = 5714.2$

Since Re < 2300 flow is turbulent

$\dfrac{L}{D} = \dfrac{2}{0.050} = 40$

$10 < \dfrac{L}{D} < 400$

For turbulent flow, (Re < 10000)

$$\text{Nusselt number Nu} = 0.036\ (Re)^{0.8}(Pr)^{0.33}\left(\frac{D}{L}\right)^{0.055}$$

$$Nu = 0.036\ (5714.2)^{0.8} \times (116)^{0.33} \times \left(\frac{0.050}{2}\right)^{0.055}$$

$$Nu = 142.8$$

$$\text{We know Nu} = \frac{hD}{K}$$

$$\Rightarrow\ 142.8 = \frac{h \times 0.050}{133.8\ \times 10^{-3}}$$

$$\Rightarrow\ h = 382.3\ W/m^2K$$

18. **A large vertical plate 4 m height is maintained at 606°C and exposed to atmospheric air at 106°C. Calculate the heat transfer is the plate is 10 m wide.**

Given :

Vertical plate length (or) Height L = 4 m

Wall temperature T_w = 606°C

Air temperature T_∞ = 106°C

Wide W = 10 m

To find: Heat transfer (Q)

Solution:

$$\text{Film temperature } T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{606 + 106}{2}$$

$$T_f = 356°C$$

Properties of air at 356°C = 350°C

ρ = 0.566 Kg/m^3

$\nu = 55.46 \times 10^{-6}\ m^2/s$

Pr = 0.676

K = 49.08×10^{-3} W/mK

$$\text{Coefficient of thermal expansion}\}\ \beta = \frac{1}{T_f \text{ in K}}$$

AMSCE - 1101

$$= \frac{1}{356+273} = \frac{1}{629}$$

$$\beta = 1.58 \times 10^{-3} K^{-1}$$

$$\text{Grashof number Gr} = \frac{g \times \beta \times L^3 \times \Delta T}{v^2}$$

$$\Rightarrow \text{ Gr} = \frac{9.81 \times 2.4 \times 10^{-3} \times (4)^3 \times (606-106)}{(55.46 \times 10^{-6})^2}$$

$Gr = 1.61 \times 10^{11}$

$Gr\ Pr = 1.61 \times 10^{11} \times 0.676$

$Gr\ Pr = 1.08 \times 10^{11}$

Since $Gr\ Pr > 10^9$, flow is turbulent

For turbulent flow,

Nusselt number $Nu = 0.10\ [Gr\ Pr]^{0.333}$

$$\Rightarrow \text{ Nu} = 0.10\ [1.08 \times 10^{11}]^{0.333}$$

$$Nu = 471.20$$

We know that,

Nusselt number $Nu = \frac{hL}{K}$

$$\Rightarrow \text{ } 472.20 = \frac{h \times 4}{49.08 \times 10^{-3}}$$

Heat transfer coefficient $h = 5.78\ W/m^2K$

Heat transfer $Q = h\ A\ \Delta T$

$$= h \times W \times L \times (T_w - T_\infty)$$

$$= 5.78 \times 10 \times 4 \times (606 - 106)$$

$$Q = 115600 \text{ W}$$

$$Q = 115.6 \times 10^3 \text{ W}$$

AMSCE - 1101

19 **Air at atmospheric pressure and 200°C flows over a plate with a velocity of 5 m/s. The plate is 15 mm wide and is maintained at a temperature 120°C. Calculated the thicknesses of hydrodynamic and thermal boundary laryer and the local heat transfer coefficient at a distance of 0.5 m from the leading edge. Assume the flow is on one side of the plate. Take $\rho = 0.815\ kg/m^3$, $\mu = 24.5 \times 10^{-6} Ns/m^2$, $Pr = 0.7$ and $k = 0.364$ W/mK.**

Given:

Temperature of air, T = 200°C

Velocity of air, U = 5 m/s

Width of the plate, W = 15 mm = 0.015 m

Temperature of plate = 120°C

To find:

1. Hydrodynamic boundary layer
2. Thermal boundary layer
3. Local heat transfer coefficient at distance = 0.5 m

Solution:

We know that,

$$\text{Film temperature} = \frac{T_w + T_\infty}{2} = \frac{200+120}{2} = \frac{320}{2}$$

$$\boxed{T_f = 160°C}$$

Properties of air at 160°C

Density, $\rho = 0.815\ kg/m^3$

Absolute viscosity, $\mu = 24.6 \times 10{-}6\ Ns/m^2$

Prandtl number, $Pr = 0.7$

Thermal conductivity, k = 0.0364 W/mK.

$$v = \frac{\mu}{\rho} = \frac{24.6 \times 10^{-6}}{0.815} = 3.018 \times 10^{-5}$$

$$\text{Reynold number} = Re = \frac{Ux}{v} = \frac{5 \times 0.5}{3.018 \times 10^{-5}}$$

$$\boxed{Re = 82836.3}$$

AMSCE - 1101

Since, Re is $< 5 \times 10^5$, hence, the flow is laminar.

For flat plate, Laminar flow [Refer HMT Data book Pg.no: 113] [Eighth Edition]

1. Hydrodynamic boundary layer thickness

$$\delta_{hx} = 5x(Re)^{-0.5} \text{ or } \frac{5x}{Re^{0.5}}$$

$$= \frac{5 \times 0.5}{\sqrt{82836.3}} = \frac{0.5 \times 5}{287.81} = 0.00869\, m$$

$$\boxed{\delta_{hx} = 8.69 \text{ mm}}$$

2. Thermal boundary layer thickness at $x = 0.5$ m

$$\delta_{th} = \frac{\delta}{Pr^{0.333}} = \frac{0.00869}{0.7^{0.333}} = 0.00979\, m$$

$$\boxed{\delta_{th} = 9.79 \text{ mm}}$$

Local heat transfer coefficient, Nu

$$Nu = 0.332\,(Re)^{0.5}\,(Pr)^{0.333}$$

$$= 0.332 \times (82836.3)^{0.5} \times (0.7)^{0.333}$$

$$\frac{h_x L}{k} = 0.332 \times (82836.3)^{0.5} \times (0.7)^{0.333}$$

$$h_x = \frac{k}{L} \times 0.332 \times (82836.3)^{0.5} \times (0.7)^{0.333}$$

$$= \frac{0.0364}{0.5} \times 0.332 \times (82836.3)^{0.5} \times (0.7)^{0.333}$$

$$\boxed{h_x = 6.177 \text{ W/m}^2\text{K}}$$

Result:

1. Hydrodynamic layer thickness, $\delta_{hx} = 8.69$ mm
2. Thermal boundary layer thickness, $\delta_{th} = 9.76$ mm
3. Local heat transfer coefficient, $h_x = 6.177$ W/m^{2}K

AMSCE - 1101

20. **A horizontal heated plate measuring 1.5 m × 1.1 m and at 215°C, facing upwards, is placed in still air at 25°C. Calculate the heat loss by natural convection. The convective film coefficient for free convection is given by the following empirical relation $h = 3.05\,(T_f)^{1/4}$ W/m^2°C where T_f in the mean temperature in degree kelvin.**

Given:

Horizontal plate size = 1.5 m × 1.1 m

Temperature of the plate = 215 + 273 = 488 K

Temperature of the air = 25 + 273 = 298 K

Heat transfer coefficient, $h = 3.05 \times (T_f)^{\frac{1}{4}}$

To find:

Heat loss by natural convection

Solution:

$$\text{Film temperature } T_f = \frac{488 + 298}{2} = \frac{786}{2} = 393\text{K}.$$

$$h = 3.05 \times (T_f)^{\frac{1}{4}}$$

$$= 3.05 \times (393)^{\frac{1}{4}}$$

$$\boxed{h = 13.58 \text{ W/m}^2\text{K}}$$

Rate of heat loss by natural convection

$$Q = h_A (T_s - T_\infty)$$

$$= 1358 \times 1.5 \times 1.1 \,(215 - 25) = 4257.33 \text{ W}$$

$$\boxed{Q = 4257.33 \text{ W}}$$

Result:

Heat loss by natural convection, Q = 4257.33 W.

AMSCE - 1101

21.

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown in Fig. 9–30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_i + T_o)/2 = (320 + 280)/2 = 300$ K $= 27°$C and 1 atm pressure are (Table A-15)

$$k = 0.02566 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7290$$

$$\nu = 1.580 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{300 \text{ K}}$$

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.729) = 4.776 \times 10^5$$

The effective thermal conductivity is

$$F_{sph} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$

$$= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4[(0.2 \text{ m}^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229$$

$$k_{eff} = 0.74k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{sph}\text{Ra}_L)^{1/4}$$

$$= 0.74(0.02566 \text{ W/m}\cdot°\text{C})\left(\frac{0.729}{0.861 + 0.729}\right)(0.005229 \times 4.776 \times 10^5)^{1/4}$$

$$= 0.1104 \text{ W/m}\cdot°\text{C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{eff}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)$$

$$= (0.1104 \text{ W/m}\cdot°\text{C})\pi\left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}}\right)(320 - 280)\text{K} = \mathbf{16.7 \text{ W}}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

AMSCE - 1101

22.

(ii) **An air stream at 0°C is flowing along a heated plate at 90°C at a speed of 75 m/s. The plate is 45 cm long and 60 cm wide. Calculate the average values of friction coefficient for the full length of the plate. Also calculate the rate of energy dissipation from the plate.**
(8)

Given data

Fluid temperature, $T_\infty = 0°C$

Speed, $U = 75$ m/sec

Surface temperature, $T_w = 90°C$

Length of the plate, $L = 45$ cm $= 0.45$ m

Width of the plate, $W = 60$ cm $= 0.60$ m

To find

1. Average friction coefficient for full length of the plate.
2. Rate of energy dissipation from the plate.

Solution

We know that, film temperature, T_f

$$T_f = \frac{T_w + T_\infty}{2} = \frac{90 + 0}{2} = 45°C$$

Properties of air at 45°C

[From HMT data book, Page No.34 (8th edition)]

$\rho = 1.1105$ kg/m^3

$\nu = 17.45 \times 10^{-6}$ m^2/sec

$Pr = 0.6985$

$k = 0.02791$ W/mK

We know, Reynold number, $Re = \dfrac{UL}{\nu}$

$$Re = \frac{75 \times 0.45}{17.45 \times 10^{-6}} = 1.93 \times 10^6$$

$$Re = 1.93 \times 10^6 > 5 \times 10^5$$

Since, $Re > 5 \times 10^5$, flow in turbulent

For flat plate, Lummar – turbulent flow

[From HMT data book, Page No.115, 8th edition]

AMSCE - 1101

SQ.15

Average friction

Coefficient $= C_{fL} = 0.074\,(Re)^{-0.2} - 1742\,(Re)^{-1.0}$

$$C_{fL} = 0.074\,(1.93 \times 10^6)^{-0.2} - 1742\,(1.93 \times 10^6)^{-1.0}$$

$$= 0.074 \times 0.0553 - 1742 \times 5.18 \times 10\text{-}7$$

$$\boxed{C_{fL} = 3.188 \times 10^{-3}}$$

Average Nusselt Number, $Nu = (Pr)^{0.333}\,[0.037\,(Re)^{0.8} - 871]$

[From HMT data book Page No.115, 8th edition

$$= (Pr)^{0.333} - [0.037\,(Re)^{0.8} - 871]$$

$$= (0.6985)^{0.333} - [0.037\,(1.93 \times 10^6)^{0.8} - 871]$$

$$= 0.887 \times [3079.45] = 2731.4$$

$$\boxed{Nu = 2731.4}$$

$$Nu = \frac{hL}{k}\,; \qquad h = \frac{Nu \times k}{L}$$

$$h = \frac{2731.4 \times 0.02791}{0.45} = 169.40$$

$$\boxed{h = 169.40\ W/m^2K}$$

Rate of energy dissipation, *

$$Q = hA\,(T_w - T_\alpha)$$

$$= 169.40 \times 0.60 \times 0.45\,(90 - 0)$$

$$\boxed{Q = 4116.42\ W}$$

Result

1. Average fraction coefficient for full length of the plate, $C_{fL} = 3.188 \times 1$
2. Rate of energy dissipation, $Q = 4116.42$ W

AMSCE - 1101

23.

The crankcase of an I.C. engine measuring 80 cm × 20 cm may be idealised as a flat plate. The engine runs at 90 km/h and the crankcase is cooled by the air flowing past it at the same speed. Calculate the heat loss from the crank surface maintained at 85°C, to the ambient air at 15°C. Due to road induced vibration, the boundary layer becomes turbulent from the leading edge itself.

Solution. *Given :* $U = 90$ km/h $= \frac{90 \times 1000}{3600} = 25$ m/s; $t_s = 85°C$; $t_\infty = 15°C$; $L = 80$ cm $= 0.8$ m; $B = 20$ cm $= 0.2$m.

The properties of air at $t_f = \frac{85 + 15}{2} = 50°C$ are :

$k = 0.02824$ W/m°C, $\nu = 17.95 \times 10^{-6}$ m²/s, $Pr = 0.698$ ... (From tables)

Heat loss from the crankcase, Q :

The Reynolds number, $Re_L = \frac{UL}{\nu} = \frac{25 \times 0.8}{17.95 \times 10^{-6}} = 1.114 \times 10^6$

Since $Re_L > 5 \times 10^5$, the nature of flow is *turbulent.*

For turbulent boundary layer,

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.036\,(Re_L)^{0.8}\,(Pr)^{0.333} = 0.036\,(1.114 \times 10^6)^{0.8}\,(0.698)^{0.333} = 2196.92$$

or, $$\bar{h} = \frac{k}{L} \times 2196.92 = \frac{0.02824}{0.8} \times 2196.92 = 77.55 \text{ W/m}^2\text{°C}$$

$$\therefore \quad Q = \bar{h}A\,(t_s - t_\infty) = 77.55 \times (0.8 \times 0.2)\,(85 - 15) = \mathbf{868.56\ W}$$

24.

AMSCE - 1101

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in. thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to air flow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Figure 7–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 75$ lbm/ft³, $C_p = 0.4$ Btu/lbm · °F, and $\varepsilon = 0.9$.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$. 3 Air is an ideal gas. 4 The local atmospheric pressure is 1 atm. 5 The surrounding surfaces are at the temperature of the room air.

Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140°\text{F}$ and 1 atm pressure are (Table A–15E)

$$k = 0.01623 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F} \qquad \text{Pr} = 0.7202$$
$$\nu = 0.7344 \text{ ft}^2/\text{h} = 0.204 \times 10^{-3} \text{ ft}^2/\text{s}$$

Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the air flow across the plastic sheet is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(10 \text{ ft/s})(4 \text{ ft})}{0.204 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.01623 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}}{4 \text{ ft}}(263.6) = 1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F}$$
$$A_s = (2 \text{ ft})(4 \text{ ft})(2 \text{ sides}) = 16 \text{ ft}^2$$

and

$$\begin{aligned}\dot{Q}_{conv} &= hA_s(T_s - T_\infty) \\ &= (1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(16 \text{ ft}^2)(200 - 80)°\text{F} \\ &= 2054 \text{ Btu/h}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) \\ &= (0.9)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)(16 \text{ ft}^2)[(660 \text{ R})^4 - (540 \text{ R})^4] \\ &= 2584 \text{ Btu/h}\end{aligned}$$

AMSCE-1101

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2054 + 2584 = \mathbf{4638\ Btu/h}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c \mathcal{V}_{plastic} = (75\ \text{lbm/ft}^3)\left(\frac{4 \times 0.04}{12}\ \text{ft}^2\right)\left(\frac{30}{60}\ \text{ft/s}\right) = 0.5\ \text{lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m}\, C_p(T_2 - T_1) \quad \rightarrow \quad T_2 = T_1 + \frac{\dot{Q}}{\dot{m} C_p}$$

Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200°\text{F} + \frac{-4638\ \text{Btu/h}}{(0.5\ \text{lbm/s})(0.4\ \text{Btu/lbm}\cdot°\text{F})}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = \mathbf{193.6°F}$$

Discussion The average temperature of the plastic sheet drops by about 6.4°F as it passes through the cooling section. The calculations now can be repeated by taking the average temperature of the plastic sheet to be 196.8°F instead of 200°F for better accuracy, but the change in the results will be insignificant because of the small change in temperature.

25.

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown in Fig. 9–30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

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Properties The properties of air at the average temperature of $T_{ave} = (T_i + T_o)/2 = (320 + 280)/2 = 300$ K $= 27°$C and 1 atm pressure are (Table A-15)

$$k = 0.02566\ \text{W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7290$$

$$\nu = 1.580 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{300\ \text{K}}$$

AMSCE - 1101

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\begin{aligned}\mathrm{Ra}_L &= \frac{g\beta(T_i - T_o)L^3}{\nu^2}\mathrm{Pr}\\ &= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.729) = 4.776 \times 10^5\end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned}F_{\mathrm{sph}} &= \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}\\ &= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4[(0.2 \text{ m}^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229\end{aligned}$$

$$\begin{aligned}k_{\mathrm{eff}} &= 0.74k\left(\frac{\mathrm{Pr}}{0.861 + \mathrm{Pr}}\right)^{1/4}(F_{\mathrm{sph}}\mathrm{Ra}_L)^{1/4}\\ &= 0.74(0.02566 \text{ W/m}\cdot{}^\circ\text{C})\left(\frac{0.729}{0.861 + 0.729}\right)(0.005229 \times 4.776 \times 10^5)^{1/4}\\ &= 0.1104 \text{ W/m}\cdot{}^\circ\text{C}\end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\begin{aligned}\dot{Q} &= k_{\mathrm{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)\\ &= (0.1104 \text{ W/m}\cdot{}^\circ\text{C})\pi\left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}}\right)(320 - 280)\text{K} = \mathbf{16.7 \text{ W}}\end{aligned}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

Discussion Note that the air in the spherical enclosure will act like a stationary fluid whose thermal conductivity is $k_{\mathrm{eff}}/k = 0.1104/0.02566 = 4.3$ times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually very significant, and should be considered in a complete analysis.

AMSCE 1101

Unit-3

PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

Part-A

1. what is meant by boiling and condonation?

Ans: The change of phase from liquid to vapour state is known as condensation.

2. Give the appliocations of boiling and condensation?

Ans: Boiling and condensation process finds wide applications as mwntioned below.

1. Thermal and nuclear power plant.
2. Rrfrigerating systems.
3. Process of heating and cooling
4. Air conditioniong systems

3. what is meanrt by pool boiling?

Ans: If heat is added to a liquid from a submerged soliod surface, the boiling process referred to as pool boiling. In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and making induced by bubble growth and detachment.

4. What is meant by film wise and Drop wise condenstaion?

Ans: The liquid condensation wets the solid surface, spreads out and forms a continuous film over the entire surface is known as film wise condensation.

5. Give the merits of drop wise condensation?

Ans: In drop wise condensation, a large portion of the area of the plate is directly exposed to vapour. The heat transfer rate in drop wise condensation is 10 times higher than in film condensation.

6. What is heat exchanger?

Ans: A heat exchanger is defined as an equipment which transfer the heat from a hot fluid to cold fluid.

7. What are the types of heat exchangers?

Ans: The types of heat exchangers are as follows:

1. Direct contact that exchangers
2. Indirect contact heat exchangers
3. Surface heat exchangers
4. Parallel flow heat exchangers
5. Counters flow heat exchangers
6. Cross flow heat exchangers
7. Shell and tube heat exchaangers
8. Compact heat exchangers.

8. What is meant by direct heat exchanger (or) open heat exchangers?

Ans: In direcrt contact heat exchanger, the heat exchange takes place by direct miding of hot and cold fluids.

9. What is meant by Indirect comtact heat exchanger?

AMSCE - 1101

Ans: In this type of heat exchangers, the transfer of heat between two fluids could be carried out by transmission through a wall which separates the two fluids.

10. What is meant by Regenerators?

Ans: In this tyype of heat exchangers, hot and cold fluids flow alternatively through the space. Examploes: IC engines, gas turbines.

11. what is meant by recuperator (or) Surface heat exchangers?

Ans: This is the most common type of heat exchangers in which the hot and cold fluid do not come into direct contact with each other but are seperated by a tube wall or a surfacxe.

12. wehat is meant by parallel flow and counter flow heat exchanger?

Ans: In this type of heat exchanger, hot and cold fluids move in the same directions.

In this type of heat exchanger hot and cold fluids parallel but opposite directions.

13. What is meant by shell and tube heat exchanger?

Ans: In this type of heat exchanger, one of the fluids move through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it moves over the outside surface of the tubes.

14. What is meant by compact heat exchangers?

Ans: There are many special purpose heat exchanegrs called compact heat exchangers called compact heat exchangers. They are generally employed when convective heat transfer co-efficient associated with one of the fluids such smaller than that associated with the other fluid.

15. What is meant by LMTD?

Ans: We know that the temperature difference between the hot and cold fluids in the heat exchanege varies from Point in addition various modes of heat transfer are involved. Therefore based on concept of appropriate mean tempoerature difference, also called logarithmic mean temperature difference, the total heat transfer rate in the heat exchanger is expressed as

$$Q = U\,A\,(\Delta T)m$$

Where U – Overall heat transfer co-efficient W/m^{2}K A – Area m^2

$(\Delta T)_m$ – Logarithmic mean temperature differnce.

16. What is meant by fouling factor?

Ans: We know the surfaces of a heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled with scaliong or deposits. The effect of these deposits the value overall heat transfer co-efficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

17. What is meant by effectiveness?

Ans: The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

$$\text{Effectiveness}\ \varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{max}}$$

AMSCE - 1101

18. Give the application of boiling and condensation?

Ans: 1. Thermal and nuclear power plant

2. Refrigerating systems

3. Process of heat and cooling

4. Air conditionaring systems.

19. What are the method of condensation?

Ans: 1. Filmwise condenstaion

2. Dropwise condenstaion

20. Draw the difference regions of boiling and what is nucleate boiling?

Ans: Nucleate boiling exists in regions ii and iii. The nuclear boiling begins at region ii. As the excess temperature is further increased, bubbles are formed more rapidly and raoid eveporartion take place. This is indicated in region iii. Nucleate boiling exists upto T = 50℃.

21. What is meant by counter flow heat exchanger?

Ans: In this types of heat exchanger, hot and cold fluids move in parallel but opposite in direction.

AMSCE - 1101

22. **What is meant by dropwise condensation?**

In dropwise condensation the vapour condenses into small liquid droplets of various sizes which fall down the surface in random fashion.

23. **Two fluids A and B exchange heat in a counter flow heat exchanger. Fluid A enters at 420°C and has a mass flow rate of 1 kg/s. fluid B enters at 20°C and has a mass flow rate of 1 kg/s. The effectiveness of heat exchanger is 75%. Determine the exit temperature of fluid B.**

Given:

Inlet temperature of fluid A = 420°C

Inlet temperature of fluid B = 20°C

Mass flow rate of A = 1 Kg/s

Mass flow rate of B = 1 Kg/s

Effectiveness of heat exchanger = 75%

To find:

Exit temperature of B

Solution:

We know that,

$$\text{Effectiveness} = \frac{T_1 - T_2}{T_1 - T_1}$$

$$0.75 = \frac{420 - T_2}{420 - 20}$$

$$0.75 \times 400 = 420 - T_2$$

$$300 = 420 - T_2$$

$$T_2 = 420 - 300$$

$$\boxed{T_2 = 120°C}$$

Result:

Exit temperature of B = 120°C.

AMSCE - 1101

22. **What is meant by dropwise condensation?**

In dropwise condensation the vapour condenses into small liquid droplets of various sizes which fall down the surface in random fashion.

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Inlet temperature of fluid A = 420°C

Inlet temperature of fluid B = 20°C

Mass flow rate of A = 1 Kg/s

Mass flow rate of B = 1 Kg/s

Effectiveness of heat exchanger = 75%

To find:

Exit temperature of B

Solution:

We know that,

$$\text{Effectiveness} = \frac{T_1 - T_2}{T_1 - T_1}$$

$$0.75 = \frac{420 - T_2}{420 - 20}$$

$$0.75 \times 400 = 420 - T_2$$

$$300 = 420 - T_2$$

$$T_2 = 420 - 300$$

$$\boxed{T_2 = 120°C}$$

Result:

Exit temperature of B = 120°C.

PART- B

1.A heating element is added with metal is 8 mm diameter and 0 emissivity is 0.92. The element is horizontally immersed in water bath. The surface temperature of the metal is 260℃ under steady state boiling conditions. Calculate the power dissipation per unit length of the herater.

Given:

Diameter, D = 8 mm = 8×10^3 m

Emissivity, $\in = 0.92$

Surface temperature, T_w = 260℃.

To find:

Power dissipation

Solution:

We know that, saturation temperature of water is 100℃.

AMSCE - 1101

i.e. $\boxed{T_{sat} = 100^\circ C}$

Excess temperature,
$$\Delta T = T_w - = T_{sat}$$
$$\Delta T = 260 - 100$$
$$\boxed{\Delta T = 160^\circ C} > 50^\circ C$$

So, this Film pool boiling

$$\text{Film temperature}, T_f = \frac{T_w + T_{sat}}{2}$$
$$= \frac{260 + 100}{2}$$
$$\boxed{T_f = 180^\circ C}$$

Properties of water Vapour at 180℃. (Saturated Steam)

[From HMT data page no.39 (Sixth edition)]

P_v = 5.16 kg/m^3

K_v = 0.03268 W/mK

C_{pv} = 2709 J/kg K

U_v = 15.10 × 10^{-6} Ns/m^2

Properties of saturated water at 100℃

[From HMT data book page No. 21 (Sixth edition]

P_t = 961 kg/m^3

From steam table At 100℃

[R.S Khurmi steam table Page No. 4]

$$h_{fg} = 2256.9\,\text{kJ / kg}$$
$$\boxed{h_{fg} = 2256.9 \times 10^3\ \text{J / kg}}$$

In film pool boiling, heat is transferred due to both convection and radiation.

Heat transfer co-efficient, h = h_{conv} + 0.75 h_{rad} ……….(1)

$$h_{conv} = 0.62\left[\frac{k^3{}_v \times \rho_v \times (\rho_t - \rho_v) \times g \times \left[h_{fg} + 0.4\left(C_{Pv}\ \Delta T\right)\right]}{\mu_v D \Delta T}\right]^{0.25}$$

[From HMT data book page No. [42]

$$h_{conv} = 0.62\left[\frac{\left(32.68 \times 10^{-3}\right)^3 \times 5.16 \times (961 - 5.16) \times 9.81 \times \left[2256.9 \times 10^3 - (0.4 \times 2709 \times 160)\right]}{15.10 \times 10^{-6} \times 8 \times 10^{-3} \times 160}\right]^{0.25}$$

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$$h_{conv} = 0.62\left[\frac{4.10\times10^{6}}{1.93\times10^{-5}}\right]^{0.25}$$

$$\boxed{h_{conv} = 421.02\,W/m^{2}K} \qquad ....(2)$$

$$h_{rad} = \sigma \in \left[\frac{T^{4}_{w} - T^{4}_{sat}}{T_{w} - T_{sat}}\right] \qquad [\text{From HMT data book page no.142}]$$

$$h_{rad} = 5.67\times10^{-8}\times0.92\times\left[\frac{(260+273)^{4}-(100+273)^{4}}{(260+273)-(100+273)}\right]$$

[∴ Stefan boltzman constant, $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4]

$$\boxed{h_{rad} = 20W/m^{2}K} \qquad ....(3)$$

Substitute (2), (3) in (1)

(1) $\Longrightarrow$ h = 421.02 + 0.75(20)

$$\boxed{h = 436.02\,W/m^{2}K}$$

Heat transferred, Q = h A ($T_w - T_{sat}$)

= h × π × D × L ($T_w - T_{sat}$)

= 436.02 × π × 8 × 10^{-3} × 1 × (260 – 100)

Q = 1753.34 W/m [∵ L = 1m]

(or)

Power dissipation, P = 1753.34 W/m

Result:

Power dissipation, P = 1753.34 W/m.

2.Hot oil C_p = 2200 J/kg K is to be cooled by water (C_p = 4180 J/kg K) in a 2-shell- pass amd 12-tube- pass heat exchanger. The tubes are thin walled and are made of copper with a diameter of 1.8 cm. The length of each tube pass in the heat exchanger is 3m, and the overall heat transfer co-efficient is 340 W/m^{2}K. Water flows through the tubes at a total rate of 0.1 kg/s, and the oil through the shell at a rate of 0.2 kg/s. The water and the oil enter at temperatures 18℃ and 160℃ respectively. Determine the rate 0, heat transfer in the heat exchanger and the outlet temperatures of the water and the oil. (16)

Given Data:

In a 2-shell- pass and 12-tube pass heat exchanger, specific heat capacity of hot oil. C_{ph}= 2200 J/kg K

Specific heat capacity of water, C_{pc} = 4180 J/kgK

Diameter of copper tube, d = 1.8 cm = 0.018 m

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Length of heat exchanger L = 3m

Overall heat transfer co-efficient, $U = 340 W/m^2K$

Mass flow rate of water, $m_c = 0.1$ kg/s

Mass flow rate of oil, $m_h = 0.2$ kg/s

Inlet temperature of cooling water, $t_1 = 18°C$

Inlet temperature of hot oil, $T_1 = 160°C$

To find:

(i) Rate of heat transfer in the heat exchanger, Q

(ii) Outlet temperature of the water. T_2

(iii) Outlet temperature of hot oil, T_2.

Solution:

Capacity rate of hot liquid,

$$C_h = m_h \times C_{ph} = 0.2 \times 2200$$

$$C_h = 440 \text{ W/K}$$

Capacity rate of cooling water,

$$C_c = m_c \times C_{pc} = 0.1 \times 4180$$

$$C_c = 418 \text{ W/K}$$

Maximum possible heat transfer,

$$Q_{max} = C_{min} (T_1 - t_1)$$

$$= 418 (160 - 18)$$

$$Q_{max} = 5.935 \times 10^4 \text{ W}$$

$C_{min} = C_c = 418$ W/K

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$C_{max} = C_h = 440$ W/K

$$\frac{C_{min}}{C_{max}} = \frac{418}{440} = 0.95$$

$$\frac{C_{min}}{C_{max}} = 0.95$$

Surface Area, $A_s = 12 \text{ tube} \times A_s$

$$= 12 \times (\pi D \ L)$$

$= 12 \times (\pi \times 0.018 \times 2)$

$= 2.04 \text{ m}^2$

Number of transfer units, $\text{NTU} = \frac{UA}{C_{min}}$

[From HMT data book, page no. 152]

$$\text{NTU} = \frac{340 \times 2.04}{418} = 1.659$$

To find effectiveness ε, refer HMT data book page No. 165.

(2 shell – 12 tube pass heat exchanger)

From graph,

$X_{axis} \longrightarrow \text{NTU} \longrightarrow 1.659$

$\text{Curve} \rightarrow \frac{C_{min}}{C_{max}} = 0.95$

Corresponding Y_{axis} value is 58%

I.e., $\varepsilon = 0.58$

$Q = \varepsilon\, Q_{max} = 0.58\ (5.935 \times 10^4)$

$= 3.442 \times 10^4$ W

$Q = m_h C_{ph} (T_1 - T_2)$

$34423 = 0.2 \times 2200\ (160 - T_2)$

$T_2 = 160 - 78.23$

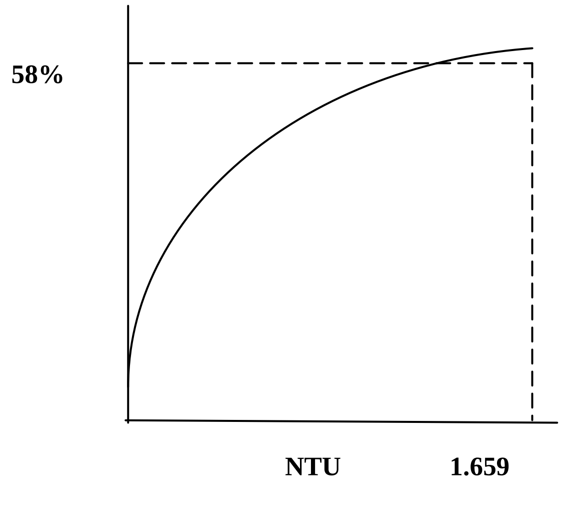
$T_2 = 81.76°C$ Effectiveness

$Q = m_c C_{pc} (t_2 - t_1)$

$34423 = 0.1 \times 4180\ (t_2 - t_1)$

$T_2 = 82.35 + 18$

$T_2 = 100.35°C$



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Result:

(i) Rate of heat transfer in the heat exchanger, Q = 34423 bW

(ii) Outlet temperature of the water, t_2 = 100 35°C

(iii) Outlet temperature of the hot oil, T_2 = 81.76°C

3.Boiling heat transfer phenomena

Boiling is a convection process involving a change of phase from liquid to vapour state. This is possible only when the temperature of the surface (T_w) exceeds the saturation temperature of liquid (T_{sat}).

According to convection law,

$$Q \qquad h\,A\,(T_w - T_{sat})$$

$$Q \qquad h\,A\,(\Delta T)$$

Where

$\Delta T = (T_w - T_{sat})$ is known as excess temperature.

If heat is added to a liquid from a submerged solid surface the boiling process referred to as pool boiling. In thios case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

Fig. 3.1 shows the temperature distribution in saturated pool boiling with a liquid – vapour interface.

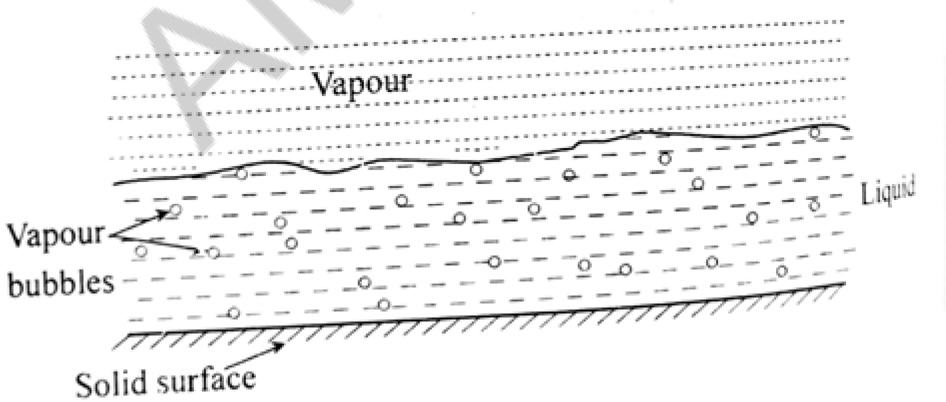

Fig.3.1 Pool Boiling

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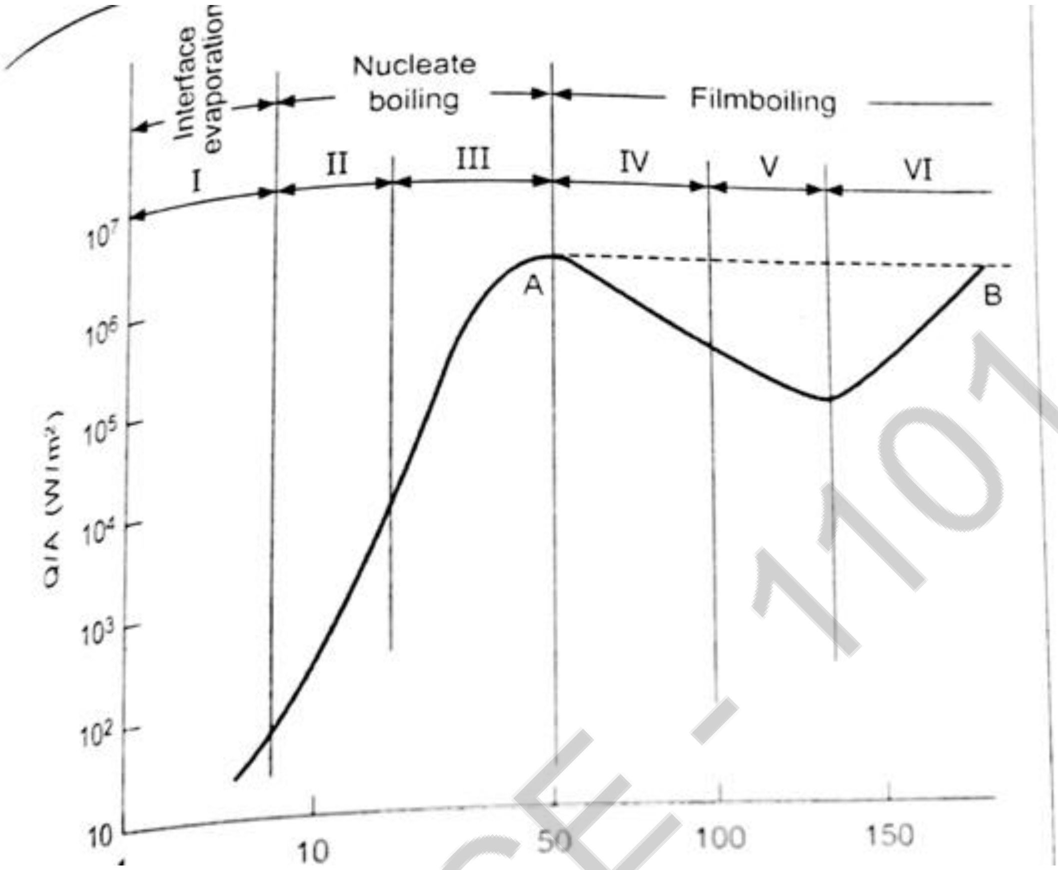
The different regions of boiling are indicated in fig. 3.2. This specific curve has been obtained from an electrically heated platinum wire submerged in a pool of water by varying its surface temperature and measuring the surface heat flux (q).



Excess Temperature $\Delta T_e = T_w - T_{sol}$

I – Free convection

II – Bubbles Condense in super heated liquid

III – Bubbles raise to surface

IV – Unstable film

V – Stable film

VI – Radiation coming into play

Fig. 3.2. pool Boiling Curve for water

4.What are the different types of fouling in heat exchangers?

The surfaces of a heat exchanger do not remain clean after has been in use for some time. The surface becomes fouls with scaling or deposits. The effect of these deposits affecting the value of overall heat transfer co-efficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

Types of fouling in heat exchanger:

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(i) Corrosion fouling,

(ii) Chemical fouling, and

(iii) Biological fouling, etc.

5. Hot exhaust gases which enter a cross-flow heat exchanger at 300℃ and leave at 100℃ are used to heat water at a flow rate of 1 kg/s from 35 to 125℃. The specific heat of the gas is 1000 J/kg. K and the overall heat transfer co-efficient based on the gas side surface is 100 W/m^2. K.

Find the required gas side surface area using the NTU method and LMTD method.

Given Data: In cross flow heat exchanger,

Hot Gas: (Hot fluid)

Inlet temperature, T_1 = 300℃

Outlet temperature, T_2 = 100℃

Water: (Cold fluid)

Inlet temperature, t_1 = 35℃

Outlet temperature, t_2 = 125℃

Overall heat transfer co-efficient, u = 100 W/m^{2}K

Specific heat of hot gas, C_{ph} = 1000 J/kgK

Specific heat of water, C_{pw} = 4186 J/kgK

To find: (i) Area (LMTD method)

(ii) Area (NTU method)

@Solution:

(i) LMTD method:

For cross-flow heat exchanger,

Logarithmic mean temperature difference, LMTD

$$= \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$(\Delta T)_{LMTD} = \frac{175 - 65}{\ln\left(\frac{175}{65}\right)} = \frac{110}{0.99} = 111.06$$

To find correction factor, F (both fluids unmixed):

[Refe3r HMT data book, Page No. 162]

From Graph,

$$\text{X} - \text{axis value, P} = \frac{t_2 - t_1}{T_1 - t_2}$$

$$= \frac{125 - 35}{300 - 35} = \frac{90}{265} = 0.34$$

$$\text{Curve value, R} = \frac{T_1 - T_2}{t_2 - t_1}$$

$$= \frac{300 - 100}{125 - 35} = \frac{200}{90} = 2.22$$

X- axis value is 0.34, curve value is 2.22, corresponding

Y – axis value is 0.87.

i.e, $\boxed{F = 0.87}$

$Q = m_c\, C_{pc}\, (t_2 - t_1)$

$= 1 \times 4186 \times (125 - 35)$

$Q = 376.74$ kW

We know that, $Q = U\, A_h\, F\, (\Delta T)_{LMTD}$

$376.74 \times 10^3 = 100 \times A_h \times 0.87 \times (111.06)$

$$\therefore \boxed{A_h = 38.99\,\text{m}^2}$$

(iii)NTU method:

$Q = m_h\, C_{phg}\, (T_1 - T_2)$

$376.74 \times 10^3 = m_h \times 1000\, (300 - 100)$

$\boxed{m_h = 1.883\,\text{kg/s}}$

$\boxed{m_C = 1\,\text{kg/s}}$

Capacity rate of hot liquid, $C_h = m_h \times C_{ph} = 1.883 \times 1000$

$C_{min} = C_h = 1883.7$ W/K

Capacity rate of cold liquid, $C_c = m_c \times C_{pc} \times = 1 \times 4186$

$C_{max} = C_c = 4186$ W/K

$$\frac{C_{min}}{C_{max}} = \frac{1883.7}{4186} = 0.45$$

$$Q_{max} = C_{min}\left(T_1 - t_1\right)$$
$$= 1883.7\left(300 - 35\right)$$
$$= 499180.5\text{W}$$

AMSCE - 1101

$$\text{Effectiveness}, \varepsilon = \frac{Q}{Q_{max}}$$
$$= \frac{m_c C_{pc}\left(t_2 - t_1\right)}{499180.5} = 0.75$$

To find NTU: **[Refer HMT data book, page no. 166]**

From graph,

$$\text{Curve} \rightarrow \frac{C_{min}}{C_{max}} = \frac{1883.7}{4186} = 0.45$$
$$Y - \text{axis} \rightarrow \varepsilon = 0.75$$

Corresponding X-axis value is NTU = 2.1

$$NTU = \frac{U_h A_h}{C_{min}}$$
$$2.1 = \frac{100 \times A_h}{1883.7}$$
$$\boxed{A_h = 39.55\,m^2}$$

6.Water is to be boiled at atmospheric pressure in a polished copper pan by means of an electric heater. The diameter of the pan is 0.38 m and is kept at 115℃. Calculate the following

1. Power required to boil the water

2. Rate of evaporation

3. Critical heat flux.

Given:

Diameter, d = 0.38 m;

Surface temperature, T_w = 115℃.

To find:

1. Power required, (P)

2. Rate of evaporation, (m)

3. Critical heat flux, $\left(\frac{Q}{A}\right)$

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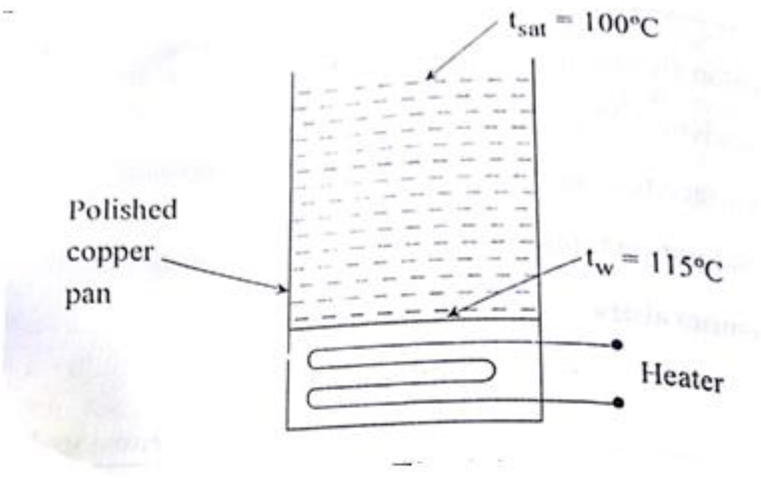

Fig. 3.3

Solution:

We know that, saturation temperature of water is 100℃

i.e. $\boxed{T_{sat} = 100^\circ C}$

Properties of water at 100℃.

[From HMT data book page no. 21, sixth edition]

Density, ρ_l = 961 kg/m^3

Kinematic viscosity, v = 0.293 × 10^{-6} m^2/s

Prandtl Number, P_r = 1.740

Specific heat, C_{pl} = 4216 J/kg k

Dynamic viscosity, $\mu_l = \rho_l \times v = 961 \times 0.293 \times 10^{-6}$

$= 281.57 \times 10^{-6}$ Ns/m^2

From steam Table [R.S. Khurmi Steam table page No 4]

At 100℃

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg

$H_{fg} = 2256.9 \times 10^3$ J/kg

Specific Volume of Vapour, v_g = 1.673 m^3/ kg

$$\text{Density of vapour}, \rho_v = \frac{1}{v_g}$$

$$= \frac{1}{1.673}$$

$$\boxed{\rho_v = 0.597\,\text{kg / m}^3}$$

ΔT = Excess temperature = $T_w - T_{sat}$ = 115° - 100 = 15℃

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$\boxed{\Delta T = 15^\circ C} < 50^\circ C.$ So this is Nucleate pool boiling process.

1. Power required to boil the water

For Nucleate pool boiling

$$\text{Heat flux}, \frac{Q}{A} = \mu_l \times h_{fg} \left[\frac{g \times (\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl} \times \Delta T}{C_{sf} \times h_{fg} P^n{}_r}\right]^3 \text{..........}(1)$$

[From HMT data book page no. 142 (sixth edition0]

Where, σ = Surface tension for Liquid vapour interface

At 100℃

$$\boxed{\sigma = 0.0588\, N/m} \qquad [\text{From HMT data book page No.144}]$$

$$\boxed{\text{For water} - \text{copper} \Rightarrow C_{sf} = \text{Surface fluid constant} = 0.013}$$

N=1 for water [From HMT data book page No. 143]

Substitute

$\mu_l, h_{fg}, \rho_l, \rho_v, \sigma, C_{pl}, \Delta T, C_{sf}, h_{fg}, n$ and P_r values in Equation (1)

$$(1) \Rightarrow \frac{Q}{A} = 281.57 \times 10^{-6} \times 2256.9 \times 10^3 \times \left[\frac{9.81 \times (961 - 0.597)}{0.0588}\right]^{0.5} \times \left[\frac{4216 \times 15}{0.013 \times 2256.9 \times 10^3 \times (1.74)^1}\right]^3$$

$$\text{Heat flux}, \frac{Q}{A} = 4.83 \times 10^5\, W/m^2$$

⇒ Heat transfer, $Q = 4.83 \times 10^5 \times A$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} d^2$$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} (0.38)^2$$

$$Q = 54.7 \times 10^3\, W$$

$$\Rightarrow \quad Q = 54.7 \times 10^3 = P$$

$$\Rightarrow \quad \boxed{\text{Power} = 54.7 \times 10^3\, W}$$

2. Rate of evaporation, (m)

We know that,

Heat transferred, $Q = m \times h_{fg}$

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$$\Rightarrow \qquad m = \frac{Q}{h_{fg}}$$

$$= \frac{54.7 \times 10^3}{2256.9 \times 10^3}$$

$$\boxed{m = 0.024\,kg/s}$$

3. critical heat flux

For Nucleate pool boiling, critical heat flux,

$$\frac{Q}{A} = 0.18 h_{fg} \times \rho_v \left[\frac{\sigma \times g \times (\rho_l - \rho_v)}{\rho^2{}_v} \right]^{0.25}$$

$$= 0.18 \times 2256.9 \times 10^3 \times 0.597 \times \left[\frac{0.0588 \times 9.81 \times (961 - 0.597)}{(0.597)^2} \right]^{0.25}$$

$$\frac{Q}{A} = 1.52 \times 10^6\ W/m^2$$

$$\boxed{\text{Critical Heat flux}, q = \frac{Q}{A} = 0.52 \times 10^6\,W/m^2}$$

Result:

1. $P = 54.7 \times 10^3$ W

2. m = 0.024kg/s

3. $\frac{Q}{A} = q = 1.52 \times 10^6\ W/m^2$.

(OR)

7. Calculate for the following cases, the surface area required for a heat exchanger which is required to cool 1200 kg hr of bezene (C_p 1.74 kJ/kg℃) from 72℃ ro 42℃. The cooling water (C_{po} 4.18 kJ/kg℃) at 15℃ has a flow rate of 200 kg/hr.

(i) Single pass counter-flow

(ii) 1- 4 exchanger (one-shell pas and four-tube passes and

(iii) Cross flow single pass with water mixed and benezene unmixed. Assume all the cases U = 0.28 kW/m^{2}K. (16)

Given:

Hot fluid-Benzene Cold fluid-Water

(T_1, T_2) (t_1, t_2)

Mass Flow rate of benzene, m_h = 3200 kg/h

= 0.889 kg/s

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Entry temperature of benzene, T_1 = 72°C

Exit temperature of benzene, T_2 = 42°C

Specific heat of benzene, C_{ph} = 1.74 kJ/kg°C

$= 1.74 \times 10^3$ J/kg°C

Specific heat water, C_{pc} = 4.18 kJ/kg°C

$= 4.18 \times 10^3$ J/kg°C

Entry temperature of water, t_1 = 15°C

Mass flow rate of water, m_c = 2200 kg/h i.e., 0.611 kg/s

Overall heat transfer co-efficient, U = 0.28 kW/m^{2}K

$= 0.28 \times 10^3$ W/m^{2}K

To find:

(i) Surface area for single pass counter flow.

(ii) Surface area for 1 – 4 exchanger.

(iii) Surface area for cross flow single pass with water mixed and benzene unmixed.

Solution:

Case (i):

Heat lost by benzene (Hot fluid)

= Heat gained by water (Cold fluid)

$Q_h = Q_c$

$M_h C_{ph} (T_1 - T_2) = m_c C_{pc} (t_2 - t_1)$

$\Rightarrow 0.889 \times 1.74 \times 10^3 (72 - 42) = 0.611 \times 4.18 \times 10^3 (t_2 - 15°)$

$\Rightarrow \boxed{t_2 = 33.2^\circ C}$

$\Rightarrow Q = m_h C_{ph} (T_1 - T_2)$ or $m_c C_{pc} (t_2 - t_1)$

$\Rightarrow Q = 0.889 \times 1.74 \times 10^3 (72 - 42)$

$\boxed{Q = 46.405 \times 10^3 W}$

We know that,

Heat transfer, $Q = U A (\Delta T)_m$ ……….(1)

[From HMT data book, page no. 151]

Where, $(\Delta T)_m$ = Logarithmic Mean Temperature

AMSCE - 1101

Difference (LMTD)

For counterflow,

$$(\Delta T)_m = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]}$$

$$= \frac{(72 - 33.2) - (42 - 15)}{\ln\left[\frac{72 - 33.2}{42 - 15}\right]}$$

$$\boxed{(\Delta T)_m = 32.5^\circ C}$$

Substitute $(\Delta T)_m$ and Q values in equation (t).

(1) ⟹ $$46.405 \times 10^3 = 0.28 \times 10^3 \times A \times (32.5^\circ)$$

$$\Rightarrow \boxed{A = 5.1 m^2}$$

Case (ii):

One shell pass and four tube passes,

To find correction factor F, refer HMT data book, page no. 158.

From graph,

$$X - \text{axis value}, P = \frac{t_2 - t_1}{T_1 - t_1} = 0.32$$

$$\text{Curve value}, R = \frac{T_1 - T_2}{t_2 - t_1} = 1.65$$

X-axis value is 0.32, curve value is 1.65, corresponding Y-axis value is (From graph) 0.9.

i.e., F = 0.9

⟹ $Q = F\,U\,A\,(\Delta T)_m$

$46.405 \times 10^3 = 0.9 \times 0.28 \times 10^3 \times A \times 32.5$

$$\Rightarrow \boxed{A = 5.66 m^2}$$

Case (iii):

Cross flow single pass with water mixed and benzene unmixed. To find correction factor, F, refer HMT data book, page no. 160.

From graph,

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$$\text{X}-\text{axis value}, P = \frac{t_2 - t_1}{T_1 - t_1}$$
$$= 0.32$$
$$\text{Curve value}, R = \frac{T_1 - T_2}{t_2 - t_1}$$
$$= 1.65$$

From graph, corresponding Y-axis value is 0.92,

i.e., $F = 0.92$

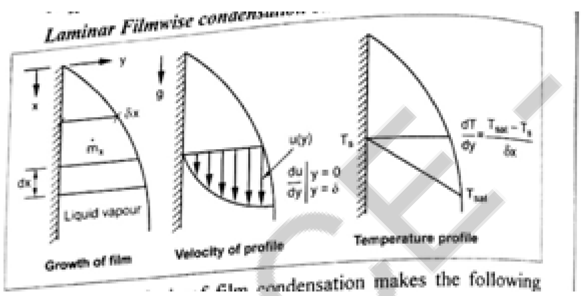
$\Rightarrow$ $Q = F\,U\,A\,(\Delta T)_m$

$46.405 \times 10^3 = 0.92 \times 0.28 \times 10^3 \times A \times 32.5$

$\Rightarrow$ $\boxed{A = 5.54\,\text{m}^2}$

8. Consider laminar film condensation of a stationary vapour on a vertical flat plate of length L and width b. Derive an expression for the average heat transfer co-efficient. State the assumptions mode.

Laminar Film wise condensation on a vertical plate:



Nusselt’s analysis of film condensation makes the following simplifying assumptiuons.

(1) The plate is maintained at a uniform temperarture T_s.

(2) Condensate flow is laminar.

(3) Fluid properties are constant.

(4) Shear stress at the liquid vapour interface is negligible.

(5) Acceleration of fluid within the condensate layer is neglected.

(6) Heat transfer is pure conduction and temperature distribution is linear.

The momentum equation is given by

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu_t}\frac{dp}{dx} - \frac{B_x}{\mu_l}$$

Where B_x is the body force in x-direction. The body force within the film is ρ_1 g, $\therefore \frac{dp}{dx} = \rho_v g$.

AMSCE - 1101

$$\therefore \frac{\partial^2 u}{\partial y^2} = -\frac{g}{\mu_1}(\rho_1 - \rho_v)$$

Integrating twice using u = 0 at y = 0 and $\frac{\partial u}{\partial y} = 0$ at $y = \delta$

$$u(y) = \frac{g(\rho_1 - \rho_v)\delta^2}{\mu_1}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

The condensate mass flow rate through any x-position of the film is given by

$$m(x) = \frac{g(\rho_1 - \rho_v)\delta^3}{3\mu_1}$$

Also $$dm = \frac{\rho_1(\rho_1 - \rho_v) g \delta^2 d\delta}{\mu_1}$$

The heat transfer at the wall is given by

$$dx.q_s = \frac{k_1(T_{sat} - T_s)dx}{\delta}$$

The heat removed by the wall is given by,

$$\delta^2 d\delta = \frac{k_1\mu_1(T_{sat} - T_s)}{g\rho_1(\rho_1 - \rho_v)h_{fg}} dx$$

The local heat transfer co-efficient may now be expressed as

$$h_x = \frac{k_1}{\delta(x)} \text{ or } h_x = \left[\frac{g\rho_1(\rho_1 - \rho_v)k^3{}_1 h_{fg}}{4\mu_1(T_{sat} - T_s)x}\right]^{0.25}$$

The average value of heat transfer co-efficient is given by

$$\bar{h}_L = \frac{1}{L}\int_0^{\alpha} h_x\, dx = \frac{4}{3} h_L$$

$$\boxed{\bar{h}_L = 0.943\left[\frac{g\rho_1(\rho_1 - \rho_v)k^3{}_1 h_{fg}}{\mu_1(T_{sat} - T_s)L}\right]^{1/4}}$$

9. what is meant by Fouling factor?

We know, the surfaces of a heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled was scaling or deposits. The effect of these deposits affecting the value of overall heat transfer co-efficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance .

10.In a cross flow both fluids unmixed heat exchanger, water at 6℃ flowing at the rate of 1.25 kg/s of air that is initially at a temperature of 50℃. Calculated the following

1. Exit temperature of air

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2. Exit temperature of water

Assume overall heat transfer co-efficient is 130 W/m^2-K and area in 23 m^2.

Given:

Cold fluid-water Hot fluid-air

Inlet temperature of water, t_l = 6°C

Mass flow rate of water, m_h = 1.2 kg/s

Initial temperature of air, T_l = 50°C

Overall heat transfer co-efficient, U = 130 W/m^2K

Surface area, A = 23m^2

To find:

1. Exit temperature of air, (T_2)

2. Exit temperature of water, (t_2)

Solution:

We know that,

Specific heat of water, C_{pc} = 4186 J/kg K

Specific heat of air, C_{-ph} = 1010J/kg K (Constant)

We know

Capacity rate of water

$C = m_c \times C_{pc}$

$= 1.25 \times 4186$

$$\boxed{C = 5232.5 W/k} \qquad (1)$$

Capacity rate of air

$$C = m_h \times C_{ph}$$
$$= 1.2 \times 1010$$
$$\boxed{C = 1212 W/K} \qquad (2)$$

From equation (1) and (2), we know that,

C_{min} = 1212 W/K

C_{max} = 5232.5 W/K

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$$\frac{C_{min}}{C_{max}} = \frac{1212}{5232.5} = 0.23$$

$$\boxed{\frac{C_{min}}{C_{max}} = 0.23} \qquad (3)$$

$$NTU = \frac{UA}{C_{min}} \qquad \text{[From HMT data book page no.151]}$$

Number of transfer units, $= \dfrac{130 \times 23}{1212}$

$$\boxed{NTU = 2.46} \qquad (4)$$

To find effectiveness ∈, refer HMT data book page No 165]

(Cross flow, both fluids unmixed)

From graph,

$X_{axis} \rightarrow NTU = 2.46$

Curve $\rightarrow \dfrac{C_{min}}{C_{max}} = 0.23$

Corresponding Y_{axis} value is 0.85

i.e, $\boxed{\in = 0.85}$

Maximum heat transfer

$$Q_{max} = C_{min}\left(T_1 - t_1\right)$$

$$= 1212\left(50 - 6\right)$$

$$\boxed{Q_{max} = 53.328\,W}$$

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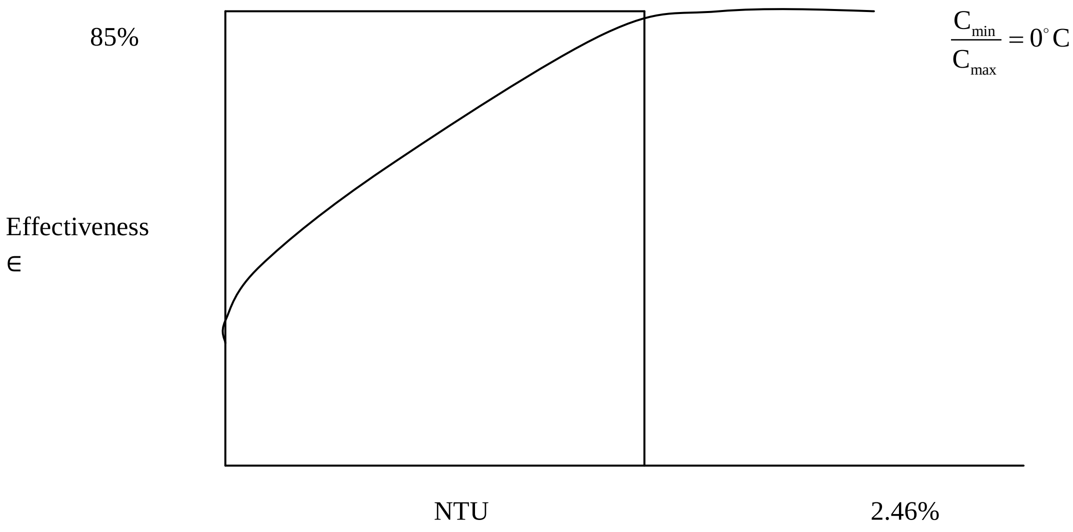

Actual heat transfer rate

$$Q = \in \times Q_{max}$$
$$= 0.85 \times 53,328$$
$$\boxed{Q = 45,328\,W}$$

Heat transfer, $\quad Q = m_c\, C_{pc}\,(t_2 - t_1)$

$45,328 = 1.25 \times 4186\,(t_2 - 6)$

$\Rightarrow \quad 45,328 = 5232.5\, t_2 - 31,395$

$\Rightarrow \quad t_2 = 14.6°C$

$$\boxed{\text{outlet temperature of water}, t_2 = 14.6^\circ C}$$

We know that,

$$\text{Heat transfer}, Q = m_h C_{ph}\left(T_1 - T_2\right)$$
$$45,328 = 1.2 \times 1010\left(50 - T_2\right)$$
$$\Rightarrow \quad 45.328 = 60,600 - 1212\, T_2$$
$$\Rightarrow \quad T_2 = 12.6^\circ C$$
$$\boxed{\text{Outlet temperature of air}, T_2 = 12.6^{\circ C}}$$

Result:

1. $T_2 = 12.6°C$

2. $t_2 = 14.6°C$

11. In a cross flow heat exchangers, both fluids unmixed, hot fluid with a specific heat of 2300 J/kg K enters at 380°C and leaves at 300°C. Cold fluids enters at 25°C and leaves at 210°C. Calculate the requirement surface area of heat exchanger. Take overall heat transfer co-efficient is 750 W/m²K. Mass flow rate of hot fluid is 1 kg/s.

Given:

Specific heat of hot fluid, $C_{ph} = 2300$ J/kg K

Entry temperature of hot fluid, $T_1 = 380°C$

Exit temperature of hot fluid, $T_2 = 300°C$

Entry temperature of cold fluid, $t_1 = 25°C$

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Exit temperature of cold fluid, $t_2 = 210°C$

Overall heat transfer co-efficient, $U = 750\ W/m^2K$

Mass flow rate of hot fluid, $m_h = 1\ kg/s$.

To find:

Heat exchanger area (A)

Solution:

This is cross flow, both fluids unmixed type heat exchanger

For cross flow heat exchanger,

$$Q = F\,U\,A\left(\Delta T\right)_{m[\text{counter flow}]} \quad (1)$$

[From HMT data book page no. 15] (sixth editiuon)

Where

F Correction factor

$(\Delta T)_m$ – Logarithmic mean temperature difference for counter flow

For Counter flow,

$$\left(\Delta T\right)_m = \frac{\left[\left(T_1 - t_2\right)\left(T_2 - t_1\right)\right]}{\ln\begin{bmatrix} T_1 & t_2 \\ T_2 & t_1 \end{bmatrix}}$$

$$= \frac{\left(380 - 210\right) - \left(300 - 25\right)}{\ln\left[\dfrac{380 - 210}{300 - 25}\right]}$$

$$\boxed{\left(\Delta T\right)_m = 218.3^\circ C}$$

$$\text{Heat transfer, } Q = m_h\, C_{ph}\left(T_1 - T_2\right)$$

$$\Rightarrow \qquad Q = 1 \times 2300\left(380 - 300\right)$$

$$\boxed{Q = 184 \times 10^3\, W}$$

To find correction factor F, refer HMT data book page no 161 (Sixth edition)

[Single pass cross flow heat exchanger – Both fluiods unmixed]

From graph,

$$X_{axis}\,\text{Value}\ P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{210 - 25}{380 - 25} = 0.52$$

$$\text{Curve value}\ R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{380 - 300}{210 - 25} = 0.432$$

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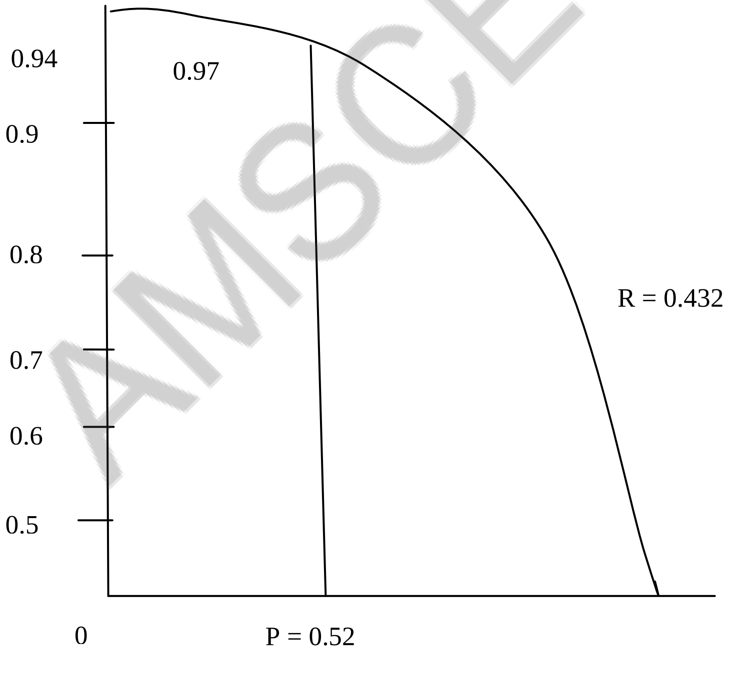
X_{axis} value is 0.52, curve value is 0.432, corresponding Y_{axis} value is 0.97,

i.e. $\boxed{F = 0.97}$



\Fig.3.15

Substitute Q, F, $(\Delta T)_m$ and U value in Equation (1)

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$$(1) \Rightarrow \quad Q = F\,U\,A\,(\Delta T)_m$$

$$184 \times 10^3 = 0.97 \times 750 \times A \times 218.3$$

$$\Rightarrow \quad \boxed{A = 1.15\,m^2}$$

Result:

Surface area, A = 1.15 m^2

12.) In a refrigerating plant water is cooled from 20℃ to 7℃ by heat solution entering at 2℃ and leaving at 3℃. The design heat loan is 5500 W and the overall heat transfer co-eficient is 800W/m^2k. What area required when using a shell and tube heat exchange with the water making one shell pass and the brine making is tube passes.

Given:

Hot fluid- water — Cold fluid- brine solution

(T_1-T_2) — ($t_1 - t_2$)

Entry temperature of water, T_1 - 20℃

Exit temperature of water, T_2 - 7℃

Entry temperature of brine solution, t_1 - 2℃

Exit temperature of brine solution, t_2 - 3℃

Heat load, Q = 5500 W

Overall heat transfer co-efficient, U = 800 W/m^2K

To find:

Area required (A)

Solution:

Shell and tube heat exchanger – One shell pass and two tube passes

For shell and tube heat exchanger (or) cross flow heat exchanger.

$$Q = F\,U\,A \times (\Delta T)_{m[\text{counter flow}]} \,.......(1)$$

[From HMT data book page No. 151]

Where

F – Correction factor

$(\Delta T)_m$ – Logaritmic mean temperature difference for counter flow.

Fpr counter flow,

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$$(\Delta T)_m = \frac{[(T_1 - t_2) - (T_2 - t_1)]}{\ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]}$$

$$= \frac{(20-3)-(7+2)}{\ln\left[\frac{20-3}{7+2}\right]}$$

$$\boxed{(\Delta T)_m = 12.57^\circ C}$$

To find correlation factor refer HMT data book page no. 158

[One shell pass and two tube passes]

$$X_{axis} \text{ value, P} \quad \frac{t_2 - t_1}{T_1 - T_2} = \frac{3+2}{20+2} = \frac{15}{22}$$

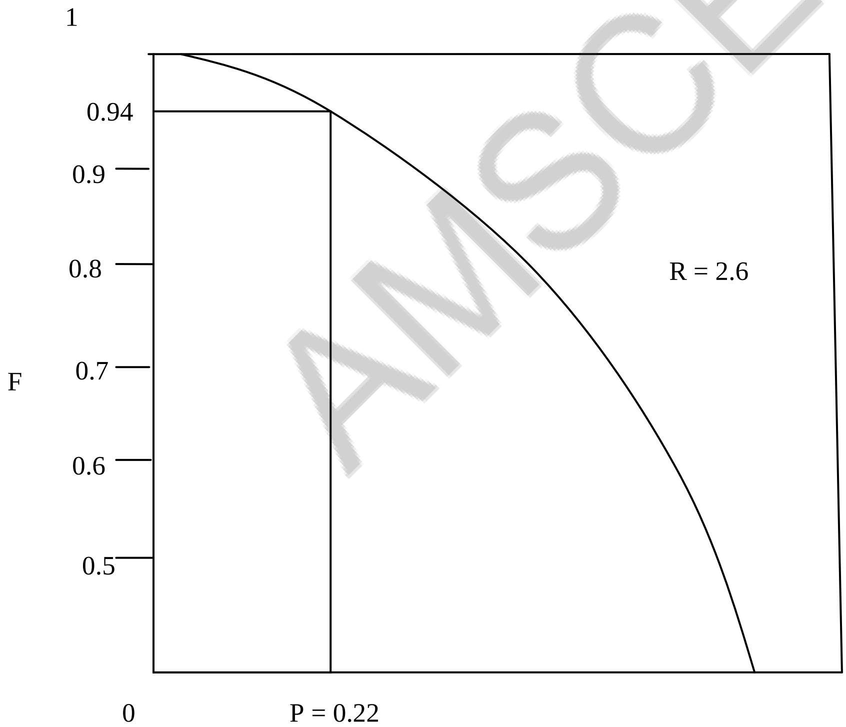
$$\boxed{P = 0.22}$$

$$\text{curve value, } R = \frac{T_1 - T_2}{t_2 + t_1} = \frac{20-7}{3+2} = \frac{13}{5}$$

$$\boxed{R = 2.6}$$

X_{axis} value is 0.22, curve value is 2.6, corresponding $Y_{axis\ value\ is\ 0.94}$,

i.e, $\boxed{F = 0.94}$



Substitute $(\Delta T)_m$, A, U and F value in Equation (1)

$(1) \Rightarrow \quad Q = F\,U\,A\,(\Delta T)_m$

$$5500 = 0.94 \times 800 \times A \times 12.57$$

$$\Rightarrow \quad \boxed{A = 0.58\,m^2}$$

Result:

Area of heat exchanger, A = 0.58 m^2.

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13. (a) (i) Discuss the different types of process for condensation of vapours on a solid surfaces.

(ii) What are the factors affecting Nucleate boiling.

(i) Different types of condensation:

Condensation occurs whenever a saturated vapour comes in contact with a surface at a lower temperature.

There are two modes of condensation:

- Filmwise condensation
- Dropwise condensation

Filmwise condensation:

The condensation wets the surface forming a continuous film which covers the entire surface.

Dropwise condensation:

The vapour condenses into small droplets of various size, which fall down the surface in a random fashion.

Filmwise condensation generally occurs on clean uncontaminated surface. In this type of condensation the film covering the entire surface grows in thickness as it moves down the surface by gravity. These exists a thermal gradient in the film and so it acts as a resistance to heat transfer.

In dropwise condensation a large portion of the area of the plate in directly exposed to the vapour, making heat transfer rates much higher than those in flimwise condensation.

Dropwise condensation can be obtained under controlled conditions with the help of certain additives to the condensate and various surface coating, but its commercial viability has not yet been proved. For this reason the condensation equipments in use are designed on the basis of filmwise condensation.

(ii) Factors affecting Nucleate boiling:

- Materials shape and condition of the heating surface
- Liquid properties
- Pressure
- Mechanical agitation.

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14. **A Counter flow heat exchanger is to heat air entering at 400°C with a flow rate of 6 kg/s by the exhaust gas entering at 800°C with a flow rate of 4 kg/s. The overall heat transfer coefficient is 100 W/m²K and the outlet temperature of air in 551.5°C. Specific heat of air Cp for both air and exhaust gas can be taken as 1100 J/kg K. Calculate.**

(i) Heat transfer area needed

(ii) Number of transfer units.

Given:

Counter flow heat exchanger

Temperature of air entering $T_1 = 400°C$

Mass flow rate of air $m_s = 6$ kg/s

Temperature of exhaust gas entering = 800°C

Mass flow rate of exhaust gas = 4 kg/s

Outlet temperature of air $T_2 = 551.5°C$

Specific heat of air and exhaust gas = 1100 J/kg K

To find:

1. Heat transfer area needed
2. Number of transfer units

Solution:

We know that,

$$\text{Capacity of air} = m_c \times C_{pc} = 6 \times 1100 = 6600$$

$$\text{Capacity of exhaust gas} = m_h \times C_{ph} = 4 \times 1100 = 4400$$

$$C_{min} = C_h = 4400$$

we know that,

Heat transferred to cold air = Heat transferred from hot gases

$$m_c C_{pc}(t_2 - t_1) = m_h C_{ph}(T_1 - T_2)$$

$$6600\,(551.5 - 400) = 4400\,(800 - T_2)$$

$$\frac{999900}{4400} = 800 - T_2$$

$$T_2 = 800 - 572.75$$

$$\boxed{T_2 = 572.75}$$

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$$Q = 999900 \text{ Joule} = 999.9 \text{ kJ}$$

$$\boxed{Q = 999.9 \text{ kJ}}$$

Heat transferred area needed [From HMT data book page no: 152]

$$Q = UA\,\Delta T_m \quad \text{--- (1)}$$

$$\Delta T_m \text{ for counter flow} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$\Delta T_m = \frac{(800 - 551.5) - (572.74 - 400)}{\ln\left(\frac{800 - 551.5}{572.74 - 400}\right)}$$

$$= \frac{75.76}{\ln\left(\frac{248.5}{172.7}\right)} = \frac{75.76}{0.3658} = 207.21°C$$

$$\boxed{\Delta T_m = 207.21°C}$$

Substitute ΔT_m, U, Q in equation (1)

$$Q = UA\,\Delta T_m$$

$$999900 = 100 \times A \times 207.21$$

$$A = \frac{999900}{100 \times 207.21} = 48 m^2$$

$$\boxed{A = 48 \text{ m}^2}$$

(ii) Number of transfer units (NTU)

$$NTU = \frac{UA}{C_{min}} = \frac{100 \times 48}{4400} = \frac{4800}{4400} = 1.09$$

$$\boxed{NTU = 1.09}$$

Result:

1. Heat transfer area needed, $A = 48 m^2$
2. Number of transfer unit = 1.09

15. **Water is boiling on a horizontal tube whose wall temperature is maintained ct 15°C above the saturation temperature of water. Calculate the nucleate boiling heat transfer coefficient. Assume the water to be at a pressure of 20 atm. And also find the change in value of heat transfer coefficient when**

1. **The temperature difference is increased to 30°C at a pressure of 10 atm.**
2. **The pressure is raised to 20 atm at Δ T = 15°C**

Given :

Wall temperature is maintained at 15°C above the saturation temperature.

$T_w = 115°C. \qquad \because T_{sat} = 100°C \; T_w = 100 + 15 = 115°C$

= p = 10 atm = 10 bar

case (i)

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$\Delta T = 30°C;\ p = 10$ atm = 10 bar

case (ii)

p = 20 atm = 20 bar; ΔT - 15°C

Solution:

We know that for horizontal surface, heat transfer coefficient

$h = 5.56\ (\Delta T)^3$ From HMT data book Page No.128

$h = 5.56\ (T_w - T_{sat})^3$

$= 5.56\ (115 - 100)^3$

$$\boxed{h = 18765\ w/m^2K}$$

Heat transfer coefficient other than atmospheric pressure

$h_p = hp^{0.4}$ From HMT data book Page No.144

$= 18765 \times 10^{0.4}$

$$\boxed{\text{Heat transfer coefficient } h_p = 47.13 \times 10^3\ W/m^2K}$$

Case (i)

P = 100 bar ΔT = 30°C From HMT data book Page No.144

Heat transfer coefficient

$$h = 5.56\ (\Delta T)^3 = 5.56(30)^3$$

$$\boxed{h = 150 \times 10^3\ W/m^2K}$$

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Heat transfer coefficient other than atmospheric pressure

$h_p = hp^{0.4}$

$= 150 \times 10^3 (10)^{0.4}$

$\boxed{h_p = 377 \times 10^3 \, W/m^2K}$

Case (ii)

P = 20 bar; ΔT = 15°C

Heat transfer coefficient h = 5.56 $(\Delta T)^3$ = 5.56 $(15)^3$

$\boxed{h = 18765 \ W/m^2K}$

Heat transfer coefficient other than atmospheric pressure

$h_p = hp^{0.4}$

$= 18765 \ (20)^{0.4}$

$\boxed{h_p = 62.19 \times 10^3 \ W/m^2K}$

16. **A vertical flat plate in the form of fin is 500m in height and is exposed to steam at atmospheric pressure. If surface of the plate is maintained at 60°C. calculate the following.**

1. The film thickness at the trailing edge
2. Overall heat transfer coefficient
3. Heat transfer rate
4. The condensate mass flow rate.

Assume laminar flow conditions and unit width of the plate.

Given :

Height ore length L = 500 mm = 5m

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Surface temperature $T^w = 60°C$

Solution

We know saturation temperature of water is 100°C

i.e. $T_{sat} = 100°C$

(From R.S. Khurmi steam table Page No.4

$h_{fg} = 2256.9 kj/kg$

$h_{fg} = 2256.9 \times 10^3$ j/kg

We know

$$\text{Film temperature } T_f = \frac{T_w + T_{sat}}{2}$$

$$= \frac{60 + 100}{2}$$

$$\boxed{T_f = 80°C}$$

Properties of saturated water at 80°C

(From HMT data book Page No.13)

ρ - 974 kg/m^3

$v = 0.364 \times 10^{-6}\ m^2/s$

$k = 668.7 \times 10^{-3}$ W/mk

$\mu = p \times v = 974 \times 0.364 \times 10^{-6}$

$$\boxed{\mu = 354.53 \times 10^{-6} Ns/m^2}$$

1. Film thickness δ_x

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We know for vertical plate

Film thickness

$$\delta x = \left(\frac{4\mu K \times x \times (T_{sat} - T_w)}{g \times h_{fg} \times \rho^2} \right)^{0.25}$$

Where

X = L = 0.5 m

$$\delta_x = \frac{4 \times 354.53 \times 10^{-6} \times 668.7 \times 10^{-3} \times 0.5 \times 100 - 60}{9.81 \times 2256.9 \times 10^3 \times 974^2}$$

$$\delta_x = 1.73 \times 10^{-4} m$$

2. Average heat transfer coefficient (h)

For vertical surface Laminar flow

$$h = 0.943 \left[\frac{k_3 \times \rho^2 \times g \times h_{fg}}{\mu \times L \times T_{sat} - T_w} \right]^{0.25}$$

The factor 0.943 may be replace by 1.13 for more accurate result as suggested by Mc Adams

$$1.13 \left(\frac{(668.7 \times 10^{-3})^3 \times (974)^2 \times 9.81 \times 2256.9 \times 10^3}{354.53 \times 10^{-6} \times 1.5 \times 100 - 60} \right)^{0.25}$$

$$h = 6164.3 \text{ W/m}^2\text{k.}$$

3. Heat transfer rate Q

We know

$$\begin{aligned} Q &= hA(T_{sat} - T_w) \\ &= h \times L \times W \times (T_{sat} - T_w) \\ &= 6164.3 \times 0.5 \times 1 \times 100\text{-}60 \end{aligned}$$

$$\boxed{Q = 123286 \text{ W}}$$

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4. Condensate mass flow rate m

We know

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}}$$

$$m = \frac{1.23.286}{2256.9 \times 10^3}$$

$$\boxed{m = 0.054 \text{ kg/s}}$$

17. Steam at 0.080 bar is arranged to condense over a 50 cm square vertical plate. The surface temperature is maintained at 20°C. Calculate the following.

a. **Film thickness at a distance of 25 cm from the top of the plate.**
b. **Local heat transfer coefficient at a distance of 25 cm from the top of the plate.**
c. **Average heat transfer coefficient.**
d. **Total heat transfer**
e. **Total steam condensation rate.**
f. **What would be the heat transfer coefficient if the plate is inclined at 30°C with horizontal plane.**

Given :

Pressure P = 0.080 bar

Area A = 50 cm × 50 cm = 50 × 050 = 0.25 m^2

Surface temperature T_w = 20°C

Distance x = 25 cm = .25 m

Solution

Properties of steam at 0.080 bar

(From R.S. Khurmi steam table Page no.7)

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$T_{satj/kg} = 41.53°C$

$h_{fg} = 2403.2 kj/kg = 2403.2\times10^{3} j/kg$

We know

$$\text{Film temperature } T_f = \frac{T_w + T_{sat}}{2}$$

$$= \frac{20+41.53}{2}$$

$$\boxed{T_f = 30.76°C}$$

Properties of saturated water at 30.76°C = 30°C

From HMT data book Page No.13

$\rho - 997\ kg/m^3$

$\nu = 0.83\times10^{-6}\ m^2/s$

$k = 612\times10^{-3} W/mK$

$\mu = p\times v = 997\times0.83\times10^{-6}$

$\mu = 827.51\times10^{-6} Ns/m^2$

a. Film thickness

We know for vertical surfaces

$$\delta x = \left(\frac{4\mu K\times x\times(T_{sat}-T_w)}{g\times h_{fg}\times\rho^2}\right)^{0.25}$$

(From HMT data book Page No.150)

$$\delta_x = \frac{4\times827.51\times10^{-6}\times612\times10^{-3}\times.25\times(41.53-20)100}{9.81\times2403.2\times10^{3}\times997^{2}}$$

$$\delta_x = 1.40\times10^{4} m$$

b. Local heat transfer coefficient h_x Assuming Laminar flow

$$h_x = \frac{k}{\delta x}$$

$$h_x = \frac{612 \times 10^{-3}}{1.46 \times 10^{-4}}$$

$$\boxed{hx = 4{,}191\ \text{W/m}^2\text{K}}$$

c. Average heat transfer coefficient h

(Assuming laminar flow)

$$h = 0.943 \left[\frac{k^3 \times \rho^2 \times g \times h_{fg}}{\mu \times L \times T_{sat} - T_w} \right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc adams

$$h = 0.943 \left[\frac{k^3 \rho^2 g\ h_{fg}}{\mu \times L \times T_{sat} - T_w} \right]^{0.25}$$

Where L = 50 cm = .5 m

$$h = 1.13 \left| \frac{(612 \times 10^{-3})^3 \times (997)^2 \times 9.81 \times 2403.2 \times 10^3}{827.51 \times 10^{-6} \times .5 \times 41.53 - 20} \right|^{0.25}$$

$$h = 5599.6\ \text{W/m}^2\text{k}$$

d. Heat transfer (Q)

We know

Q = hA(T_{sat} – T_w)

$$h \times A \times (T_{sat} - T_w)$$

$$= 5599.6 \times 0.25 \times (41.53 - 20$$

$$Q = 30.139.8\ \text{W}$$

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e. Total steam condensation rate (m)

We know

Heat transfer

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}}$$

$$m = \frac{30.139.8}{2403.2 \times 103}$$

$$m = 0.0125 \ kg/s$$

f. If the plate is inclined at θ with horizontal

$$h_{inclined} = h_{vertical} \times \sin\theta^{1/4}$$

$$h_{inclined} = h_{vertical} \times (\sin 30)^{1/4}$$

$$h_{inclined} = 5599.6 \times \left(\frac{1}{2} \right)^{1/4}$$

$$h_{inclined} = 4.708.6 \ W/m^2k$$

Let us check the assumption of laminar film condensation

We know

$$\text{Reynolds Number } R_e = \frac{4m}{w\mu}$$

where

W = width of the plate = 50cm = .50m

$$R_e = \frac{4 \times .0125}{0.50 \times 827.51 \times 10^{-6}}$$

$$R_e = 120.8 < 1800$$

So our assumption laminar flow is correct.

AMSCE - 1101

18. *A cross flow heat exchanger with both fluids unmixed is used to heat water flowing at a rate of 20 kg/s from 25°C to 75°C using gases available at 300°C to be cooled to 180°C. The overall heat transfer coefficient has a value of 95 W/m²K. Determine the area required. For gas c_p = 1005 J/kgK.*

Solution. The properties of gas can be taken as equal to that of air

Heat transfer rate $Q = 20 \times 4180\,(75 - 25) = 4.18 \times 10^6$ W

$$\textbf{LMTD counter flow} = \frac{(300-75)-(180-25)}{\ln\left(\dfrac{300-75}{180-25}\right)} = 187.831°\text{C}$$

To find correction factor F:

$$\text{P} = \frac{t_2 - t_1}{\text{T}_1 - t_1} = \frac{75-25}{300-25} = 0.1818$$

$$\text{R} = \frac{300-180}{75-25} = \frac{120}{50} = 2.4$$

Reading from chart Fig. 8.9 (*c*)

$F = 0.97$

$\therefore$ $A = Q/U.$ LMTD.

$A = 4.18 \times 10^6/95 \times 187.831 \times 0.97 = \mathbf{241.5\ m^2}$

Flow rate of air : $4.18 \times 10^6/1005 \times (300 - 180) = 34.66$ kg/s.

AMSCE - 1101

UNIT-4

RADIATION

PART-A

1. Define emissive power [E].

Ans: The emissive power is defined as the total amount of radiation emitted by a body per unit area. It is expressed in W/m^2.

2. What is meant by absorptivity ?

Ans: Absorptivity is defined as the ratio between radiation absorbed and incident radiation.

3. What is black body?

Ans: Black body is an ideal surface having the following properties.

A black body absorps all incident radiation, regardless of wave length and direction. For a prescribed temperature and wave length, no surface can emit more energy than black body.

4. State planck's distribution law.

Ans: The relationship between thee monochromatic emissive power of a black body and wave length of a radiation at a particular temperature is given by the following expression, by planck.

$$E_{b\lambda} = \frac{C_1\lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right)}-1}$$

Where $E_{b\lambda}$ = Monochromatic emissive power W/m^2

λ = wave length – m

$C_1 = 0.374 \times 10^{-15}$ W m^2

$C_2 = 14.4 \times 10^{-3}$ mK.

5. State Wien's displacement law.

Ans: The Wien's law gives the relationship between temperature and wavelength corresponding to the maximum spectral emissive power of the black body at that temperature.

$$\lambda_{mas} T = c_3$$

where $c_3 = 2.9 \times 10^{-3}$ [Radiation constant]

$\Rightarrow \lambda_{mas} T = 2.9 \times 10^{-3}$ mK.

6. State Stefan-Boltzmann law.

Ans: The emissive power of a black body is proportional to the fourth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

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Where E_b = Emissive power, w/m^2

σ = Stefan, Boltzmann constant

$= 5.67 \times 10^{-8}\ W/m^2\ K^4$

T = Temperature, K.

7. Define Emissivity.

Ans: It is defined as the ability of the surface of a body to radiate heat. It is also defined as the radio of emissive power of any body to the emissive power of a black body of equal temperature.

$$\text{Emissivity } \varepsilon = \frac{E}{E_b}$$

8. State Kirchoff's law of radiation.

Ans: This law states that the ratio of total emissive power to the absorptivity is constant for all surfaces which are in thermal equilibrium with the surroundings. This can be written as:

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}$$

It also states that the emissivity of the body is always equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

$\alpha_1 = E_1$; $\alpha_2 = E_2$ and so on.

9. Define intensity of radiation (I_b).

Ans: It is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

$$I_n = \frac{E_b}{\pi}$$

10. State Lambert's cosine law.

Ans: It states that the total emissive power E_b from a radiating plane surface in any direction proportional to the cosine of the angle of emission

$E_b \infty \cos\theta$.

11. What is the purpose of radiation shield?

Ans: Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the neet radiation transfer between two surface.

12. Define irradiation (G).

Ans: It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m^2.

13. What is meant by shape factor?

AMSCE - 1101

Ans: The shape factor is defined as the fraction of the radiative energy that is diffused from on surface element and strikes the other surface directly with no intervening reflections. It is represented by F_n . Other names for radiation shape factor are view factor, angle factor and configuration factor.

14. What is meant by reflectivity?

Ans: Reflectivity is defined as the ratio of radiation reflected to the incident radiation.

15. What is meant transmissivity?

Ans: Transmissivity is defined as the ratio of radiation transmitted to the incident radiation.

16. What is gray body?

Ans: If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

17. Define monochromatic emissive power.[E_{b2}]

Ans: The energy emitted by the surface at a given length per unit time per unit area in all directions is known as monochromatic emissive power.

18. Define emissivity.

Ans: It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of emission power of any body to the emissive power of a black body of equal temperature. Emissivity, * = E/E_b.

19. Define radiosity (J).

Ans: It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in W/m^2.

20. What is meant by shape factor and mention its physical significance?

Ans: The shape factor is defined as " The fraction of the radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections".

AMSCE - 1101

21. **The effective temperature of a body having an area of 0.12 m^2 is 527°C. Calculate the wavelength of the maximum monochromatic emissive power.**

Given:

Area of a body = 0.12 m^2

Effective temperature of a body = 527°C

To find:

Maximum wavelength of monochromatic emissive power

Solution:

From Wien's displacement law, we know that

$$\lambda_{max} \cdot T = 2898 \mu mK$$

[From HMT data book, Page No.82 (Eighth Edition)]

$$\lambda_{max} \cdot 800 = 2898$$

$$\lambda_{max} = \frac{2898}{800} = 3.622 \mu m$$

$$\boxed{\lambda_{max} = 3.622 \mu m}$$

Result:

22. Maximum wavelength of monochromatic emissive power = 3.622 μm

What are the properties of black body?

The properties of back body are

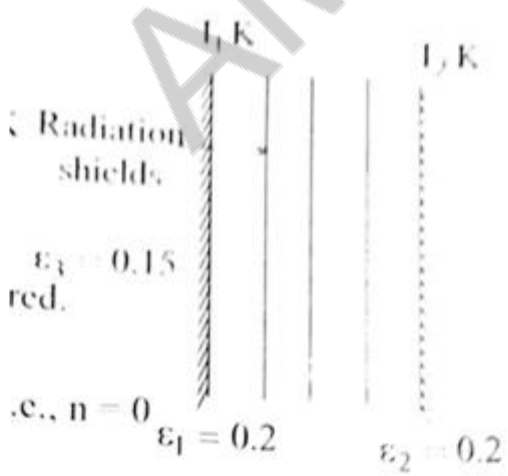
(i) It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wave length and direction.

(ii) It emits maximum amount of amount of thermal radiation at all wavelength at any specific temperature.

(iii) It is a diffuse emitter (the radiation emitted by a black body is independent of direction).

PART-B

1) Two very parallel plates are maintained at uniform temperature of T_1 = 100 K and T_2 = 800 K and have emissivities of $\varepsilon_1 = \varepsilon_2$ = 0.2 respectively. It is desired to reduce the net rate of radiation heat transfer between the two plates to one-fifth by placing thin aluminium sheets with an emissivity of 0.15 on both sides between the plates. Determine the number of sheets that need to be inserted. (10)



$\varepsilon_1 = 0.2$; $\varepsilon_2 = 0.2$

T_1 = 1000K : T

$\varepsilon_s = \varepsilon_3$ = 0.15

To find:

Number of shields required

Solution:

Heat transfer without shield, i.e, n = 0

$$\underset{(\text{no shield})}{Q_{12}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{1 \times 5.67 \times 10^{-8}\left(1000^4 - 800^4\right)}{\frac{1}{0.2} + \frac{1}{0.2} - 1}$$

$$= \frac{3.34 \times 10^4}{9}$$

$$= 0.37 \text{ x } 10^4$$

$$\underset{(\text{no shield})}{Q_{12}} = 3711.12\, \text{W/m}^2 \text{ ,}$$

we know that,

$$\frac{1}{5}\text{th of } \underset{(\text{no shield})}{Q_{12}} = \frac{1}{5} \times 3711.12 = 742.2\, \text{W/m}^2$$

Heat transfer with n shield is given by.

$$\underset{\text{with shield}}{Q_{12}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \left(\frac{2n}{\varepsilon_s}\right) - (n+1)}$$

$$742.4 = \frac{1 \times 5.67 \times 10^8 \times \left(1000^4 - 800^4\right)}{\frac{1}{0.2} + \frac{1}{0.2} + \frac{2n}{0.15} - (n+1)}$$

$$742.4\left[10 + \frac{2n}{0.15} - (n+1)\right] = 3.34 \times 10^4$$

$$7424 + \frac{1484.8n}{0.15} - 742.4n - 742.4 = 3.34 \times 10^4$$

1113.6 + 1484.8n – 111.36n – 111.36 = 0.15(3.34 x 10^4)

= 5010

1373.44 + 1002.24 = 5010

$$n = \frac{4007.76}{1373.44} = 2.91 \approx 3$$

Result :

Number of aluminium sheets = 3

2. Define the following terms:

(1) Monochromatic emissivity

(2) Gray body (3) Shape factor

It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of emissive power of any body to the emissive power of a black body of equal temperature.

$$\text{Emissivity, } \varepsilon = \frac{E}{E_b}$$

If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

The shape factor is defined as " The fraction of the radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflection". It is represented by F_{ij}. Other names for radiation shape factor are view factor, angle factor and configuration factor. The shape factor is used in the analysis of radiative heat exchange between two surfaces.

3) The spectral emissivity function of an opaque surface at 1000 K is approximated as

$\boldsymbol{\varepsilon_{\lambda 1} = 0.4, 0 \leq \lambda < 2\ \mu m;}$

$\boldsymbol{\varepsilon_{\lambda 2} = 0.7, 2\ \mu m \leq \lambda < 6\ \mu m;}$

$\boldsymbol{\varepsilon_{\lambda 3} = 0.3, 6\ \mu m \leq \lambda < \infty}$

Determine thee average emissivity of the surface and the rate of radiation emission from the surface, in W/m^2.

Given data :

$\varepsilon_1 = 0.4$: $\varepsilon_2 = 0.7$; $\varepsilon_3 = 0.3$

T = 1000K

$\lambda_1 = 2\ \mu m$

$\lambda_2 = 6\ \mu m$

To find :

(i) Average emissivity of the surface.

(ii) Rate of radiation emission from the surface, in W/m^2

Solution:

(i) Average emissivity of the surface,

AMSCE - 1101

$$\bar{\varepsilon} = \varepsilon_1 \frac{E_b(0-\lambda_1 T)}{\sigma T^4} + \varepsilon_2 \frac{E_b(\lambda_1-\lambda_2)}{\sigma T^4} + \varepsilon_3 \frac{E_b(\lambda_2-\lambda_\infty)}{\sigma T^4} \quad ....(1)$$

$\lambda_1 T$ = 2 x 1000 = 2000 μmK

λ_2 T = 6 x 1000 = 6000 μmK

λ_1 T = 2000 μm K, corresponding Fractional Emission.

From HMT data book, page. No.83 (seventh edition)

$$F_{\lambda 1} = \frac{E_b(0-\lambda_1 T)}{\sigma T^4} = 0.066728$$

$\lambda_2 T$ = 6000 μm K, corresponding Fractional Emission.

From HMT data book, page No.83 (seventh edition)

$$F_{\lambda 2} = \frac{E_b(0-\lambda_2 T)}{\sigma T^4} = 0.737818$$

$$\text{i.e,.} F_{\lambda 2} - F_{\lambda 1} = \frac{E_b(\lambda_1-\lambda_2)}{\sigma T^4} = 0.737818 - 0.066728$$
$$= 0.67109$$

$$\frac{E_b(\lambda_2-\lambda_\infty)}{\sigma T^4} = F_{\lambda\infty} - F_{\lambda 2} = 1 - F_{\lambda 2} \quad [\therefore F_{\lambda\infty} = 1]$$
$$= 1 - 0.73781$$
$$= 0.26219$$

Equation (1) $\Rightarrow$

$$\bar{\varepsilon} = \varepsilon_1(F_{\lambda 1}) + \varepsilon_2(F_{\lambda 2} - F_{\lambda 1}) + \varepsilon_3(1 - F_{\lambda 2})$$

$$\bar{\varepsilon} = 0.4(0.066728) + (0.7)(0.737818 - 0.066728) + 0.3(1 - 0.73781)$$
$$= 0.02669 + 0.46976 + 0.07865$$
$$\bar{\varepsilon} = 0.575$$

(ii) Rate of emission = $\bar{\varepsilon}\sigma T^4$

$$= 0.575(5.67\times10^{-8})(1000)^4$$

= 32.6 x 10^3 W/m^2

Result:

(i) Average emissivity of the surface, $\bar{\varepsilon} = 0.575$

(ii) Rate of emission = 32.6 KW / m^2

4) Emissivities of two large parallel plate maintained at 800°C and 300°C are 0.3 and 0.5 respectively. Find net radiant heat exchange per square metre for these plates. Find the percentage reduction in heat

AMSCE - 1101

transfer when a polished aluminium radiation shield of emissivity 0.06 is placed between them. Also find the temperature of the shield.

Given : $T_1 = 800°C + 273$

$= 1073$ K

$T_2 = 300°C + 273$

$= 573$ K

$\varepsilon_1 = 0.3$

$\varepsilon_2 = 0.5$

Shield emissivity, $\varepsilon_3 = 0.06$

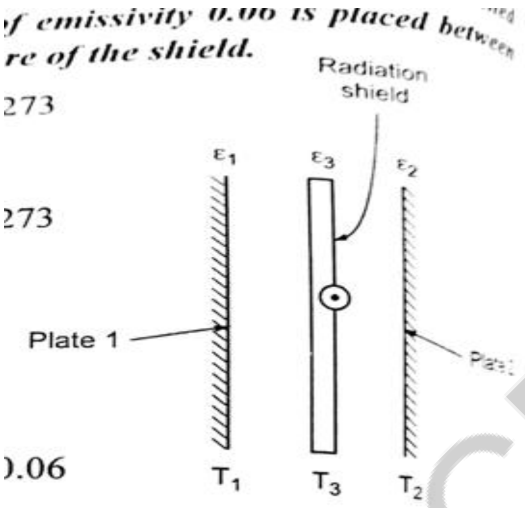


Fig. 4.13.

To find:

1. Net radiant heat exchange per square metre. (Q/A)

2. Percentage reduction in heat transfer due to radiation shield.

3. Temperature of the shield (T_3).

Solution: Heat exchange between two large parallel plates without radiation shield is given by

$$Q_{12} = \bar{\varepsilon}\sigma A\left[T_1^4 - T_2^4\right]$$

[From equation no.(4.28)]

Where, $$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{1}{\frac{1}{0.3}+\frac{1}{0.5}-1}$$

$$\boxed{\bar{\varepsilon} = 0.230}$$

AMSCE - 1101

$$Q_{12} = 0.230 \times \sigma \times A\left[T_1^4 - T_2^4\right]$$

$$\Rightarrow \qquad = 0.230 \times 5.67 \times 10^{-8} \times A \times \left[(1073)^4 - (573)^4\right]$$

$$\frac{Q_{12}}{A} = 15{,}880.7\,\text{W/m}^2 = 15.88\,\text{kW/m}^2$$

Heat transfer per square metre without radiation shield

$$\boxed{\frac{Q_{12}}{A} = 15.88\,\text{kW/m}^2} \qquad \ldots(1)$$

Heat exchange between plate 1 and radiation shield 3 is given by

$$\Rightarrow \quad Q_{13} = \bar{\varepsilon}\sigma A\left[T_1^4 - T_3^4\right]$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$\Rightarrow Q_{13} = \frac{\sigma \times A\left[T_1^4 - T_3^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3 - 1}} \qquad \ldots..(A)$$

Heat exchange between radiation shield 3 and plate 2 is given by

$$Q_{32} = \bar{\varepsilon}\sigma A\left[T_3^4 - T_2^4\right]$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow Q_{32} = \frac{\sigma \times A\left[T_3^4 - T_2^4\right]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2 - 1}} \qquad \ldots..(B)$$

We know that, $\qquad Q_{13} = Q_{32}$

$$\Rightarrow \frac{\sigma A\left[T_1^4 - T_3^4\right]}{\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_3} - 1} = \frac{\sigma A\left[T_3^4 - T_2^4\right]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow \frac{\left[T_1^4 - T_3^4\right]}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = \frac{\left[T_3^4 - T_2^4\right]}{\frac{1}{0.06} + \frac{1}{0.5} - 1}$$

$$\Rightarrow \frac{(1073)^4 - (T_3)^4}{19} = \frac{T_3^4 - (573)^4}{17.6}$$

$$\Rightarrow T_3^4 = \frac{17.6\left[(1073)^4 - (T_3)^4\right]}{19} + (573)^4$$

AMSCE - 1101

$$\Rightarrow T_3^4 = 0.926\left[(1073)^4 - (T_3)^4\right] + (573)^4$$

$$\Rightarrow T_3^4 = 0.926\times(1073)^4 - 0.926\times(T_3)^4 + (573)^4$$

$$(T_3)^4 + 0.926(T_3)^4 = 1.33\times10^{12}$$

$$(1.926)(T_3)^4 = 1.33\times10^{12}$$

$$(T_3)^4 = 6.90\times10^{11}$$

$$\boxed{T_3 = 911.5\,K}$$

$$\boxed{\text{Radiation shield temperature}, T_3 = 911.5\,K}$$

Substituting T_3 value in equation (A) (or) equation (B),

Heat transfer with radiation shield

$$\Rightarrow Q_{13} = \frac{5.67\times10^{-8}\times A\times\left[(1073)^4 - (911.5)^4\right]}{\frac{1}{0.3}+\frac{1}{0.06}-1}$$

$$\frac{Q_{13}}{A} = 1895.76\,W/m^2$$

Heat transfer with radiation shield

$$\Rightarrow \boxed{\frac{Q_{13}}{A} = 1.89\,kW/m^2} \qquad ....(2)$$

Reduction in heat transfer due to radiation shield

$$= \frac{Q_{\text{without shield}} - Q_{\text{with shield}}}{Q_{\text{without shield}}} = \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{15.88 - 1.89}{15.88}$$

$$= 0.88 = 88\%$$

Result:

1. Heat exchanger per square meter without radiation shield

Q_{12} = 15.88 kW/m^2

2. Percentage reduction in heat transfer = 88%

3. Temperature of radiation shield T_3 = 911.5 K

5) State and prove Kirchhoff's law of thermal radiation.

KIRCHOFF'S LAW OF RADIATION

The law states that the ratio of total emissive power to the absorptivity is constant for all surfaces which are in thermal equilibrium with the surroundings. This can be written as

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}.........$$

It also states that the emissivity of the body is always equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

$\alpha_1 = E_1$; $\alpha_2 = E_2$ and so on.

6) What is a black body? A 20 cm diameter spherical bull at 527° C is suspended in the air. The ball closely approximates a black body. Determine the total black body emissive power, and spectral black body emissive power at a wavelength of 3 μm

Black body : Refer Page no.4.3, section 4.6.

Given data: In sphere, (Black body)

Diameter of sphere, d = 20cm = 0.2 m

Temperature of spherical ball, T = 527°C + 273 = 800 K

To find:

(i) Total black body emissive power, E_b

(ii) Spectral black body emissive power at a wavelength 3 μm

Solution:

(i) Total black body emissive power, E_b

$$E_b = \sigma A T^4 = 5.67 \times 10^{-8} \times \pi \times (0.2)^2 \times (800)^4$$

$$= 23224.32 \times 0.12573$$

$$\boxed{E_b = 2920\,W}$$

(ii) Spectral black body emissive power: @ $\lambda = 3$ μm

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}$$

$$= \frac{0.374 \times 10^{-15}}{\left(3\times10^{-6}\right)^5\left[\exp\left(\frac{14.14\times10^{-3}}{3\times10^{-6}\times800}\right)-1\right]}$$

$E_{b\lambda} = 3824.3 \times 10^6$ W/m^2 (or) 3824.3 W/m^2 μm

7) An oven is approximated as a long equilateral triangular which has a heated surface maintained at a temperature of 1200 K. The other surface is insulated while third surface is at 500 K. The duct has a width of a 1 m a side and the heated and insulated surfaces have an emissivity of 0.8. The emissivity of the third surface is 0.4. Steady state operation find the rate at which energy must applied to the heated side per unit length of the duct to maintain its temperature at 1200 K. What is the temperature the insulated surface?

Solution: $A_1 = A_2 = A_R$

AMSCE - 1101

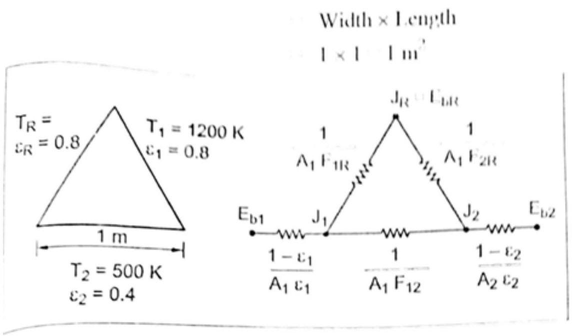


Fig. 3. Electrical network diagram

From electrical network diagram,

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.8}{0.8(1)} = 0.25$$

$$\frac{1-\varepsilon_R}{\varepsilon_R A_R} = \frac{1-0.8}{0.8(1)} = 0.25$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.4}{0.4(1)} = 1.5$$

We know that, $F_{11} + F_{12} + F_{1R} = 1$

From symmetric of three surface enclosure, from one surface shares two equal radiations into two different surfaces.

i.e., $F_{12} = 0.5$

But $F_{11} = 0$

$0 + 0.5 + F_{1R} = 1$

$$\boxed{F_{1R} = 0.5}$$

Similarly, $F_{21} + F_{22} + F_{2R} = 1$

$0.5 + 0 + F_{2R} = 1$

$$\boxed{F_{2R} = 0.5}$$

From electrical network diagram,

$$\frac{1}{A_1 F_{12}} = \frac{1}{0.5} = 2$$

$$\frac{1}{A_1 F_{1R}} = \frac{1}{0.5} = 2$$

$$\frac{1}{A_2 F_{2R}} = \frac{1}{0.5} = 2$$

From Stefan Boltzmann law,

$E_b = \sigma T^4$

$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8} \times (1200)^4 = 117573\ W/m^2$

AMSCE - 1101

$E_{b2} = \sigma T_2^4 = 5.67 \times 10^{-8} \times (500)^4 = 3544 \text{ W/m}^2$

$E_{bR} = J_R = \sigma T_R^4$

The radiosities J_1 and J_2 can be calculated by using Kirchoff's law.

At node J_1:

$$\frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{J_1 - J_R}{\frac{1}{A_1 F_{1R}}}$$

$$\frac{117573 - J_1}{0.25} = \frac{J_1 - J_2}{2} + \frac{J_1 - J_R}{2}$$

$$10J_1 - J_2 - J_R = 940584 \qquad \text{...(1)}$$

At node J_2 :

$$\frac{E_{b2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \frac{J_2 - J_1}{\frac{1}{A_2 F_{21}}} + \frac{J_2 - J_R}{\frac{1}{A_2 F_{2R}}}$$

$$\frac{3544 - J_2}{1.5} = \frac{J_2 - J_1}{2} + \frac{J_2 - J_R}{2}$$

$$-J_1 + 3.33 J_2 - J_R = 4725 \qquad \text{.......(2)}$$

At node J_R :

$$\frac{E_{bR} - J_R}{\frac{1-\varepsilon_3}{\varepsilon_3 A_R}} = \frac{J_R - J_1}{\frac{1}{A_R F_{R1}}} + \frac{J_R - J_2}{\frac{1}{A_R F_{2R}}}$$

$$0 = \frac{J_R - J_1}{2} + \frac{J_R - J_2}{2} \qquad \left[\therefore E_{bR} = J_R\right]$$

$$-J_1 - J_2 + 2J_R = 0 \qquad \text{.......(3)}$$

By solving equations (1), (2) and (3), we get

$J_1 = 1{,}08{,}339 \text{ W/m}^2$

$J_2 = 59{,}093 \text{ W/m}^2$

$J_R = 83{,}716.33 \text{ W/m}^2$

We know that,

$E_{bR} = J_R = \sigma T_R^4$

$$T_R = \left(\frac{83716.33}{5.67 \times 10^{-8}}\right)^{1/4}$$

$$\boxed{T_R = 1102.31\text{K}}$$

8) Derive wien's displacement law of radiation from planck's law. (8)

Solution:

AMSCE - 1101

We know that, planck's distribution law.

$$(E_\lambda)_b = \frac{c_1\lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right)-1}$$

$(E_\lambda)_b$ becomes maximum (if T remains constant)

$$\frac{d(E_\lambda)_b}{d\lambda} = 0$$

When

$$\Rightarrow \frac{d(E_\lambda)_b}{d\lambda} = \frac{d}{d\lambda}\left[\frac{c_1\lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right)-1}\right] = 0$$

$$\Rightarrow \frac{\left[\exp\left(\frac{c_2}{\lambda T}\right)-1\right]\left(-5c_1\lambda^{-6}\right)-c_1\lambda^{-5}\left[\text{ex}[\left(\frac{c_2}{\lambda T}\right)\frac{c_2}{T}\left(\frac{-1}{\lambda^2}\right)\right]}{\left[\exp\left(\frac{c_2}{\lambda T}\right)-1\right]^2} = 0$$

$$\Rightarrow -5c_1\lambda^{-6}\exp\left(\frac{c_2}{\lambda T}\right)+5c_1\lambda^{-6}+c_1c_2\lambda^{-5}\frac{1}{\lambda^2 T}\exp\left(\frac{c_2}{\lambda T}\right) = 0$$

Dividing both sides by 5 c_1 λ^{-6}, we get

$$-\exp\left(\frac{c_2}{\lambda T}\right)+1+\frac{1}{5}c_2\times\frac{1}{\lambda T}\exp\left(\frac{c_2}{\lambda T}\right) = 0$$

Solving this equation by trial and error method, we get

$$\frac{c_2}{\lambda T} = \frac{c_2}{\lambda_{max}T} = 4.965$$

$$\Rightarrow \lambda_{max}T = \frac{c_2}{4.965}$$

$$= \frac{1.439\times10^4}{4.965}\,\mu\text{mK}$$

$$= 2898\ \mu\text{mK}$$

$$\Rightarrow \boxed{\lambda_{max}T = 2.9\times10^{-3}\,\text{mK}}$$

9) A black body at 3000 K emits radiation.

Calculate the following :

1. Monochromatic emissive power at 1 μm wave length.

2. Wave length at which emission is maximum.

3. Maximum emissive power.

4. Total emissive power,

AMSCE - 1101

5. Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to 0.85.

Given : Surface temperature, T = 3000 K

To find :

1. Monochromatic emissive power $E_{b\lambda}$ at

   $\lambda = 1\ \mu = 1 \times 10^{-6}$ m.

2. Maximum wave length, (λ_{max}).

$\therefore$ Maximum emissive power. $(E_{b\lambda})_{max}$

4. Total emissive power, E_b.

5. Emissive power of real surface at $\varepsilon = 0.85$.

Solution:

1. Monochromatic Emissive Power:

From Planck's distribution law, we know that,

$$E_{b\lambda} = \frac{c_1 \lambda^{-5}}{e^{\left[\frac{c_2}{\lambda T}\right]} - 1}$$

[From HMT data book, page no.81]

Where $c_1 = 0.374 \times 10^{-15}$ W m^2

$c_2 = 14.4 \times 10^{-3}$ mK

$\lambda = 1 \times 10^{-6}$ m [Given]

$$\Rightarrow \quad E_{b\lambda} = \frac{0.374 \times 10^{-15} \left[1 \times 10^{-6}\right]^{-5}}{e^{\left[\frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 3000}\right]} - 1}$$

$$\boxed{E_{b\lambda} = 3.10 \times 10^{12}\ \text{W/m}^2}$$

2. Maximum wave length, (λ_{max}):

From wien's law, we know that,

$\lambda_{max}\, T = 2.9 \times 10^{-3}$ mK

$$\Rightarrow \quad \lambda_{max} = \frac{2.9 \times 10^{-3}}{3000}$$

$$\boxed{\lambda_{max} = 0.966 \times 10^{-6}\ \text{m}}$$

AMSCE - 1101

3. Maximum emissive power $(E_{b\lambda})_{max}$:

Maximum emissive power

$(E_{b\lambda})_{max}$ = $1.307 \times 10^5\ I^5$

$= 1.307 \times 10^5 \times (3000)^5$

$$\boxed{(E_{bl})_{max} = 3.17 \times 10^{12}\ W/m^2}$$

4. Total emissive power (E_b) :

From Stefan-Boltzmann law, we know that

$E_b = \sigma T^4$

[From HMT data book, page no 81]

Where σ = Stefan-Boltzman constant

$= 5.67 \times 10^{-8}\ W/m^2\ K^4$

$\Rightarrow E_b = (5.67 \times 10^{-8}) \times (3000)^4$

$$\boxed{E_b = 4.59 \times 10^6\ W/m^2}$$

5. Total emissive power of a real surface:

$(E_b)_{real} = \varepsilon\ \sigma\ T^4$

Where ε - Emissivity = 0.85

$(E_b)_{real} = 0.85 \times 5.67 \times 10^{-8} \times (3000)^4$

$$\boxed{(E_b)_{real} = 3.90 \times 10^6\ W/m^2}$$

Result:

1. $E_{b\lambda} = 3.10 \times 10^{12}\ W/m^2$
2. $\lambda_{max} = 0.966 \times 10^{-6}$ m
3. $(E_{b\lambda})_{max} = 3.17 \times 10^{12}\ W/m^2$
4. $E_b = 4.59 \times 10^6\ W/m^2$
5. $(E_b)_{real} = 3.90 \times 10^6\ W/m^2$

10)Two parallel plates of size 1 m x 1 m are spaced 0.5 m apart are located in a very large room, the walls of which are maintained at a temperature of 27°C. One plate is maintained at a temperature of 900°C and the other at 400°C. Their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and to the room. Consider only the plate surfaces facing each other.

Solution:

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Size of the plates = 1 m × 1 m

Distance between plates = 0.5 m

Room temperature, T_3 = 27°C + 273 = 300 K

First plate temperature, T_1 = 900°C + 273 = 1173 K

Second plate temperature, T_2 = 400°C + 273 = 673 K

Emissivity of first plate, $\varepsilon_1 = 0.2$

Emissivity of second plate, $\varepsilon_2 = 0.5$

To find: 1. Net heat transfer to each side plate.

2. Net heat transfer to room.

Solution: In this problem, heat exchange take place between two plates and the room. So, this is three surface problem and the corresponding radiation network is given below.

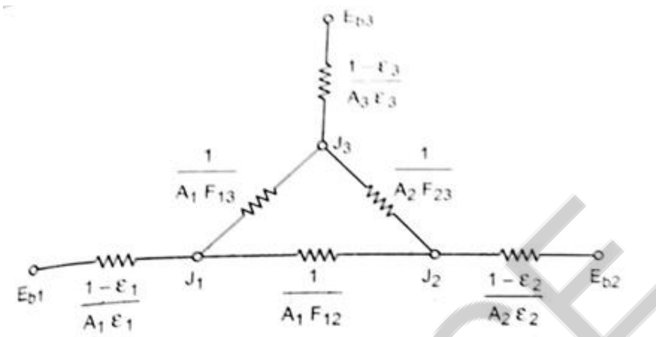

Fig. 4.62. Electrical network diagram

Area, $A_1 = 1 \times 1 = 1\ m^2$

$$\Rightarrow \quad \boxed{A_1 = A_2 = 1 m^2}$$

Since the room is large, $A_3 = \infty$

From electrical network diagram.

$$\frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.2}{1\times 0.2} = 4$$

$$\frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.5}{1\times 0.5} = 1$$

$$\frac{1-\varepsilon_3}{A_3\varepsilon_3} = 0 \qquad [\because A_3 = \infty]$$

Apply $\frac{1-\varepsilon_1}{A_1\varepsilon_1} = 4, \frac{1-\varepsilon_2}{A_2\varepsilon_2} = 1, \frac{1-\varepsilon_3}{A_3\varepsilon_3} = 0$ values in electrical

AMSCE - 1101

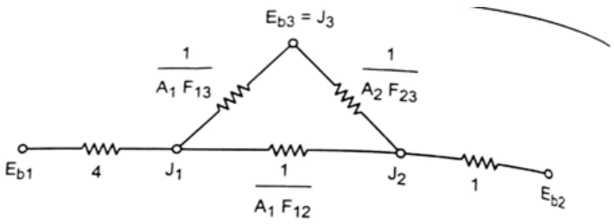

Fig.4.63 Electrical network diagram

To find shape factor F_{12}, refer HMT data book page no.91 & 92 (sixth edition).

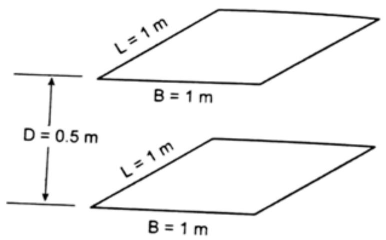

Fig.4.64

$$X = \frac{L}{D} = \frac{1}{0.5} = 2$$
$$Y = \frac{B}{D} = \frac{1}{0.5} = 2$$

X value is 2, Y value is 2. From that, we can find corresponding shape factor value is 0.41525.

i.e., $F_{12} = 0.41525$

we know that,

$$F_{11} + F_{12} + F_{13} = 1$$

But $F_{11} = 0$

$\Rightarrow$ $F_{13} = 1 - F_{12}$

$\Rightarrow$ $F_{13} = 1 - 0.41525$

$$\boxed{F_{13} = 0.5847}$$

Similarly, $F_{21} + F_{22} + F_{23} = 1$

We know that, $F_{22} = 0$

$\Rightarrow$ $F_{23} = 1 - F_{21}$

$= 1 - F_{12}$

$= 1 - 0.41525$

$$\boxed{F_{23} = 0.5847}$$

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From electrical network diagram,

$$\frac{1}{A_1F_{13}} = \frac{1}{1\times 0.5847} = 1.7102$$

$$\frac{1}{A_2F_{23}} = \frac{1}{1\times 0.5847} = 1.7102$$

$$\frac{1}{A_1F_{12}} = \frac{1}{1\times 0.41525} = 2.408$$

From Stefan-Boltzman law,

$$E_b = \sigma T^4$$

$$E_{b1} = \sigma T_1^4$$

$$= 5.67 \text{ x } 10^{-8}[1173]^4$$

$$\boxed{E_{b1} = 107.34\times 10^3 \text{ W/m}^2}$$

$$= 5.67 \text{ x } 10^{-8}[300]^4$$

$$E_{b3} = 459.27 \text{ W/m}^2$$

From electrical network diagram, we know that,

$$\boxed{E_{b3} = J_3 = 459.27 \text{ W/m}^2}$$

The radiosities J_1 and J_2 can be calculated by using Kirchoff's law.

$\Rightarrow$ The sum of current entering the node J_1 is zero.

At Node J_1 :

$$\frac{E_{b1}-J_1}{4} + \frac{J_2 - J_1}{\frac{1}{A_1F_{12}}} + \frac{E_{b3}-J_1}{\frac{1}{A_1F_{13}}} = 0$$

[From electrical network diagram]

$$\Rightarrow \frac{107.34\times 10^3 - J_1}{4} + \frac{J_2 - J_1}{2.408} + \frac{459.27 - J_1}{1.7102} = 0$$

$$\Rightarrow 26835 - \frac{J_1}{4} + \frac{J_2}{2.408} - \frac{J_1}{2.408} + 268.54 - \frac{J_1}{1.7102} = 0$$

At Node J_2:

$$\frac{J_1 - J_2}{\frac{1}{A_1F_{12}}} + \frac{E_{b3}-J_2}{\frac{1}{A_2F_{23}}} + \frac{E_{b2}-J_2}{2} = 0$$

$$\Rightarrow \quad \frac{J_1 - J_2}{2.408} + \frac{459.27 - J_2}{1.7102} + \frac{11.63\times 10^3 - J_2}{2} = 0$$

AMSCE - 1101

$$\Rightarrow \frac{J_1}{2.408} - \frac{J_2}{2.408} + 268.54 - \frac{J_2}{1.7102} + \frac{11.63 \times 10^3}{2} - \frac{J_2}{2} = 0$$

$$\Rightarrow \left.\begin{array}{r} 0.415J_1 - 0.415J_2 + 268.54 - 0.5847J_2 \\ 5.815 \times 10^3 - 0.5J_2 \end{array}\right\} = 0$$

$\Rightarrow 0.415J_1 - 1.499J_2 + 6.08 \times 10^3 = 0$

$\Rightarrow \quad 0.415J_1 - 1.4997J_2 = -6.08 \times 10^3 \quad ....(2)$

Solving equation (1) and (2),

$$\begin{array}{r} -1.2497J_1 + 0.415J_2 = -27.10 \times 10^3 \\ 0.415J_1 \quad -1.4997J_2 = -6.08 \times 10^3 \\ \hline \text{By solving,} \quad J_2 = 11.06 \times 10^3 \ \text{W/m}^2 \\ J_1 = 25.35 \times 10^3 \ \text{W/m}^2 \end{array}$$

Heat lost by plate (1), $Q_1 = \dfrac{E_{b1} - J_1}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}}$

[From electrical network diagram]

$$= \frac{107.34 \times 10^3 - 25.35 \times 10^3}{\dfrac{1-0.2}{1 \times 0.2}}$$

$$\boxed{Q_1 = 20.49 \times 10^3 \, \text{W}}$$

Heat lost by plate (1), $Q_1 = \dfrac{E_{b2} - J_2}{\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$

$$= \frac{11.63 \times 10^3 - 11.06 \times 10^3}{\dfrac{1-0.5}{1 \times 0.5}}$$

$$\boxed{Q_2 = 570 \text{W}}$$

$$\left.\begin{array}{l} \text{Total heat lost by the} \\ \text{plates (1) and (2)} \end{array}\right\} Q = Q_1 + Q_2$$

$$= 20.49 \times 10^3 + 570$$

$$\boxed{Q = 21.06 \times 10^3 \, \text{W}}$$

$$\left.\begin{array}{l} \text{Total heat received or} \\ \text{absorbed by the room} \end{array}\right\} Q = \frac{J_1 - J_3}{\dfrac{1}{A_1F_{13}}} + \frac{J_2 - J_3}{\dfrac{1}{A_2F_{23}}}$$

AMSCE - 1101

$$= \frac{25.35\times10^3 - 459.27}{1.7102} + \frac{11.06\times10^3 - 459.27}{1.7102}$$

$$\left[\because E_{b3} = J_3 = 459.27\,\mathrm{W/m^2}\right]$$

$$\boxed{Q = 20.752\times10^3\ \mathrm{W}}$$

[**Note:** Heat lost by the plates is equal to heat received by the room.]

Result: 1. Net heat lost by each plates

$Q_1 = 20.49 \times 10^3$ W

$Q_2 = 570$ W

2. Net heat transfer to the room

$Q = 20.752 \times 10^3$ W

11) Distinguish between irradiation and radiosity. (4)

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m^2.

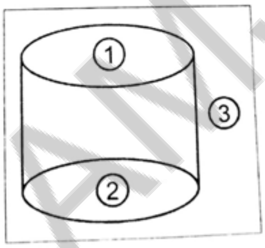
It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in W/m^2.

11) ii Consider a cylindrical furnace with outer radius = height = 1m. The top (surface 1) and the base (surface 2) of the furnace have emissivities 0.8 & 0.4 and are maintained at uniform temperatures of 700 K and 500 K respectively. The side surface closely approximates a black body and is maintained at a temperature of 400 K. Find the net rate of radiation heat transfer at each surface during steady state operation. Assume the view factor from the base to the top surface as 0.38. (12)

The view factor F_{1-3} can be obtained from summation rule.

$F_{1-1} + F_{1-2} + F_{1-3} = 1$

$F_{1-3} = 1 - 0 - 0.38 = 0.62$



Radiation resistance,

$$R_1 = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.8}{0.8\times3.141} = 0.0796\mathrm{m}^{-2}$$

$$R_2 = \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.4}{0.4\times3.141} = 0.4777\mathrm{m}^{2}$$

$$R_{1-2} = \frac{1}{A_2 F_{1-2}} = \frac{1}{3.141\times0.3} = 0.8381\mathrm{m}^{-2}$$

$$R_{1-3} = \frac{1}{A_1 F_{13}} = \frac{1}{3.141\times0.62} = 0.5137\mathrm{m}^{-2}$$

Substituting the numerical values,

$$\frac{13614-J_1}{0.0796}+\frac{J_2-J_1}{0.8881}+\frac{1452-J_1}{0.5137}=0 \quad ...(1)$$

$$\frac{354-J_2}{0.4777}+\frac{J_1-J_2}{0.5137}+\frac{1543-J_1}{0.8381}=0 \quad ....(2)$$

Solving these equations

$$\boxed{J_1=11418\,W/m^2} \qquad \boxed{J_2=4562\,W/m^2}$$

Net rate of radiation heat transfer for top surface (1) and base surface (2).

$$Q_1=\frac{Eb_1-J_1}{R_1}=\frac{13614-11418}{0.0796}\Rightarrow\boxed{Q_1=27588\,W}$$

$$Q_2=\frac{Eb_2-J_2}{R_2}=\frac{3544-4562}{0.4777}\Rightarrow\boxed{Q_2=-2131\,W}$$

Net rate of heat transfer at cylindrical surface

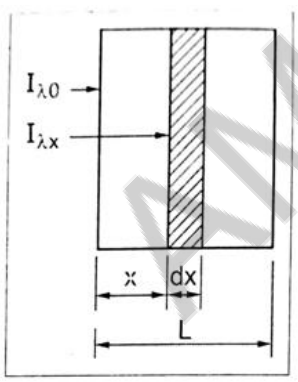
$$Q_3=\frac{J_3-J_1}{R_1-3}+\frac{J_3-J_2}{R_2-3}$$

$$=\frac{1452-11418}{0.5137}+\frac{1452-4562}{0.5137}\Rightarrow\boxed{Q_3=-25455\,W}$$

12) Considering radiation in gases, derive the exponential decay formula.

Exponential – Decay formula:

Consider absorption of thermal radiation by a gas layer.



$I_{\lambda 0}$ is radiation intensity at the left face and $I_{\lambda x}$ propagates in a gas layer is proportional to the thickness dx.

$$dI_\lambda(x) = -m_\lambda I_\lambda(x)\,dx$$

where the proportionality constant m_λ is spectral absorption coefficient of gas. Integrating both sides

$$\int_{I_{\lambda 0}}^{I_{\lambda L}}\frac{dI_\lambda(x)}{I_\lambda(x)}=-m_\lambda\int_0^L dx \quad \text{or} \quad \ln\left(\frac{I_{\lambda l}}{I_{\lambda 0}}\right)=-m_\lambda L$$

or $\boxed{I_{\lambda L}=I_{\lambda 0}\exp(-m_\lambda L)}$

AMSCE - 1101

The radiation intensity $I_{\lambda L}$ decreases exponentially with thickness of gas layer.

13) Two very large parallel plates with emissivities 0.5 exchange heat. Determine the percentage reduction in the heat transfer rate if a polished aluminium radiation shield of ε = 0.04 is placed in between the plates.

Given:

Emissivity of plate 1, $\varepsilon_1 = 0.5$

Emissivity of plate 2, $\varepsilon_2 = 0.5$

Emissivity of radiation shield, $\varepsilon_3 = 0.04 = \varepsilon_s$

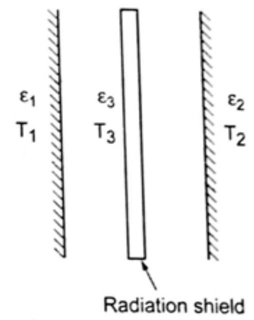


Fig.4.23.

To find: Percentage of reduction in heat transfer due to radiation shield.

Solution:

Case 1 : Heat transfer without radiation shield:

Heat exchange between two large parallel plates without radiation shield is given by,

$$Q_{12} = \bar{\varepsilon}\sigma A\left[T_1^4 - T_2^4\right]$$

Where, $$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{1}{\frac{1}{0.5} + \frac{1}{0.5} - 1}$$

$$\boxed{\bar{\varepsilon} = 0.333}$$

$$Q_{12} = 0.333\sigma A\left[T_1^4 - T_2^4\right]$$

$$Q_{\text{without shield}} = 0.333\sigma A\left[T_1^4 - T_2^4\right] \;.....(1)$$

Case 2 : Heat transfer with radiation shield :

We know that,

AMSCE - 1101

Heat transfer with n shield,

$$Q_{\text{with shield}} = \frac{A\,\sigma\left[T_1^4 - T_2^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_s} - (n+1)}$$

Where, ε_s – Emissivity of radiation shield.

n - Number of radiation shield.

$$Q_{\text{with shield}} = \frac{A\,\sigma\left[T_1^4 - T_2^4\right]}{\frac{1}{0.5} + \frac{1}{0.5} + \frac{2(1)}{0.04} - (1+1)}$$

$$\Rightarrow \qquad = \frac{A\,\sigma\left[T_1^4 - T_2^4\right]}{52}$$

$$Q_{\text{with shield}} = 0.0192\,A\,\sigma\left[T_1^4 - T_2^4\right] \quad ....(2)$$

We know that,

$$\left.\begin{array}{l}\text{Reduction in heat transfer}\\ \text{due to radiation shield}\end{array}\right\} = \frac{Q_{\text{without shield}} - Q_{\text{with shield}}}{Q_{\text{without shield}}}$$

$$= \frac{0.333A\,\sigma\left[T_1^4 - T_2^4\right] - 0.0192\,A\,\sigma\left[T_1^4 - T_2^4\right]}{0.333\,A\,\sigma\left[T_1^4 - T_2^4\right]}$$

$$= \frac{0.333 - 0.0192}{0.333}$$

= 0.942 = 94.2%

Result : Percentage of reduction in heat transfer rate

14. Two black square plates of size 2 by 2 m are placed parallel to each other at a distance of 0.5 m. One plate is maintained at a temperature of 1000°C and the other at 500°C. Find the heat exchange between the plates.

Given: Area A = 2 × 2 = 4 m^2

T_1 = 1000°C + 273

= 1273 K

T_2 = 500°C + 273

= 773 K

Distance = 0.5 m

To find : Heat transfer (Q)

Solution : We know Heat transfer general equation is

AMSCE - 1101

where $Q_{12} = \dfrac{\sigma\left[T_1^4 - T_2^4\right]}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_1\varepsilon_2}}$ [From equation No.(6)]

For black body $\varepsilon_1 = \varepsilon_2 = 1$

$$\Rightarrow Q_{12} = \sigma[T_1^4 - T_2^4] \times A_1F_{12}$$
$$= 5.67 \times 10^{-8}\left[(1273)^4 - (773)^4\right] \times 4 \times F^{12}$$
$$\boxed{Q_{12} = 5.14 \times 10^5 F_{12}} \quad ......(1)$$

Where F_{12} – Shape factor for square plates

In order to find shape factor F_{12}, refer HMT data book, Page No.76.

$$\text{X axis} = \frac{\text{Smaller side}}{\text{Distance between planes}}$$
$$= \frac{2}{0.5}$$
$$\boxed{\text{X axis} = 4}$$

Curve $\rightarrow$ 2 [Since given is square plates]

X axis value is 4, curve is 2. So corresponding Y axis value is 0.62.

i.e., $\boxed{F_{12} = 0.62}$

$$(1) \Rightarrow Q_{12} = 5.14 \times 10_5 \times 0.62$$
$$\boxed{Q_{12} = 3.18 \times 10^5 \text{ W}}$$

15. Two parallel plates of size 3 m × 2 m are placed parallel to each other at a distance of 1 m. One plate is maintained at a temperature of 550°C and the other at 250°C and the emissivities are 0.35 and 0.55 respectively. The plates are located in a large room whose walls are at 35°C. If the plates located exchange heat with each other and with the room, calculate.

1. Heat lost by the plates.

2. Heat received by the room.

Given: Size of the plates = 3 m × 2 m

Distance between plates = 1 m

First plate temperature T_1 = 550°C + 273 = 823 K

Second plate temperature T_2 = 250°C + 273 = 523 K

Emissivity of first plate ε_1 = 0.35

AMSCE - 1101

Emissivity of second plate ε_2 = 0.55

Room temperature $T_3 = 35°C + 273 = 308$ K

To find: 1. Heat lost by the plates

2. Heat received by the room.

Solution: In this problem, heat exchange takes place between two plates and the room. So this is three surface problems and the corresponding radiation network is given below. Area $A_1 = 3 \times 2 = 6$ m^2

$$\boxed{A_1 = A_2 = 6m^2}$$

Since the room is large $\boxed{A_3 = \infty}$

From electrical network diagram.

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.35}{0.35 \times 6} = 0.309$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.55}{0.55 \times 6} = 0.136$$

$$\frac{1-\varepsilon_3}{\varepsilon_3 A_3} = 0 \qquad [\because A_3 = \infty]$$

Apply $\frac{1-\varepsilon_3}{\varepsilon_3 A_3} = 0$, $\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 0.309$, $\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 0.136$ values in electrical network diagram.

To find shape factor F_{12} refer HMT data book, Page No.78.

$$X = \frac{b}{c} = \frac{3}{1} = 3$$

$$Y = \frac{a}{c} = \frac{2}{1} = 2$$

X value is 3, Y value is 2, corresponding shape factor [From table]

$F_{12} = 0.47$

$$\boxed{F_{12} = 0.47}$$

We know that,

$F_{11} + F_{12} + F_{13} = 1$ But, $F_{11} = 0$

$$\Rightarrow F_{13} = 1 - F_{12}$$

$$\Rightarrow F_{13} = 1 - 0.47$$

$$\boxed{F_{13} = 0.53}$$

AMSCE - 1101

Similarly, $F_{21} + F_{22} + F_{23} = 1$ We know $F_{22} = 0$

$$\Rightarrow \quad F_{23} = 1 - F_{21}$$

$$\Rightarrow \quad F_{23} = 1 - F_{12}$$

$$F_{13} = 1 - 0.47$$

$$\boxed{F_{23} = 0.53}$$

From electrical network diagram,

$$\frac{1}{A_1F_{13}} = \frac{1}{6 \times 0.53} = 0.314 \qquad ....(1)$$

$$\frac{1}{A_2F_{23}} = \frac{1}{6 \times 0.53} = 0.314 \qquad ....(2)$$

$$\frac{1}{A_1F_{12}} = \frac{1}{6 \times 0.47} = 0.354 \qquad ....(3)$$

From Stefan – Boltzmann law, we know

$$E_b = \sigma\ T^4$$

$$E_{b1} = \sigma T_1^4$$

$$= 5.67 \times 10^{-8} \left[823\right]^4$$

$$\boxed{E_{b1} = 26.01 \times 10^3 W/m^2} \qquad .....(4)$$

$$E_{b2} = \sigma\ T_2^4$$

$$= 5.67 \times 10^{-8} \left[823\right]^4$$

$$\boxed{E_{b2} = 4.24 \times 10^3 W/m^2} \qquad .....(5)$$

$$E_{b3} = \sigma T_3^4$$

$$= 5.67 \times 10^{-8} \left[308\right]^4$$

$$\boxed{E_{b3} = J_3 = 510.25\ W/m^2} \qquad .....(6)$$

[From diagram]

The radiosities, J_1 and J_2 can be calculated by using Kirchoff's law.

$\Rightarrow$ The sum of current entering the node J_1 is zero.

At Node J_1:

AMSCE - 1101

$$\frac{E_{b1}-J_1}{0.309}+\frac{J_2-J_1}{\frac{1}{A_1F_{12}}}+\frac{E_{b3}-J_1}{\frac{1}{A_1F_{13}}}=0$$

[From diagram]

$$\Rightarrow \quad \frac{26.01\times10^3-J_1}{0.309}+\frac{J_2-J_1}{0.354}+\frac{510.25-J_1}{0.314}=0$$

$$\Rightarrow \quad 84.17\times10^3-\frac{J_1}{0.309}+\frac{J_2}{0.354}+\frac{J_1}{0.354}+1625-\frac{J_1}{0.354}=0$$

$$\Rightarrow \quad -9.24J_1+2.82J_2=-85.79\times10^3 \quad .....(7)$$

At node j_2

$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}}+\frac{E_{b3}-J_2}{\frac{1}{A_2F_{23}}}+\frac{E_{b2}-J_2}{0.136}=0\text{-+*}$$

$$\frac{J_1-J_2}{0.354}+\frac{510.25-J_2}{0.314}+\frac{4.24\times10^3-J_2}{0.136}=0$$

$$\frac{J_1}{0.354}-\frac{J_2}{0.354}+\frac{510.25}{0.314}-\frac{J_2}{0.314}+\frac{4.24\times10^3}{0.136}-\frac{J_2}{0.136}=0$$

$$\Rightarrow \quad 2.82J_1-13.3J_2=-32.8\times10^3 \quad ....(8)$$

Solving equation (7) and (8),

$$\Rightarrow \quad -9.24J_1+2.82J_2=-85.79\times10^3 \quad .....(7)$$

$$\Rightarrow \quad 2.82J_1-13.3J_2=-32.8\times10^3 \quad .....(8)$$

$$\boxed{\begin{aligned}J_2&=4.73\times10^3\,W/m^2\\ J_1&=10.73\times10^3\,W/m^2\end{aligned}}$$

Heat lost by plate (1) is given by

$$Q_1=\frac{E_{b1}-J_1}{\left(\frac{1-\varepsilon_1}{\varepsilon_1A_1}\right)}$$

$$Q_1=\frac{26.01\times10^3-10.73\times10^3}{\frac{1-0.35}{0.35\times6}}$$

$$\boxed{Q_1=49.36\times10^3\ W}$$

AMSCE - 1101

Heat lost by plate 2 is given by

$$Q_2 = \frac{E_{b2} - J_2}{\left(\frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)}$$

$$Q_2 = \frac{4.24\times10^3 - 4.73\times10^3}{\frac{1-0.55}{6\times0.55}}$$

$$\boxed{Q_2 = -3.59\times10^3 \text{ W}}$$

Total heat lost by the plates

$$Q = Q_1 + Q_2$$

$$= 49.36 \times 10^3 - 3.59 \times 10^3$$

$$\boxed{Q = 45.76\times10^3 \text{ W}} \qquad \text{......(9)}$$

Heat received by the room

$$Q = \frac{J_1 - J_3}{\frac{1}{A_1F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_1F_{12}}}$$

$$= \frac{10.73\times10^3 - 510.25}{0.314} = \frac{4.24\times10^3 - 510.25}{0.314}$$

$$[\because E_{b1} = J_1 = 512.9]$$

$$\boxed{Q = 45.9\times10^3 \text{ W}} \qquad \text{.....(10)}$$

From equation (9), (10), we came to know heat lost by the plates is equal to heat received by the room.

16. **A gas mixture contains 20% CO_2 and 10% H_2o by volume. The total pressure is 2 atm. The temperature of the gas is 927°C. The mean beam length is 0.3 m. Calculate the emissivity of the mixture.**

Given : Partial pressure of CO_2, P_{CO_2} = 20% = 0.20 atm

Partial pressure of H_2o, $P_{H_2 0}$ = 10% = 0.10 atm.

Total pressure P = 2 atm

Temperature T = 927°C + 273

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= 1200 K

Mean beam length L_m = 0.3 m

To find: Emissivity of mixture (ε_{mix}).

Solution : To find emissivity of CO_2

$$P_{CO_2} \times L_m = 0.2 \times 0.3$$

$$\boxed{P_{CO_2} \times L_m = 0.06 \text{ m - atm}}$$

From HMT data book, Page No.90, we can find emissivity of CO_2.

From graph, Emissivity of CO_2 = 0.09

$$\boxed{\varepsilon_{CO_2} = 0.09}$$

To find correction factor for CO_2

Total pressure, P = 2 atm

$$P_{CO_2} L_m = 0.06 \text{ m - atm.}$$

From HMT data book, Page No.91, we can find correction factor for CO_2

From graph, correction factor for CO_2 is 1.25

$$\boxed{C_{CO_2} = 1.25}$$

$$\varepsilon_{CO_2} \times C_{CO_2} = 0.09 \times 1.25$$

$$\boxed{\varepsilon_{CO_2} \times C_{CO_2} = 0.1125}$$

To find emissivity of H_2o :

$$P_{H_2o} \times L_m = 0.1 \times 0.3$$

$$\boxed{P_{H_2o} L_m = 0.03 \text{ m - atm}}$$

From HMT data book, Page No.92, we can find emissivity of H_2o.

From graph Emissivity of H_2o = 0.048

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$$\boxed{\varepsilon_{H_2o} = 0.048}$$

To find correction factor for H_2o:

$$\frac{P_{H_2o} + P}{2} = \frac{0.1 + 2}{2} = 1.05$$

$$\frac{P_{H_2o} + P}{2} = 1.05,$$

$$P_{H_2o} \ L_m = 0.03 \text{ m - atm}$$

From HMT data book, Page No.92 we can find emission of H_20

17. **Two black square plates of size 2 by 2 m are placed parallel to each other at a distance of 0.5 m. One plate is maintained at a temperature of 1000°C and the other at 500°C. Find the heat exchange between the plates.**

Given: Area A = 2 × 2 = 4 m^2

T_1 = 1000°C + 273 = 1273 K

T_2 = 500°C + 273 = 773 K

Distance = 0.5 m

To find : Heat transfer (Q)

Solution : We know Heat transfer general equation is

where $$Q_{12} = \frac{\sigma\left[T_1^4 - T_2^4\right]}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_1\varepsilon_2}}$$

[From equation No.(6)]

For black body $\varepsilon_1 = \varepsilon_2 = 1$

$$\Rightarrow \ Q_{12} = \sigma[T_1^4 - T_2^4] \times A_1F_{12}$$

$$= 5.67 \times 10^{-8}\left[(1273)^4 - (773)^4\right] \times 4 \times F^{12}$$

$$\boxed{Q_{12} = 5.14 \times 10^5 F_{12}} \quad ......(1)$$

Where F_{12} – Shape factor for square plates

In order to find shape factor F_{12}, refer HMT data book, Page No.76.

AMSCE - 1101

$$\text{X axis} = \frac{\text{Smaller side}}{\text{Distance between planes}}$$

$$= \frac{2}{0.5}$$

$$\boxed{\text{X axis} = 4}$$

Curve $\rightarrow$ 2 [Since given is square plates]

X axis value is 4, curve is 2. So corresponding Y axis value is 0.62.

i.e., $\boxed{F_{12} = 0.62}$

$$(1) \Rightarrow Q_{12} = 5.14 \times 10_5 \times 0.62$$

$$\boxed{Q_{12} = 3.18 \times 10^5 \text{ W}}$$

From graph,

Correction factor for H_2o = 1.39

$$\boxed{C_{H_2O} = 1.39}$$

$$\varepsilon_{H_2O} \times C_{H_2O} = 0.048 \times 1.39$$

$$\boxed{\varepsilon_{H_2O} \times C_{H_2O} = 0.066}$$

Correction factor for mixture of CO_2 and H_2O:

$$\frac{P_{H_2o}}{P_{H_2o} + P_{CO_2}} = \frac{0.1}{0.1 + 0.2} = 1.05$$

$$\boxed{\frac{P_{H_2o}}{P_{H_2o} + P_{CO_2}} = 0.333}$$

$$P_{CO_2} \times L_m + P_{H_2O} \times L_m = 0.06 + 0.03$$

$$\boxed{P_{CO_2} \times L_m + P_{H_2O} \times L_m = 0.09}$$

From HMT data book, Page No.95, we can find correction factor for mixture of CO_2 and H_2o.

AMSCE - 1101

18.i) **The filament of a 75 W light bulb may be considered a black body radiating into a black enclosure at 70°C. The filament diameter is 0.10 m and length in 5 cm. considering the radiation, determine the filament temperature.**

Given:

Capacity of filament = 75 W

Temperature of black enclosure = 70°C

Diameter of the filament= 0.10 m

Length of the filament = 5 cm = 0.05 m

To find:

Filament temperature

Solution:

Heat exchange between two black enclosures

$$Q = \bar{\varepsilon} A \sigma \left(T_1^4 - T_2^4\right) \qquad \text{Assume} = \varepsilon = 1 \text{ for black body}$$

$$75 = 5.67\times10^{-8}\times1\times\pi\times0.1\times0.05\left(T_1^4 - 343^4\right)$$

$$\left(T_1^4 - 343^4\right) = \frac{75}{5.67\times10^{-8}\times\pi\times0.1\times0.05}$$

$$\boxed{T_1 = 3029 \text{ K}}$$

$$t_1 = 3029 - 273 = 2756°C$$

Result:

Filament temperature = 2756°C

ii) **Explain the following:**

(i) Total Emissive power

(ii) Stefan–Boltzmann law

(iii) Definition of Geometric factor, and air expression for the geometric factor $F_{1,2}$ for the inside surface of a black hemispherical cavity of

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(i) Total Emissive Power:

It is defined as the total amount of radiation emitted by a body per unit area and time. It is expressed in W/m^2.

(ii) Stefan's Boltzmann Law:

It states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

$$E = \sigma T^4$$

(iii) Geometric factor:

It is defined as the fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflection.

Geometric factor F_{1-1} for the inside surface of a black hemispherical cavity of radius R.

With respect to itself

$$F_{1-1} = 1 - \left(\frac{A_2}{A_1}\right)$$

$$= 1 - \left(\frac{\pi R_2}{2\pi R_1}\right)$$

$$= 1 - \frac{1}{2}$$

$$\boxed{F_{1-1} = 0.5}$$

19. *A solar concentrator causes a heat flux of 2000 W/m² on tube of 60 mm ID. Pressurised water flows through the tube at a rate of 0.01 kg/s. If the bulk temperature at inlet is 20°C, what will be the length required to heat the water to a bulk temperature of 80°C. Also find the wall temperature at exit.*

Solution: Bulk mean temperature = (80 + 20)/2 = 50°C.

The property values are

$$\rho = 990,\ \nu = 0.5675 \times 10^{-6},\ Pr = 3.68,\ k = 0.64 \text{ W/mK},\ c = 4181 \text{ J/kg K}.$$

$$\text{Re} = \frac{4G}{\pi D\mu} = \frac{4 \times 0.01}{\pi \times 0.06 \times 990 \times 0.5675 \times 10^{-6}} = 377.7$$

$\therefore$ **Flow is laminar.**

$$\pi DLq = mc\,\Delta T_m, \quad \therefore \quad L = \frac{mc\,\Delta T_m}{\pi Dq}$$

$$L = \frac{0.01 \times 4181 \times (80 - 20)}{\pi \times 0.06 \times 2000} = 6.65 \text{ m.}$$

Assuming fully developed condition,

$$Nu = 4.364, \quad \therefore \quad h = \frac{4.364 \times 0.64}{0.06} = 46.55 \text{ W/m}^2\text{ K}$$

To find the temperature at exit :

$$T_{so} = \frac{q}{h} + T_{mo} = \frac{2000}{46.55} + 80 = 122.97°\text{C}.$$

20.

AMSCE - 1101

A 4 kg/s product stream from a distillation column is to be cooled by a 3 kg/s water stream in a counterflow heat exchanger. The hot and cold stream inlet temperatures are 400 K and 300 K respectively, and the area of the exchanger is 30 m^2. If the overall heat transfer coefficient is estimated to be 820 W/m^2K, determine the product stream outlet temperature, if its specific heat is 2500 J/kgK and the coolant outlet temperature.

Solution:

The effectiveness of counterflow heat exchanger is given by

$$\varepsilon = \frac{1 - \exp[-NTU(1-R)]}{1 - R\exp[-NTU(1-R)]}$$

where
$$R = \frac{C_{min}}{C_{max}}, \quad NTU = \frac{U_0 A_0}{C_{min}}$$

$$C_h = (\dot{m}\, c_p)_h = 4 \times 2500 = 10{,}000 \text{ W/K}$$

$$C_c = (\dot{m}\, c_p)_c = 3 \times 4180 = 12{,}540 \text{ W/K}$$

$\therefore$
$$C_{min} = 10{,}000 \text{ W/K}$$

$$R = \frac{10{,}000}{12{,}540} = 0.797$$

$$NTU = \frac{U_0 A_0}{C_{min}} = \frac{820 \times 30}{10{,}000} = 2.46$$

$\therefore$
$$\varepsilon = \frac{1 - \exp[-2.46(1-0.797)]}{1 - 0.797\exp[-2.46(1-0.797)]} = 0.761$$

Also,

$$\varepsilon = \frac{C_h(T_{h_1} - T_{h_2})}{C_{min}(T_{h_1} - T_{c_1})}$$

or,
$$0.761 = \frac{10{,}000(400 - T_{h_2})}{10{,}000(400 - 300)}$$

$\therefore$
$$T_{h_2} = 323.9 \text{ K} \quad \textit{Ans.}$$

By energy balance,

$$C_h(T_{h_1} - T_{h_2}) = C_c(T_{c_2} - T_{c_1})$$

$$10{,}000\,(400 - 323.9) = 12{,}540\,(T_{c_2} - 300)$$

$\therefore$
$$T_{c_2} = 360.7 \text{ K} \quad \textit{Ans.}$$

AMSCE- 1101

UNIT 5 MASS TRANSFER

1. What is mass transfer?

Ans: The process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer.

2. Give the examples of mass transfer.

Ans: Some examples of mass transfer.

1) Humidification of air in cooling tower

2) Evaporation of petrol in the caburetor of an IC engine

3) The transfer of water vapour into dry air.

3. What are the modes of mass transfer?

Ans: There are basically two modes of mass transfer.

1) Diffusion mass transfer

2) Convective mass transfer.

4. What is molecular diffusion?

Ans: The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

5. What is Eddy diffusion?

Ans: When one of the diffusion fluids is in turbulent motion, eddy, diffusion takes place.

6. What is convective mass transfer?

Ans : convective mass transfer is a process of mass transfer that will occur between surface and a fluid medium when they are at different concentration.

7. State Fick's law of diffusion.

The diffusion rate is given by the Fick's law. which states that molar flux of an element per unit area is directly proportional to concentration gradient..

$$\frac{m_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

$$\text{where, } \frac{m_a}{A} - \text{Molar flux, } \frac{kg-mole}{s-m^2}$$

D_{ab} Diffusion co-efficient of species a and b, m^2/s

$\frac{dC_a}{dx}$ - concentration gradient, kg/m^3.

8. What is free convective mass transfer?
Ans: If the fluid motion is produced due to change in density resulting from concentration gradients, the mode of mass transfer is said to be free or natural convection mass transfer. **Example:** Evaporation of alcohol.

AMSCE - 1101

9. Define forced convective mass transfer.

Ans: If the fluid motion is artificially created by means of an external force like a blower or fan, that type of mass transfer is known as convective mass transfer. **Example**: The evaluation if water from an ocean w hen air blows over it.

10. Define Schmidt Number.

Ans: It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$S = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass}}$$

11. Define Scherwood Number.

Ans : It is defined as the ratio of concentration gradients at the boundary

$$S_c = \frac{h_m x}{D_{ab}}$$

hm – Mass transfer co – efficient, m/s

D_{ab} – Diffusion co – efficient , m^2/s

x - Length, m

12. Give two example convective mass transfer.

Ans:

1) Evaporation of alcohol

2) Evaporation of water from an ocean when air blows over it.

13. Define mass concentration or mass density.

Ans: Mass of a component per unit volume of the mixture lt is expressed in Kg/m^3.

14. Define molar concentration or molar density.

Ans: Number of molecules of a component per unit volume of the mixture, it is expressed in Kg mole/m^3.

15. Define mass fraction.

Ans: The mass fraction is defined as the ratio of mass concentration of species to the total mass density of the mixture.

16.. Define mode fraction.

Ans: The mode fraction is defined as the ratio of mole concentration of species to the total molar concentration.

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17. Given any two examples of mass transfer in day to day life.

Examples of mass transfer

1. Dissolution of sugar added to a cup of coffee.
2. Humidification of air in cooling tower.
3. The transfer of water vapour into dry air, drying and evaporation.

18. What do you mean by equimolar counter diffusion.

Equimolar counter diffusion between species A and B of a binary gas mixture in defined as isothermal diffusion process in which each molecule of component A in replaced by each molecule of constituent B and vice versa.

PART B

1.A 3 –cm diameter Stefan tube is used to measure the binary diffusion coefficient of water vapour in air at 20°C at an elevation of 1600 m where the atmospheric pressure is 83.5 kPa. The tube is partially filled with water, and the distance from the water surface to the open end of the tube is 40 cm. Dry air blown over the open end of the tube so that water vapour rising to the top is removed immediately and the concentration of vapour at the top of the tube is zero. In 15 days of continuous operation at constant pressure and temperature, the amount of water that has evaporated is measured to be 1.23 g. Determine the diffusion coefficient of water vapour in air at 20°C and 83.5 kPa. (10)

Given data:

Diameter d = 3 cm = 0.03 m

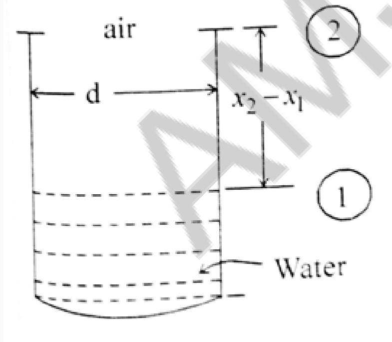
Deep, $(x_2 - x_1)$ = 40 cm = 0.4 m

Temperature, T = 20°C + 273 = 293 K

Atmospheric pressure, P = 83.5 KPa

Dry saturated = 83.5×10^3 N/m^2

m_a = 1.23 g.



To find :

Diffusion coefficient of water vapour, D_{ab},

Solution :

We know that, for isothermal evaporation.

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$$\text{Molar flux, } \frac{m_d}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_2 - x_1)} . \ln\left(\frac{P - P_{w2}}{P - P_{w1}}\right) \qquad .....(1)$$

$$\text{Area. } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times \left(9 \times 10^{-4}\right)$$

$$A = 7.068 \times 10^{-4}\, m^2$$

$m_a = 1.23 \times 10^{-3}$ kg/15 days.

$$m_a = \frac{1.23 \times 10^{-3}}{15 \times 24 \times 3600} \text{kg/sec}$$

$m_a = 9.49 \times 10^{-10}$ kg/sec

$$\text{Molar flux, } N_{ax} = \frac{m_a}{\text{M.wt of vapour} \times \text{Area}}$$

$$= \frac{9.49 \times 10^{-10}}{18 \times 7.068 \times 10^{-4}}$$

$= 0.0745 \times 10^{-6} = 7.45 \times 10^{-8}$ Kmol/m^2

Where,

$$G - \text{Universal gas constant} = 8314 \frac{J}{Kg - mole - K}$$

P_{w1} – Partial pressure at the bottom of the (water vapour) Stefan tube corresponding to saturation temperature at 20°C.

At 20°C,

P_{w1} = 0.0234 bar (From steam table, page, No.2)

P_{w1} = 2.34 kPa

Partial pressure at the top of the Stefan tube.

$P_{w2} = 0$

$$\Rightarrow N_{ax} = \frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_1 - x_2)} \ln\left(\frac{83.5 - 0}{83.5 - 2.34}\right)$$

$D_{ab} = 3.06 \times 10^{-5}$ m^2/sec

Result:

Diffusion coefficient of water vapour.

$D_{ab} = 3.06 \times 10^{-5}$ m^2/sec

2.State some analogies between heat and mass transfer.

S.No	**Parameter**	**Heat transfer**	**Mass transfer**
1.	Driving force	Temperature gradient.	Concentration gradient.

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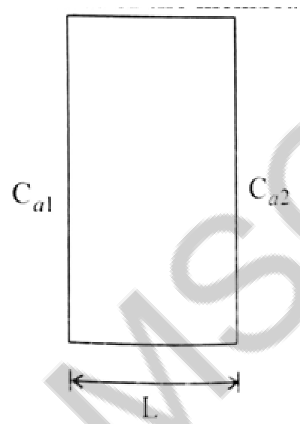
2.	Proportionality constant.	Thermal conductivity (Fourier's Law).	Diffusion coefficient (Fick's Law).
3.	Modes	Conduction, convection and Radiation	Conduction (Diffusion) and convection only.
4.	Internal heat generation	.Heat generation takes place in systems.	Species generation takes place.
5.	Convection coefficients.	Heat transfer coefficient (h).	Mass transfer coefficient (h_m)
6.	Dimensionless numbers.	Depends upon Nusselt number and parallel number.	Depends upon Sherwood number and Schmidt number.

3.A thin plastic membrane separate hydrogen from air. The molar concentrations of hydrogen in the membrane at the inner and outer surfaces are determined to be 0.045 and 0.002 kmol/m^3 , respectively. The binary diffusion coefficient of hydrogen in plastic at the operation temperature is 5.3 × 10^{-10} m^2/s. Determine the mass flow rate of hydrogen by diffusion through the membrane under steady conditions if the thickness of the membrane is (8)

(1) 2 mm and

(2) 0.5 mm.



Given data :

Case (i)

Thickness of the membrane, I = 2 mm = 0.002 m

Concentration at inner side, C_{a1} = 0.045 kmol / m^3

Concentration at outer side, C_{a2} = 0.002 kmol / m^3

Diffusion coefficient of hydrogen in plastic,

$$C_{ab} = 5.3 \times 10^{-10} \text{ m}^2 / \text{sec}$$

Case (ii)

Thickness of the membrane, L = 0.5 mm = 0.0005 m

To find :

AMSCE - 1101

(1) The mass flow rate of hydrogen by diffusion through the membrane. (m_a)

Case (i) L = 0.002 m

Case (ii) L = 0.0005 m

Solution:

Case(i) L =0.002 m

We know that, for plane membrane

$$\text{Molar flux, } \frac{m_a}{A} = N_{ax} = \frac{D_{ab}\left(C_{a1} - C_{a2}\right)}{L}$$
$$= \frac{5.3\times10^{-10}\times\left(0.045 - 0.002\right)}{0.002}$$

$N_{ax} = 1.139 \times 10^{-8}$ kmol/m^2.sec

Mass flow rate of hydrogen, m = N_{ax} × (Mol.wt of H_2)

= $1.139 \times 10^{-8} \times 2$

m = 2.278×10^{-8} kg/m^2.sec

Case (ii) L = 0.5 mm = 0.0005 m

We know that, for plane membrane.

$$\text{Molar flux, } \frac{m_a}{A} = N_{ax} = \frac{D_{ab}\left(C_{a1} - C_{a2}\right)}{L}$$
$$= \frac{5.3\times10^{-10}\times\left(0.045 - 0.002\right)}{0.0005}$$

$N_{ax} = 4.558 \times 10^{-8}$ kmol/m^2.sec

Mass flow rate of hydrogen, m = N_{ax} × (Mol.wt of H_2)

= $4.558 \times 10^{-8} \times 2$

= 9.116×10^{-8} (or)

m = 0.9116×10^{-7}

Result : Mass flow rate of hydrogen by diffusion through membrane.

Case (i) L = 0.002 m ; m = 2.278×10^{-8} kg/m^2 sec

Case (ii) L = 0.0005 m; m = 0.9116×10^{-7} kg/m^2 sec.

4.Dry air at 15°C and 92 kPa flows over a 2 m long wet surface with a free stream velocity of 4 m/s. Determine the average mass transfer coefficient. (8)

Given data:

Fluid temperature, T_∞ = 15°C

Velocity, U = 4m/sec

Length, X = 2m

Air flow pressure, P_2 = 92 kPa

Atmospheric pressure, P_1 = 100 kPa

To find :

Average mass transfer coefficient, h_m

Solution :

Properties of air at 15°C.

[From HMT data book, page No.34] (seventh edition)

Kinematic viscosity, $v_1 = 1.47 \times 10^{-5}$ m²/sec

We know that

$$\frac{P_1}{P_2} = \frac{v_2}{v_1}$$

$$\frac{100}{92} = \frac{v_2}{1.47 \times 10^{-5}}$$

$$V_2 = 2.57 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Reynolds number, } Re = \frac{Ux}{v} = \frac{1.47 \times 10^{-5}}{2.57 \times 10^{-5}} = 544217.6$$

$$\text{Schmidt number, } Sc = \frac{v}{D_{ab}} = \frac{1.47 \times 10^{-5}}{2.57 \times 10^{-5}} = 0.5719$$

Sherwood number, sh = $0.664\ Re^{0.5}\ Sc^{0.333}$

$$= 0.664\ (544217.6)^{0.5}\ (0.5719)^{0.333}$$

$$= 0.664\ (737.71)\ (0.8302)$$

$$Sh = 406.6$$

$$\text{Sherwood number, } sh = \frac{h_m.L}{D_{ab}} \Rightarrow h_m = \frac{sh.D_{ab}}{L}$$

$$h_m = \frac{406.6 \times 2.57 \times 10^{-5}}{2} = 0.00522\ \text{m/sec}.$$

Result :

Average mass transfer coefficient = 0.00522 m/sec.

5.Explain equimolal counter diffusion in gases.

STEADY STATE EQUIMOLAR COUNTER DIFFUSION

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Consider two large chambers a and b connected by a passage as shown in Fig.5.3

N_a and N_b are the steady state molar diffusion rates of components a and b respectively.

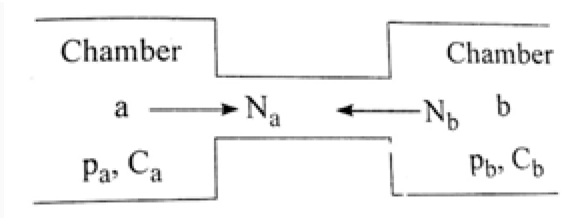

Fig.5.3

Equimolar diffusion is defined as each molecules o, ‘a’ is replaced by each molecule of ‘b’ and vice versa. The total pressure $P = P_a - P_o$ is uniform throughout the system.

$$P = P_a + P_b$$

Differentiating with respect to x

$$\frac{dp}{dx} = \frac{dp_a}{dx} + \frac{dp_b}{dx}$$

Since the total pressure of the system remains constant under steady state conditions.

$$\frac{dp}{dx} = \frac{dp_a}{dx} + \frac{dp_b}{dx} = 0$$

$$\Rightarrow \boxed{\frac{dp_a}{dx} = -\frac{dp_b}{dx}}$$

Under steady state conditions, the total molar flux is zero

$$\Rightarrow N_a + N_b = 0$$

$$N_a = -N_b$$

$$\Rightarrow \quad D_{ab}\frac{A}{GT}\frac{dp_a}{dx} = -D_{ba}\frac{A}{G_1}\frac{dp_b}{dx} \quad ....(5.5)$$

From Fick’s law,

$$N_a = D_{ab}\frac{A}{GT}\frac{dp_a}{dx}$$

$$N_b = D_{ba}\frac{A}{GT}\frac{dp_b}{dx}$$

We know,

$$\frac{dp_b}{dx} = -\frac{dp_a}{dx} \qquad [\text{From equations 5.4}]$$

Substitute in equation (5.5)

$$(5.5) \Rightarrow -D_{ab}\frac{A}{GT}\frac{dp_a}{dx} = -D_{ba}\frac{A}{GT}\frac{dp_a}{dx}$$

AMSCE - 1101

$$\Rightarrow \boxed{D_{ab} = D_{ba} = D}$$

6.Discuss briefly the Analog between heat and mass transfer.

In a system consisting of two or more components whose concentrations vary from point to point, there is a natural tendency for species (particles) to be transferred from a region of higher concentration side (higher density side) to a region of lower concentration side (lower density side).

This process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer.

Some examples of mass transfer are

1. Humidification of air in cooling tower.
2. Evaporation of petrol in the carburetter of an IC engine
3. The transfer of water vapour into dry air.
4. Dissolution of sugar added to a cup of coffee.

7.Define mass transfer coefficient. Air at 1 bar pressure and 25°C containing small quantities of iodine flows with a velocity of 5.2 m/s. Inside a tube having an inner diameter of 3.05 cm. Find the mass transfer coefficient for iodine transfer from the gas stream to the wall surface. If c_m is the mean concentration of iodine in kg.mol/m^3 in the air stream. Find the rate of deposition on the tube surface by assuming the wall surface is a perfect sink for iodine deposition. Assume D = 0.0834 cm^2/s.

Solution:

Mass transfer coefficient :

The mass transfer coefficient is a diffusion rate constant that relates the mass transfer rate, mass transfer area and concentration gradient as driving force. (Unit is m/s).

$$h_m = \frac{\dot{m}_{conv}}{A\left(\rho_{A,s} - \rho_{A,\infty}\right)}$$

Where, h_m = Mass transfer coefficient;

A = Area

$\dot{m}$ = Mass transfer rate

$\rho_{A,s} - \rho_{A,\infty}$ = Concentration gradient

Mass transfer coefficient determines the rate of mass transfer across a medium in response to a concentration gradient.

Given data:

Fluid temperature, T_∞ = 25°C

Velocity, u = 5.2 m/s

d = 3.05 cm = 0.0305 m

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Diffusion coefficient, $D_{ab} = 0.0834\ cm^2/s$

$= 0.0834 \times 10^{-4}\ m^2/s$

$D_{ab} = 8.34 \times 10^{-6}\ m^2/s$

To find:

(i) Mass transfer coefficient for iodine transfer, h_m

(ii) Rate of deposition of tube, $N = h_m(C_m - C_{A,\infty})$

8.Air at 25°C flows over a tray full of water with a velocity of 2.8 m/s. The tray measures 30 cm along the flow direction and 40 cm wide. The partial pressure of water present in the air is 0.007 bar. The partial pressure of water present in the air is 0.007 bar. Calculate the evaporation rate of water if the temperature on the water surface is 15°C. Take diffusion co-efficient is 4.2 × 10⁻⁵ m²/s.

Given :

Fluid temperature, $T_\infty = 25°C$

Speed, $U = 2.8\ m/s$

Flow direction is 30 cm side. So, $x = 30\ cm = 0.30\ m$

Area, $A = 30\ cm \times 40\ cm = 0.30 \times 0.40\ m^2$

Partial pressure of water, $p_{w2} = 0.007\ bar$

$$\boxed{p_{w2} = 0.007 \times 10^5\ N/m^2}$$

Water surface temperature, $T_w = 15°C$

Diffusion co-efficient, $D_{ab} = 4.2 \times 10^{-5}\ N/m^2$

To find:

Evaporation rate of water,(m_w)

Solution:

We know that,

Film temperature, $$T_f = \frac{T_w + T_\infty}{2} = \frac{15+25}{2}$$
$$\boxed{T_f = 20°C}$$

Properties of air at 20°C

[From HMT data book, page no 33]

Kinematic viscosity, $v = 15.06 \times 10^{-6}\ m^2/s$

We know that,

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$$\text{Reynolds Number, Re} = \frac{Ux}{v}$$

$$= \frac{2.8 \times 0.30}{15.06 \times 10^{-6}}$$

$$Re = 0.557 \times 10^5 < 5 \times 10^5$$

Since, $Re < 5 \times 10^5$, flow is laminar.

For flat plate, Laminar flow :

Sherwood Number, (Sh) = $[0.664\ (Re)^{0.5}\ (Sc)^{0.333}]$ .....(1)

[From HMT data book, page no 175]

Where,

$$\text{Sc – Schmidt Number} = \frac{v}{D_{ab}}$$

$$Sc = \frac{15.06 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boxed{Sc = 0.358}$$

Substitute Sc, Re values in equation (1)

(1) $\Rightarrow Sh = [0.664\ (0.557 \times 10^5)^{0.5}\ (0.358)^{0.333}]$

$\boxed{Sh = 111.37}$ We know that,

$$\text{Sherwood Number, Sh} = \frac{h_m x}{D_{ab}}$$

$$\Rightarrow 111.37 = \frac{h_m \times 0.30}{4.2 \times 10^{-5}}$$

$$\boxed{\text{Mass transfer co-efficient, } h_m = 0.0155\ m/s}$$

Mass transfer co-efficient based on pressure difference is given by.

$$h_{mp} = \frac{h_m}{RT_w} = \frac{0.0155}{287 \times 288}$$

$$[\because T_w = 15°C + 273 = 288\,K, R = 287\ J/kg\,K]$$

$$\boxed{h_{mp} = 1.88 \times 10^{-7}\ m/s}$$

Saturation pressure of water at 15°C

P_{w1} = 0.017 bar [From steam table (R.S. Khumi) page no.1]

$$\boxed{p_{w1} = 0.017 \times 10^5\ N/m^2}$$

The evaporation rate of water is given by,

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$M_w = h_{mp} \times A[p_{w1} - p_{w2}]$

$= 1.88 \times 10^{-7} \times (0.30 \times 0.40) \times [0.017 \times 10^5 - 0.007 \times 10^5]$

$\boxed{m_w = 2.25 \times 10^{-5}\ \text{kg/s}}$

Result:

Evaporation rate of water, $m_w = 2.25 \times 10^{-5}$ kg/s

9.O_2 and air experience equimolar counter diffusion in a circular tube whose length and diameter are 1.2 m and 60 mm respectively. The system is at a total pressure of 1 atm and a temperature of 273 K. The ends of the tube are connected to large chambers. Partial pressure of CO_2 at one end is 200 mm of Hg while at the other end is 90 mm of Hg. Calculate the following

1. Mass transfer rate of Co_2 and

2. Mass transfer rate of air

Given :

Diameter, d = 60 mm = 0.060 m

Length. $(x_2 - x_1) = 1.2$ m

Total pressure. P = 1 atm = 1 bar

Temperature, T = 273 K

Partial pressure of CO_2 at one end

$$P_{a1} = 200\,\text{mm of Hg} = \frac{200}{760}\,\text{bar}$$

$\Rightarrow$ $P_{a1} = 0.263$ bar [$\because$ 1 bar = 760 MM OF Hg]

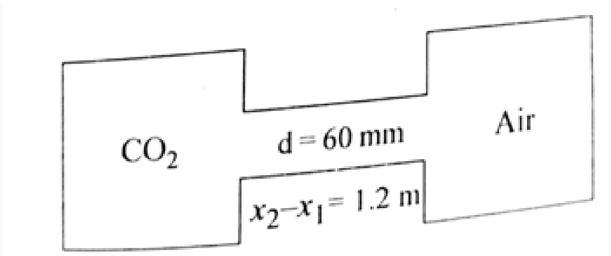
$\Rightarrow$ $\boxed{P_{a1} = 0.263 \times 10^5\ \text{N/m}^2}\left[\because 1\,\text{bar} = 10^5\ \text{N/m}^2\right]$

Partial pressure of CO_2 at other end

$$P_{a2} = 90\ \text{mm of Hg} = \frac{90}{760}\,\text{bar}$$

$\Rightarrow P_{a2} = 0.118$ bar

$\Rightarrow$ $\boxed{P_{a2} = 0.118 \times 10^5\ \text{N/m}^2}$



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To find :

1. Mass transfer rate of CO_2

2. Mass transfer rate of air

Solution:

We know that, for equimolar counter diffusion

$$\text{Molar flux.} \frac{m_a}{A} = \frac{D_{ab}}{GT}\left[\frac{P_{a1} - P_{a2}}{x_2 - x_1}\right] \qquad ....(1)$$

Where,

D_{ab} = Diffusion co – efficient – m^2/s

The diffusion co-efficient for CO_2 – Air combination 11.89×10^{-6} m^2/s

[from HMT data book page no.180 (sixth edition)

$$\boxed{D_{ab} = 11.89 \times 10^{-6}\ m^2/s}$$

$$g - \text{universal gas constant} - 8314 \frac{J}{kg - mole - k}$$

$$A - \text{Area} = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4}(0.060)^2$$

$$\boxed{A = 2.82 \times 10^{-3} m^2}$$

$$(1) \Rightarrow \frac{m_a}{2.82 \times 10^{-3}} = \frac{11.89 \times 10^{-6}}{8314 \times 273} \times \left[\frac{0.263 \times 10^5 - 0.118 \times 10^5}{1.2}\right]$$

$$\boxed{\text{Molar transfer rate of } CO_2, m_a = 1.785 \times 10^{-10} \frac{kg - mole}{s}}$$

We know,

Mass transfer rate of CO_2 = Molar transfer × Molecular weight

$= 1.785 \times 10^{-10} \times 44.01$

[Molecular weight of CO_2 = 44.01, refer HMT data, page no.182 (sixth editional)]

$$\boxed{\text{Mass transfer rate of } CO_2 = 7.85 \times 10^{-9}\ kg/s}$$

We know,

$$\text{Molar transfer rate of air, } m_b = -1.785 \times 10^{-10} \frac{kg - mole}{s}$$

[$\because m_a = - m_b$]

Mass transfer rate of air = Molar transfer × Molecular weight of air

$$= -1.785 \times 10^{-10} \times 29$$

$$\boxed{\text{Mass transfer rate of air} = -5.176 \times 10^{-9}\ \text{kg/s}}$$

Result :

1. Mass transfer rate of CO_2 = 7.85×10^{-9} kg/s

2. Mass transfer rate of air = -5.176×10^{-9} kg/s

10.Air at 25°C flows over a tray full of water with a velocity of 2.8 m/s. The tray measures 30 cm along the flow direction and 40 cm wide. The partial pressure of water present in the air is 0.007 bar. The partial pressure of water present in the air is 0.007 bar. Calculate the evaporation rate of water if the temperature on the water surface is 15°C. Take diffusion co-efficient is 4.2×10^{-5} m²/s.

Given :

Fluid temperature, T_∞ = 25°C

Speed, U = 2.8 m/s

Flow direction is 30 cm side. So, x = 30 cm = 0.30 m

Area, A = 30 cm × 40 cm = 0.30×0.40 m²

Partial pressure of water, p_{w2} = 0.007 bar

$$\boxed{p_{w2} = 0.007 \times 10^5\ \text{N/m}^2}$$

Water surface temperature, T_w = 15°C

Diffusion co-efficient, $D_{ab} = 4.2 \times 10^{-5}$ N/m²

To find:

Evaporation rate of water,(m_w)

Solution:

We know that,

Film temperature, $T_f = \dfrac{T_w + T_\infty}{2} = \dfrac{15 + 25}{2}$

$$\boxed{T_f = 20°C}$$

Properties of air at 20°C

[From HMT data book, page no 33]

Kinematic viscosity, $v = 15.06 \times 10^{-6}$ m²/s

We know that,

$$\text{Reynolds Number, Re} = \frac{Ux}{v}$$

$$= \frac{2.8 \times 0.30}{15.06 \times 10^{-6}}$$

$$Re = 0.557 \times 10^5 < 5 \times 10^5$$

Since, $Re < 5 \times 10^5$, flow is laminar.

For flat plate, Laminar flow :

$$\text{Sherwood Number, (Sh)} = [0.664\,(Re)^{0.5}\,(Sc)^{0.333}] \quad .....(1)$$

[From HMT data book, page no 175]

Where,

$$\text{Sc – Schmidt Number} = \frac{v}{D_{ab}}$$

$$Sc = \frac{15.06 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boxed{Sc = 0.358}$$

Substitute Sc, Re values in equation (1)

(1) $\Rightarrow Sh = [0.664\,(0.557 \times 10^5)^{0.5}\,(0.358)^{0.333}]$

$\boxed{Sh = 111.37}$ We know that,

$$\text{Sherwood Number, } Sh = \frac{h_m x}{D_{ab}}$$

$$\Rightarrow 111.37 = \frac{h_m \times 0.30}{4.2 \times 10^{-5}}$$

$$\boxed{\text{Mass transfer co-efficient, } h_m = 0.0155\,m/s}$$

Mass transfer co-efficient based on pressure difference is given by.

$$h_{mp} = \frac{h_m}{RT_w} = \frac{0.0155}{287 \times 288}$$

$$[\because T_w = 15°C + 273 = 288\,K,\ R = 287\,J/kg\,K]$$

$$\boxed{h_{mp} = 1.88 \times 10^{-7}\ m/s}$$

Saturation pressure of water at 15°C

$P_{w1} = 0.017$ bar [From steam table (R.S. Khumi) page no.1]

$$\boxed{p_{w1} = 0.017 \times 10^5\ N/m^2}$$

The evaporation rate of water is given by,

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$M_w = h_{mp} \times A[p_{w1} - p_{w2}]$

$= 1.88 \times 10^{-7} \times (0.30 \times 0.40) \times [0.017 \times 10^5 - 0.007 \times 10^5]$

$$\boxed{m_w = 2.25 \times 10^{-5}\ \text{kg/s}}$$

Result:

Evaporation rate of water, $m_w = 2.25 \times 10^{-5}$ kg/s

Analogy between heat and mass transfer.

There is similarity among heat and mass transfer. The three basic equations dealing with these are

(i) Newtonian equation of momentum

(ii) Fourier law of heat transfer

(iii) Fick law of mass transfer

∴ The momentum, heat and mass transfer equation can be written as

Continuity equation, $\frac{\partial u}{dx} + \frac{\partial v}{dy} = 0$

Momentum Transfer, $u\frac{\partial u}{dx} + v\frac{\partial v}{dy} = v\frac{\partial^2 u}{\partial y^2}$

Heat transfer, $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}u$

Mass transfer, $u\frac{\partial c_a}{\partial x} + v\frac{\partial c_n}{\partial y} = D\frac{\partial^2 c_a}{\partial y^2}$

11.Evaporation process in the atmosphere.

Ans. Refer Page no.5.34, section 5.16.

12. ISOTHERMAL EVAPORATION OF WATER INTO AIR

Consider the isothermal evaporation of water from a water surface and its diffusion through the stagnant air layer over it as shown in Fig.5.4. The free surface of the water is exposed to air in the tank.

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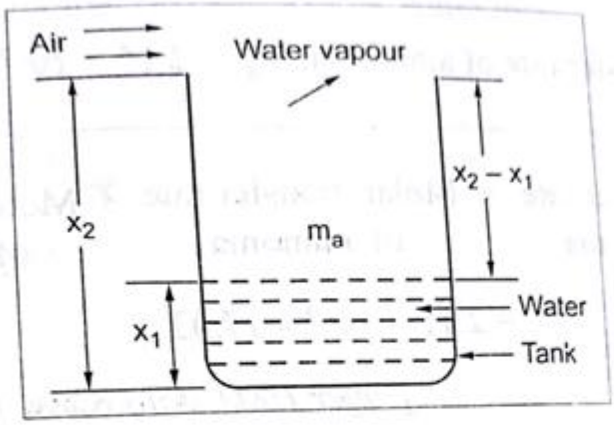

Fig.5.4

For the analysis of this type of mass diffusion, following assumptions are made,

1. The system is isothermal and total pressure remains constant.

2. System is in steady state condition.

3. There is slight air movement over the top of the tank to remove the water vapour which diffuses to that point.

4. Both the air and water vapour behave as ideal gases.

From Fick's law of diffusion. We can find

$$\text{Molar flux. } \frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{p}{(x_2 - x_1)} \ln\left[\frac{P - P_{w2}}{P - P_{w1}}\right] \quad ....(5.9)$$

Where,

$\frac{m_A}{A}$ – Molar flux – $\frac{\text{kg} - \text{mole}}{\text{s} - \text{m}^2}$

D_{ab} – Diffusion co- efficient – m^2/s

G – universal gas constant – $8314 \frac{J}{\text{kg} - \text{mole} - \text{k}}$

T – Temperature – K

P – Total pressure in bar

P_{w1} – Partial pressure of water vapour corresponding to saturation temperature at 1 in N/m^2

P_{w2} – Partial pressure of dry air at 2 in N/m^2

13.A mixture of O_2 and N_2 with their partial pressures in the ratio 0.21 to 0.79 is in a container at 25°C. Calculate the molar concentration, the mass density, and the mass fraction of each species for a total pressure of 1 bar. What would be the average molecular weight of the mixture?

[Dec 2004 & 2005, Anna Univ]

AMSCE - 1101

Given:

Partial pressure of O_2, P_{O_2} p = 0.21 × Total pressure

= 0.21 × 1 bar

= $0.21 \times 1 \times 10^5$ N/m^2

Partial pressure of N_2, P_{N_2} = 0.79 × Total pressure

= 0.79 × 1 bar

= $0.79 \times 1 \times 10^5$ N/m^2

Temperature, T = 25° C + 273

= 298 K

To find :

1. Molar concentrations, C_{o_2}, C_{N_2}
2. Mass densities, P_{O_2}, P_{N_2}
3. Mass fractions, $\dot{m}_{O_2}, \dot{m}_{N_2}$
4. Average molecular weight, M

Solution:

We know that,

Molar concentration, $C = \dfrac{P}{GT}$

$$\Rightarrow \quad C_{O_2} = \frac{P_{O_2}}{GT}$$

$$= \frac{0.21 \times 1 \times 10^5}{8314 \times 298}$$

[∵ Universal gas constant, G = 8314 J/Kg – mole – K]

$$\boxed{C_{O_2} = 8.476 \times 10^{-3} \text{kg} - \text{mole} / \text{m}^3}$$

$$\Rightarrow \quad C_{N_2} = \frac{P_{N_2}}{GT}$$

$$= \frac{0.79 \times 1 \times 10^5}{8314 \times 298}$$

AMSCE - 1101

$$\boxed{C_{N_2} = 31.88 \times 10^{-3} \text{kg} - \text{mole}/\text{m}^3}$$

We know that,

$$\text{Molar concentration, } C = \frac{\rho}{M} \text{ F}$$

$$\Rightarrow \qquad \rho = C \times M$$

$$\rho_{O_2} = c_{O_2} \times M_{O_2}$$

$$\Rightarrow \qquad = 8.476 \times 10^{-3} \times 32$$

$$[\because \text{Molecular weight of } O_2 \text{ is } 32]$$

$$\boxed{\rho_{O_2} = 0.271 \text{kg}/\text{m}^3}$$

$$\rho_{N_2} = c_{N_2} \times M_{N_2}$$

$$\Rightarrow \qquad = 31.88 \times 10^{-3} \times 28$$

$$[\because \text{Molecular weight of } N_2 \text{ is } 28]$$

$$\boxed{\rho_{N_2} = 0.893 \text{kg}/\text{m}^3}$$

$$\text{Overall density} \quad \rho = \rho_{O_2} + P_{N_2}$$

$$= 0.271 + 0.893$$

$$\boxed{\rho = 1.164 \text{kg}/\text{m}^3}$$

Mass fractions :

$$\dot{m}_{O_2} = \frac{\rho_{O_2}}{\rho} = \frac{0.271}{1.164}$$

$$\boxed{\dot{m}_{O_2} = 0.233}$$

$$\dot{m}_{N_2} = \frac{\rho N_2}{\rho} = \frac{0.893}{1.164}$$

$$\boxed{\dot{m}_{N_2} = 0.767}$$

Average Molecular weight

$$M = P_{O_2} M_{O_2} + P_{N_2} M_{N_2}$$

$$= 0.21 \times 32 + 0.79 \times 28$$

$$M = 28.84$$

Result:

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$$1.\ C_{O_2} = 8.476 \times 10^{-3}\ \text{kg} - \text{mole}/\text{m}^3$$
$$C_{N_2} = 31.88 \times 10^{-3}\ \text{kg} - \text{mole}/\text{m}^3$$

$$2.\ \rho_{O_2} = 0.271\,\text{kg}/\text{m}^3$$
$$\rho_{N_2} = 0.893\,\text{kg}/\text{m}^3$$

$$3.\ \dot{m}_{O_2} = 0.233$$
$$\dot{m}_{N_2} = 0.767$$

4. M = 28.84

14.Consider air inside a tube of surface area 0.5 m^2 and wall thickness 10 mm. The pressure of air drops from 2.2 bar to 2.18 bar in 6 days. The solubility of air in the rubber is 0.072 m^3 of air per m^3 rubber at 1 bar. Determine the diffusivity of air in rubber at the operating temperature of 300 K if the volume of air in the tube is 0.028 m^3.

Given:

$A = 0.5\ \text{m}^2$

$L = 10\ \text{mm} = 0.010\ \text{m}$

$P_i = 2.2\ \text{bar} = 2.2 \times 10^5\ \text{N/m}^2$

$P_d = 2.18\ \text{bar} = 2.18 \times 10^5\ \text{N/m}^2$

$S = 0.072\ \text{m}^3$

$T = 300\ \text{K}$

$V = 0.028\ \text{m}^3$

To find : Diffusivity of air in rubber [D]

Solution: Initial mass of air in the tube,

$$m_i = \frac{P_i V}{R\,T} = \frac{2.2 \times 10^5 \times 0.028}{287 \times 300}$$
$$\boxed{m_i = 0.0715\,\text{kg}}$$

Final mass of air in the tube

$$m_d = \frac{P_d V}{R\,T} = \frac{2.18 \times 10^5 \times 0.028}{287 \times 300}$$
$$\boxed{m_d = 0.07089\,\text{kg}}$$

Man of air escaped = 0.0715 – 0.07089 = 0.00061 kg

The man flux of air escaped is given by

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$$N_a = \frac{m_a}{A} = \frac{\text{Man of air escaped}}{\text{Time elapsed} \times \text{Area}}$$
$$= \frac{0.00061}{(6 \times 24 \times 3600) \times 0.5}$$

$= 2.35 \times 10^{-9}$ kg/s – m^2.

The solubility of air should be calculated at the mean operating pressure,

i.e $\frac{2.2 + 2.18}{2} = 2.19$ bar

The solubility of air i.e, volume at the mean inside pressure,

$S = 0.072 \times 2.19$

$= 0.1577$ m^3/m^3 of rubber

The air which escapes to atmospheric will be at 1 bar pressure and its solubility will remain at 0.072 m^3 of air per m^3 of rubber.

The corresponding mass concentrations at the inner and outer surfaces of the tube, from characteristic gas equation, are calculated as;

$$C_{a1} = \frac{p_1 V_1}{RT_1} = \frac{2.19 \times 10^5 \times 0.1577}{287 \times 300}$$
$$= 0.4011 \text{ kg/m}^3$$

$$C_{a2} = \frac{P_2 V_2}{RT_2} = \frac{1 \times 10^5 \times 0.072}{287 \times 300}$$
$$= 0.0836 \text{ kg/m}^3$$

The diffusion flux rate of air through the rubber is given by

$$N_a = \frac{m_a}{A} = \frac{D[C_{a1} - C_{a2}]}{(x_2 - x_1)}$$
$$= \frac{D[C_{a1} - C_{a2}]}{L}$$

$$\Rightarrow \quad 2.35 \times 10^{-9} = \frac{D[0.4011 - 0.0836]}{0.01}$$

$\Rightarrow \quad D = 0.74 \times 10^{-10}$ m^2/s

Result : The diffusivity of air in rubber

$D = 0.74 \times 10^{-10}$ m^2/s

15. Derive an expression for mass flux in steady state molecular diffusion:

(a) A through non diffusing B

(b) Equimolar Counter Diffusion.

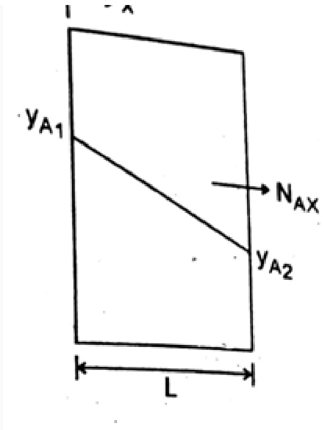
AMSCE - 1101

Ans: **(a) Steady state molecular diffusion, A through non-diffusion, A through non – diffusing B**

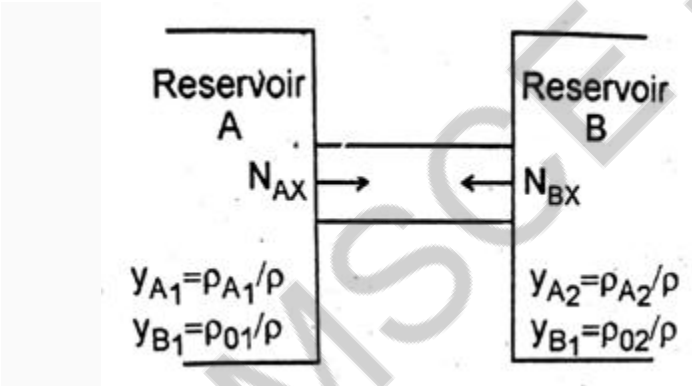
$$N_{A_x} = -CD_{AB}\frac{\left({}^{y}A_2 - {}^{y}A_1\right)}{L}$$

Mass current

$$Q_{mA} = \frac{D_{AB}.A_S}{L}\left(C_{A_1} - C_{A_1}\right)$$



(b) Steady state equimolar counter diffusion:



$$N_{A_x} = -N_{B_x}$$

$$N_{A_x} = -CD_{AB}\frac{dy_A}{dx}$$

$$N_{B_x} = -CD_{BA}\frac{dy_B}{dx}$$

$$\frac{dy_A}{dx} = -\frac{dy_B}{dx}$$

$$N_{A_x} = \frac{CD_{AB}}{\left(x_2 - x_1\right)}\left(y_{A_1 - yA_2}\right)$$

$$N_{A_x} = \frac{D_{AB}\left(cA_1 - cA_2\right)}{\left(x_2 - x_1\right)}$$

AMSCE - 1101

16.NH_2 gas (A) diffuses through N_2(B) under steady state condition with non – diffusing N_2 . The total pressure is 101.325 kPa and temperature is 298 K. The diffusion thickness is 0.15 m the partial pressure of NH_3 at one. Point is 1.5×10^4 Pa and at the outer point is 5×10^3 Pa. The D_{AB} for mixture at 1 atm and 298 K is 2.3×10^{-5} m^2/sec. (i) Calculate flux of NH_3. (A through non diffusing B) . Calculate flux for equimodal counter diffusion.(8)

Given : P = 102.325 KPa.

T = 298 k.

L = 0.15 m

$P_{a1} = 1.5 \times 10^4$ Pa.

$P_{a2} = 5 \times 10^3$ Pa.

Ans : (a) Equimolar counter diffusion

$$\left(\frac{m_a}{A}\right) = -\frac{\eta_{ab}}{R_a T} \times \frac{P_{a_1} - P_{a_2}}{x_2 - x_1}$$

$$R_a = \frac{8314}{17} = 489\,\text{J/kg K.}$$

$$= 1.052 \times 10^{-5}\ \text{kg/gm}^2$$

(b) A through non – diffusing B

$$\frac{m_a}{A} = D_{ab} \cdot \frac{P}{R_a T (Y_2 - Y_1)} \ln\left(\frac{P_{b_2}}{P_{b_1}}\right)$$

$$P_{a_1} + P_{b_1} = P$$

$$1.5 \times 10^4 + P_{b_1} = 101.325 \times 10^3$$

$$P_{b_1} = 86325\,\text{Pa.}$$

$$P_{a_2} + P_{b_2} = P$$

$$5 \times 10^3 + P_{b_2} = 101.325 \times 10^3$$

$$P_{b_2} = 96325\,\text{Pa.}$$

$$\Rightarrow \frac{m_a}{A} = \frac{2.3 \times 10^{-5} \times 101.325 \times 10^3}{\left(\frac{8314}{17}\right)(298)(0.15)\,\text{LN}\left(\frac{96325}{86325}\right)}$$

$$= 1.168 \times 10^{-5}\ \text{kg/gm}^2.$$

17.Write a note on the convective mass transfer coefficients for liquids and gases. Ans: Convective Mass Transfer Co-efficient:

A fluid of species molar concentration, $C_{A\infty}$, flouring a surface at which species concentration is C_{AS}.

AMSCE - 1101

As long as $C_{AS} \neq CA_\infty$, mass transfer by convection will occur:

$$N_A = h_m (C_{AS} - C_{A\infty})$$

$$h_m = \frac{N_A}{C_{AS} - C_{A\infty}} \text{ m/s.}$$

ii) **Give a brief description on heat, momentum and mass transfer analogies.**

Ans : Heat, Momentum and Mass Transfer Analogy:

In has been sent that there is a marked similarly between the laws governing the boundary layer growth of the three transport phenomena, of momentum, heat and mass. These equations for a laminar boundary layer over a flat plate are:

(i) Momentum Transfer

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \qquad .....(1)$$

(ii) Heat Transfer

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad ....(2)$$

(iii) Mass Transfer

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2} \qquad ....(3)$$

It was shown that the momentum and thermal boundary layers are identical for $v = \alpha$ or when prandtl number is unity.

i.e., $$P_r = \frac{v}{\alpha} = 1$$

Note the similarity between Equations. The velocity and concentration profiles will have the same shape when $v = D_{AB}$. The dimensionless ratio v/D_{AB} is called the Schmidt number

$$S_C = \frac{v}{D_{AB}} \qquad .....(4)$$

The Schmidt number is important in problems involving both momentum and convection mass transfer. It assumes the same importance in mass transfer as does the Prandtl number to convection heat transfer problems. Table 14.1 gives the values of the Schmidt for same common gases diffusing into air at 25°C and 1 atmosphere.

Obviously, the temperature and concentration profiles will be similar when $\alpha = D_{AB}$. The dimensionless ratio α/D_{AB} is called the Lewis number.

$$L_e = \frac{\alpha}{D_{AB}}$$

Table : Schmidt Number for some Gases Diffusing

AMSCE - 1101

Into Air at 1 atm, and 25°C

Gas	Schmidth Number
Ammonia	**0.66**
Carbon dioxide	**0.94**
Hydrogen	**0.22**
Oxygen	**0.75**
Water Vapour	**0.6**
Ethyl ether	**1.66**
Benzene	**1.76**

The Lewis number is of significance in problem involving both heat and mass transfer. All the three boundary layer profiles will become identical when

$$Pr = Sc = Le = 1$$

Just like the Nusselt number in convective heat transfer,. We define a non-dimensional parameter called Sherwood number as:

$$Sh = \frac{h_m x}{D_{AB}} \qquad ....(5)$$

Where x is characteristic length.

Similarly, corresponding to Stanton number,

$$St = \frac{N_u}{Re.Pr} = \frac{h}{pu_\infty C_P}$$

We have a dimensionless number in mass transfer St_m, given by:

$$St_m = \frac{S_h}{Re.Sc} = \frac{h_m}{\mu_\infty}$$

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$$St_m = \frac{S_h}{Re.Sc} = \frac{h_m}{\mu_\infty}$$

As seen, the forced heat transfer correlations are of thee form:

$$N_u\ (Re, Pr)$$

Likewise in forced convection mass transfer, the correlations would be of the form:

$$Sh = sh(Re, Sc)$$

The free convection heat transfer correlations have been seen to be the form:

$$Nu = Nu(Gr, Pr)$$

Where $Gr = \dfrac{\rho^2 \beta_g L^3\,\Delta T}{\mu^2} G$

AMSCE - 1101

We need to define a new ,ass Grashof number, Gr_m, because the density variation in mass transfer is due to concentration difference and not temperature difference., The buoyancy force in mass transfer is given by:

$$\frac{g}{\rho_\infty}(\rho-\rho_\infty) = -g\beta_m(m_A - m_{Aw})$$

Where $\beta_m = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial m_A}\right)$ is a quantity analogous to β.

It indicates the variations of density with composition. The mass Grashof number is then defined as :

$$Gr_m = \frac{\beta_m g L^3 (m_{A_w} - m_{Aw})}{v^2} \qquad \text{......(6)}$$

or $$Gr_m = \frac{gL^3(\rho_\infty - \rho_w)}{v^2\rho_\infty} \qquad \text{........(7)}$$

Where subscript w refers to the wall. The correlations for natural mass transfer can then be written in the form:

Sh = sh $Gr_{m,}$ Pr).

18. Explain in detail the various modes of mass transfer.

Various modes of mass transfer:

There are 3 modes of mass transfer

1. Mass transfer by diffusion
2. Mass transfer by convection
3. Mass transfer by change of phase.

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1. Mass transfer by diffusion:

The transport of water on a microscopic level as a result of diffusion from a region of high concentration to a region of low concentration in a system/mixture of liquids or gases in called molecular diffusion. It occurs when a substance diffuses through a layer of stagnant fluid and may be due to concentration temperature or pressure gradients. In a gaseous mixture, molecular diffusion occurs due to random motion of the molecules.

2. Mass transfer by convection:

Mass transfer by convection involves transfer between a moving fluid and a surface or between two relatively immiscible moving fluids. The convective mass transfer depends on the transport properties and on the dynamic characteristics of the flowing fluid. Example: The evaporation of ether.

3. Mass transfer by change of phase:

Mass transfer occurs whenever a change from one phase to another phase takes place. The mass transfer in such a case occurs due to simultaneous action of convection and diffusion some example are.

Hot gases escaping frcm the chimney rise by convect'on and then diffuse into the air above the chimney, mixing of water vapour with air during vaporation of water from the lake surface.

19. Hydrogen gas in maintained at 5 bar and 1 bar on opposite sides of a plastic membrane, which is 0.3 mm thick. The temperature is 25°C and the binary diffusion coefficient of hydrogen in the plastic is 8.7×10^{-8} m²/s. The solubility of hydrogen in the membrane is 1.5×10^{-3} kg mol/m³ bar. What is the mass flux of hydrogen by diffusion through the membrane.

Given:

Inside pressure of hydrogen = 5 bar

Outside pressure of hydrogen = 1 bar

Thickness of plastic membrane = 0.3 mm thick

$= 0.3 \times 10^{-3}$ m

Diffusion coefficient, D_{ab} $= 8.7 \times 10^{-8}$ m²/s

Solubility of hydrogen = 1.5×10^{-3} kg mol/m³ bar

Temperature = 25°C

To find:

Mass flux of hydrogen

Solution:

1. Molar concentration on inner side

$$C_{a1} = \text{Solubility} \times \text{Inner pressure}$$
$$= 1.5 \times 10^{-3} \times 5 = 7.5 \times 10^{-3} \text{ kg-mol/m}^3$$

$$\boxed{C_{a1} = 7.5 \times 10^{-3} \text{kg mol/m}^3.}$$

Molar concentration on outer side.

$$C_{a2} = \text{Solubility} \times \text{outer pressure}$$
$$= 1.5 \times 10^{-3} \times 1$$

$$\boxed{C_{a2} = 1.5 \times 10^{-3} \text{kg mol/m}^3.}$$

we know that,

$$\text{Molar flux, } \frac{ma}{A} = \frac{D_{ab}}{L}\left[C_{a1} - C_{a2}\right]$$

$$\frac{ma}{A} = \frac{8.7 \times 10^{-8}}{0.3 \times 10^{-3}}\left[7.5 \times 10^{-3} - 1.5 \times 10^{-3}\right]$$

$$\boxed{\text{Molar flux, } \frac{ma}{A} = 17.4 \times 10^{-7} \text{kg mol/m}^2\text{s}}$$

$$\text{Mass flux} = \text{Molar flux} \times \text{Molecular weight}$$
$$= 17.4 \times 10^{-7} \times 2/\text{mole}$$
$$= 34.8 \times 10^{-7} \text{kg/m}^2\text{s}$$

$$\boxed{\text{Mass flux} = 34.8 \times 10^{-7} \text{kg/m}^2\text{s}}$$

Result:

Mass flux hydrogen = 34.8×10^{-7} kg/m²s.

20.

A vessel contains a binary mixture of O_2 and N_2 with partial pressures in the ratio 0.21 and 0.79 at 20°C. If the total pressure of the mixture is 1.1 bar, calculate the following:

i) Molar concentrations

ii) Mass densities

iii) Mass fractions

iv) Molar fractions of each species

AMSCE - 1101

Given:

Partial pressure of O_2, p_{O_2} = $0.21 \times$ Total presure

$= 0.21 \times 1.1$ bar

$= 0.21 \times 1.1 \times 10^5\ N/m^2$

Partial pressure of N_2, p_{N_2} = $0.79 \times$ Total presure

$= 0.79 \times 1.1$ bar

$= 0.79 \times 1.1 \times 10^5\ N/m^2$

Temperature, $T = 20^oC + 273$

$= 293$ K

To find:

1. Molar concentrations, C_{O_2}, C_{N_2}
2. Mass densities, ρ_{O_2}, ρ_{N_2}
3. Mass fractions, $\dot{m}_{O_2}$, $\dot{m}_{N_2}$
4. Molar fractions, x_{O_2}, x_{N_2}

Solution:

We know that,

Molar concentration, $C = \dfrac{p}{GT}$

$$\Rightarrow \quad C_{O_2} = \frac{p_{O_2}}{GT} = \frac{0.21 \times 1.1 \times 10^5}{8314 \times 293}$$

[$\because$ Universal gas constant, G = 8314 J/kg – mole – K]

$$\boxed{C_{O_2} = 9.48 \times 10^{-3}\ kg - mole/m^3}$$

$$\Rightarrow \quad C_{N_2} = \frac{p_{N_2}}{GT} = \frac{0.79 \times 1.1 \times 10^5}{8314 \times 293}$$

$$\boxed{C_{N_2} = 35.67 \times 10^{-3}\ kg - mole/m^3}$$

Molar concentration, $C = \dfrac{\rho}{M}$

$$\Rightarrow \quad \rho = C \times M$$

$$\Rightarrow \quad \rho_{O_2} = C_{O_2} \times M_{O_2} = 9.48 \times 10^{-3} \times 32$$

[$\because$ Molecular weight of O_2 is 32]

$$\boxed{\rho_{O_2} = 0.303\ kg/m^3}$$

$$\Rightarrow \quad \rho_{N_2} = C_{N_2} \times M_{N_2} = 35.67 \times 10^{-3} \times 28$$

[$\because$ Molecular weight of N_2 is 28]

$$\boxed{\rho_{N_2} = 0.9987\ kg/m^3}$$

Overall density, $\rho = \rho_{O_2} + \rho_{N_2}$

$= 0.303 + 0.9987$

$$\boxed{\rho = 1.302\ kg/m^3}$$

Mass fractions :

$$\dot{m}_{O_2} = \frac{\rho_{O_2}}{\rho} = \frac{0.303}{1.302}$$

$$\boxed{\dot{m}_{O_2} = 0.233}$$

$$\dot{m}_{N_2} = \frac{\rho_{N_2}}{\rho} = \frac{0.9987}{1.302}$$

$$\boxed{\dot{m}_{N_2} = 0.767}$$

We know that,

Total concentration, $C = C_{O_2} + C_{N_2}$

$= 9.48 \times 10^{-3} + 35.67 \times 10^{-3}$

$$\boxed{C = 0.045}$$

Mole fractions:

$$x_{O_2} = \frac{C_{O_2}}{C}$$

AMSCE - 1101

$$= \frac{9.48 \times 10^{-3}}{0.045}$$

$$\boxed{x_{O_2} = 0.210}$$

$$x_{N_2} = \frac{C_{N_2}}{C}$$

$$= \frac{35.67 \times 10^{-3}}{0.045}$$

$$\boxed{x_{N_2} = 0.792}$$

Result:

1. $C_{O_2} = 9.48 \times 10^{-3}\, kg-mole/m^3$

   $C_{N_2} = 35.67 \times 10^{-3}\, kg-mole/m^3$

2. $\rho_{O_2} = 0.303\, kg/m^3$

   $\rho_{N_2} = 0.9987\, kg/m^3$

3. $\dot{m}_{O_2} = 0.233$

   $\dot{m}_{N_2} = 0.767$

4. $x_{O2} = 0.210$

   $x_{N2} = 0.792$

AMSCE-1101

21.

Air at 25°C flows over a tray full of water with a velocity of 2.8 m/s. The tray measures 30 cm along the flow direction and 40 cm wide. The partial pressure of water present in the air is 0.007 bar. Calculate the evaporation rate of water if the temperature on the water surface is 15°C. Take diffusion co-efficient is 4.2 × 10⁻⁵ m²/s

Given:

Fluid temperature, $T_\infty = 25^\circ C$

Speed, $U = 2.8\ m/s$

Flow direction is 30 cm side. So, $x = 30\ cm = 0.30\ m$

Area, $A = 30\ cm \times 40\ cm = 0.30 \times 0.40\ m^2$

Partial pressure of water, $p_{w2} = 0.007$ bar

$p_{w2} = 0.007 \times 10^5\ N/m^2$

Water surface temperature, $T_w = 15^\circ C$

Diffusion co-efficient, $D_{ab} = 4.2 \times 10^{-5}\ N/m^2$

To find:

Evaporation rate of water, (m_w)

Solution:

We know that,

Film temperature, $T_f = \frac{T_w + T_\infty}{2} = \frac{15 + 25}{2}$

$$\boxed{T_f = 20^\circ C}$$

Properties of air at 20°C

[From HMT data book, page no.34]

Kinematic viscosity, $\nu = 15.06 \times 10^{-6}\ m^2/s$

We know that,

$$\text{Reynolds Number, } Re = \frac{Ux}{\nu}$$

$$= \frac{2.8 \times 0.30}{15.06 \times 10^{-6}}$$

$$Re = 0.557 \times 10^5 < 5 \times 10^5$$

Since, $Re < 5 \times 10^5$, flow is turbulant

For flat plate, Laminar flow ,

Sherwood Number (Sh) = $[0.664\ (Re)^{0.5}\ (Sc)^{0.333}]$ ...(1)

[From HMT data book, page no.176]

where,

$$\text{Sc} - \text{Schmidt Number} = \frac{\nu}{D_{ab}} \quad \ldots(2)$$

$$\text{Sc} = \frac{15.06 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boxed{\text{Sc} = 0.358}$$

Substitute Sc, Re values in equation (1)

(1)⇒ $$\text{Sh} = [0.664\,(0.557 \times 10^5)^{0.5}\,(0.358)^{0.333}]$$

$$\boxed{\text{Sh} = 111.37}$$

We know that,

$$\text{Sherwood Number, Sh} = \frac{h_m x}{D_{ab}}$$

⇒ $$111.37 = \frac{h_m \times 0.30}{4.2 \times 10^{-5}}$$

$$\boxed{\text{Mass transfer co-efficient, } h_m = 0.0155 \text{ m/s}}$$

Mass transfer co-efficient based on pressure difference is given by,

$$h_{mp} = \frac{h_m}{R\,T_w} = \frac{0.0155}{287 \times 288}$$

$$[\because T_w = 15^\circ\text{C} + 273 = 288 \text{ K}, R = 287 \text{ J/kg K}$$

$$\boxed{h_{mp} = 1.88 \times 10^{-7} \text{ m/s}}$$

Saturation pressure of water at 15°C

$p_{w1} = 0.017$ bar *[From steam table R.S.Khurmi) page no.*

$$\boxed{p_{w1} = 0.017 \times 10^5 \text{ N/m}^2}$$

The evaporation rate of water is given by,

$$m_w = h_{mp} \times A\,[p_{w1} - p_{w2}]$$

$$= 1.88 \times 10^{-7} \times (0.30 \times 0.40) \times [0.017 \times 10^5 - 0.007 \times 10^5]$$

$$\boxed{m_w = 2.25 \times 10^{-5} \text{ kg/s}}$$

Result :

Evaporation rate of water, $m_w = 2.25 \times 10^{-5}$ kg/s

AMSCE - 1101