UNIT - 3

DEFLECTION

PART-A

1. **State the two theorems in moment area method.**
(AU April/May 2017)

Theorem-I

The change of slope between two points of a loaded beam is equal to the area of BMD between two points divided by EI.

Theorem-II

The deflection of a point with respect to tangent at second point is equal to the first moment of area of BMD between two points about the first point divided by EI.

2. **Write the maximum value of deflection for a cantilever beam of length L constant EI and carrying concentrated load W at the end.**
(AU April/May 2017)

$$y_B = \frac{WL^3}{3EI}$$

3. **Write the maximum value of deflection for a simply supported beam of constant EI, span L carrying central concentrated load W.** **(AU Nov/Dec 2016)**

$$y_c = \frac{WL^3}{48EI}$$

4. **Where the maximum deflection will occur in a simply supported beam loaded with UDL of w kN/m run.** **(AU Nov/Dec 2016)**

It will occur at centre

$$y_c = \frac{5}{384} - \frac{WL^3}{EI}$$

5. What are the advantages of Macaulay's method over double integration method? (AU Nov/Dec 2015)

In double integration method for finding slope and deflection for a simply supported beam loaded with many point loads and UDL is very tedious and laborious

In Macaulay's method integrating the continuous expression for bending moment in such a way that the constants of integration are valid for all sections of the beam.

6. What are the methods for finding out the slope and deflection at a section? (AU Nov/Dec 2014)

The important methods used for finding out the slope and deflection at a section in a loaded beam are

- Double integration method
- Moment area method
- Macaulay's method
- Conjugate beam method

The first two methods are suitable for a single load, where as the last one is suitable for several loads.

7. Why moment area method is more useful, when compared with double integration?

Moment area method is more useful, as compared with double integration method because many problems which do not have a simple mathematical solution can be simplified by the moment area method.

8. Explain the Theorem for conjugate beam method?

Theorem I : "The slope at any section of a loaded beam, relative to the original axis of the beam is equal to the shear in the conjugate beam at the corresponding section"

Theorem II: "The deflection at any given section of a loaded beam, relative to the original position is equal to the Bending moment at the corresponding section of the conjugate beam"

9. Define method of Singularity functions?

In Macaulay's method a single equation is formed for all loading on a beam, the equation is constructed in such away that the constant of

Integration apply to all portions of the beam. This method is also called method of singularity functions.

10. What are the points to be worth for conjugate beam method?

1. This method can be directly used for simply supported Beam

2. In this method for cantilevers and fixed beams atrificial contrains need to be supplied to the conjugate beam so that it is supported in a manner. Consistent with the contrsints of the real beam.

11. Define: Mohr's Theorem for slope

The change of slope between two points of a loaded beam is equal to the area of BMD between two points divided by EI.

12. Define: Mohr's Theorem for deflection

The deflection of a point with respect to tangent at second point is equal to the first moment of area of BMD between two points about the first point divided by EI.

13. Define slope.

Slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.

14. Define deflection.

The deflection at any point on the axis of the beam is the distance between its position before and after loading.

15. Write the maximum value of deflection for a cantilever beam of length L constant EI and carrying UDL for the entire span.

$$y_B = \frac{\omega L^4}{8EI} = \frac{WL^3}{8EI}$$

16. A cantilever of length 2.5m carries a uniformly distributed load of 16.4 kN per meter length over the entire length. If the moment of inertia of the beam = 7.95 × 10^2 mm^4 and value of E = 2 × 10^5 n/mm^2, determine the deflection at the free end.

Sol, Given:

Length, L = 2.5mm = 2500mm

U.d.l, $\omega = 16.4$ kN/m

Total load, $W = \omega \times L = 16.4 \times 2.5 = 41$kN =41000 N

Value of $I = 7.95 \times 10^7$ mm^4

Value of $E = 2 \times 10^5$ N/mm^2

Let y_B = Deflection at the free end,

Using equation (13.6), we get

$$y_B = \frac{WL^3}{8EI} = \frac{41000 \times 2500^3}{8 \times 2 \times 10^5 \times 7.95 \times 10^7}$$
$$= 5.036\,\text{mm.} \quad \text{Ans}$$

17. A cantilever of length 3m is carrying a point load of 25kN at the free end. If the moment of inertia of the beam = 10^8 mm^4 and value of E = 2.1×10^5 N/mm^2 , Find (i) Slope of the cantilever at the free end and (ii) deflection at the free end.

Sol. Given:

Length, L =3m = 3000mm

Point load, W = 25kN = 25000N

M.O.I, $I = 10^8$mm^4

Value of $E = 2.1 \times 10^5$ N/mm^2

(i) Slope at the free end is given by equation (13.1 A)

$$\therefore \quad \theta_B = \frac{WL^3}{2EI} = \frac{25000 \times 3000^2}{2 \times 2.1 \times 10^5 \times 10^8} = 0.005357\,\text{rad. Ans.}$$

(ii) Deflection at the free end is given by equation (13.2 A).

$$y_B = \frac{WL^3}{3EI} = \frac{25000 \times 3000^3}{3 \times 2.1 \times 10^5 \times 10^8} = 10.71\text{mm. Ans}$$

18. A beam 3m long, simply supported at its ends, is carrying a point load W at the centre. If the slope at the ends of the beam should not exceed 1”, find the deflection at the centre of the beam.

Sol, Given:

Length, $L = 3\text{m} = 3 \times 1000 = 3000$ mm

Point load at centre $= W$

Slope at the ends, $\theta_A = \theta_B = 1"$

$$= \frac{1 \times \pi}{180} = 0.01745 \text{ Radians}$$

Let, Y_C = Deflection at the centre

Using equation (12.6), We get

$$\theta_A = \frac{WL^2}{16EI} \quad \text{or} \quad 0.01745 = \frac{WL^2}{16EI}$$

Now using equation (12.7), we get

$$y_c = \frac{WL^3}{48EI} = \frac{WL^2}{16EI} \times \frac{L}{3}$$

$$= 0.0175 \times \frac{L}{3}$$

$$= 0.0175 \times \frac{3000}{3}$$

$$= 17.45 \text{ mm.} \quad \text{Ans.}$$

19. Where the maximum slope will occur in a simply supported beam loaded with UDL of w kN/m run.

It occurs at supports.

20. What are the units of slope and deflection?

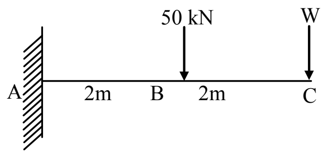
- Slope in radians
- Deflection in millimeter.

PART-B

1. A cantilever beam 4m long carries a load of 50kN at a distance of 2m from the free end, and a load of W at the free end. If the deflection at the free end is 25mm, Calculate the magnitude of the load W, and the slope at the free end. E = 200kN/mm²,m I = 5 × 10⁷ mm⁴.

(AU April/may 2017)

Given Data:



$L = 4m = 4000mm$

$W_1 = 50kN = 50 \times 10^3$ N

$Y_C = 25mm$

$E = 200kN/mm^2 = 200 \times 10^3 N/mm^2$

$I = 5 \times 10^7 mm^4$

To find:

1. Load at the free end, W =?
2. Slope at the free end, θ_C = ?

Solution:

BM equation

$$EI\frac{d^2y}{dx^2} = Wx - \qquad 50000(x-2000)$$

Integrating above equation twice,

$$EI\frac{dy}{dx} = -W\frac{x^2}{2} + C_1 \qquad -\frac{50000(x-2000)^2}{2}$$

Deflection equation,

$$EI.y = -W\frac{x^3}{6} + C_1x + C_2 \qquad -\frac{50000(x-2000)^3}{6}$$

Applying the following boundary conditions are

(i) When x = 4000mm, Slope $\frac{dy}{dx} = 0$

(ii) When x =4000mm, Deflection y =0

Applying first B.C to the slope equation

$$0=-\frac{W\times(4000)^2}{2}+C_1-\frac{50000(4000-2000)^2}{2}$$

$$C_1=8\times10^6\,W+1\times10^{11}$$

Applying second B.C to the deflection equation

$$0=-\frac{w\times(4000)^3}{6}+\left(8\times10^6\,W+1\times10^{11}\right)4000+C_2-\frac{50000(4000-2000)^3}{6}$$

$$C_2=-2.133\times10^{10}\,W-3.33\times10^{14}$$

Substitute C_1 and C_2 values in the slope and deflection equations

$$\Rightarrow\quad EI\frac{dy}{dx}=-W\frac{x^2}{2}+8\times10^6W+1\times10^{11}-\frac{50000(x-2000)^2}{2}$$

$$EI.y=-W\frac{x^3}{6}+\left(8\times10^6W+1\times10^{11}\right)x-2.133\times10^{10}W-3.33\times10^{14}-\frac{50000(x-2000)^3}{6}$$

But Y_C =25mm at x = 0

$$200\times10^3\times5\times10^7\times25=-W\frac{(0)^3}{6}+\left(8\times10^6\,W+1\times10^{11}\right)\times0-2.133\times10h10W$$
$$-3.33\times10^{14}-\frac{50000(0-2000)^3}{6}$$

$$W=24.21kN=24210N$$

Slope at the free end, x = 0

$$\frac{dy}{dx}=\theta_C=\frac{1}{200\times10^3\times5\times10^7}\left[\frac{24210(0)^2}{2}+8\times10^6\times24210+1\times10^{11}-\frac{50000\times(0-2000)^2}{2}\right]$$

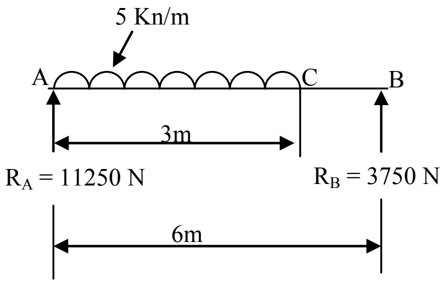
$$=-0.0194\,\text{radians}$$

Result:

1. The load at the free end, W = 24.21kN
2. Slope at the free end, θ_c = 0.0194 radians.

2. A SSB of spam 6m carries UDL 5kN/m over a length of 3m extending from left end. Calculate deflection at mid span E =2 × 10⁵ N/mm² . I = 6.2 × 10⁶mm⁴. (AU Nov / Dec 2015 & 2016)

Given Data:



L = 6m = 6000mm

W = 5kN/m = 5N/mm

$E = 2 \times 10^5$ N/mm^2

$I = 6.2 \times 10^6$ mm^4

To find : Deflection at mid-span, Y_c =?

Solution: Taking moment about A,

$$R_B \times 6000 = 5 \times 3000 \times \frac{3000}{2}$$

$$R_B = 3750N$$

$$R_A + R_B = 5 \times 3000$$

$$R_A = 11250N$$

$$EI\frac{d^2y}{dx^2} = 3750x \qquad - \frac{5(x-3000)^2}{2}$$

Integrating the above equation twice,

$$EI\frac{dy}{dx} = \frac{3750x^2}{2} + C_1 \qquad - \frac{5(x-3000)^3}{6}$$

$$EI.y = \frac{3750x^2}{6} + C_1x + C_2 \qquad - \frac{5(x-3000)^4}{24}$$

Applying the following boundary conditions

(i) When x = 0, y =0

(ii) When x = 6000 m, y = 0

Applying first B.C

$$\theta = \frac{3750(0)^3}{6} + C_1(0) + C_2 - \frac{5(0-3000)^4}{24}$$

$$C_2 = 1.6875 \times 10^{13}$$

For the second B.C

$$0 = \left[\frac{3750(6000)^3}{6} + C_1(6000) + 1.6875\times10^{13} - \frac{5(6000-3000)^4}{24}\right]$$

$$C_1 = -2.25\times10^{10}$$

Deflection at mid-span x = 3000mm

$$y_{max} = \frac{1}{2\times10^5\times6.2\times10^6}\left[\frac{3750(3000)^3}{6} - 2.25\times10^{10}\times3000 + 1.6875\times10^{13} - \frac{5(3000-3000)^4}{24}\right]$$

= -27.22mm (Take only its magnitude)

Result: Maximum deflection, Y_{max} = 27.22mm

3. A beam of length 6m is simply supported at its ends and carries two point loads of 48kN at a distance of 1m and 3m respectively from the left support. (AU Nov / Dec 2014)

Find:

(i) Deflection under each load.

(ii) Maximum deflection, and

(iii) the point at which maximum deflection occurs.

Given E = 2× 10^5 N/mm^2 and I = 85 × 10^6mm^4.

Sol, Given:

I = 85 × 10^4 mm^4; E = 2 × 10^5 N/mm^2

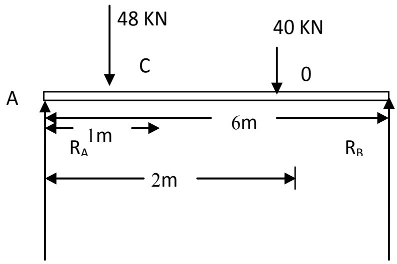
Find calculation the reaction R_A and R_B.

Taking moments about A, we get

$$R_B \times 6 = 48\times1 + 40\times3 = 168$$

$$\therefore \quad R_B = \frac{168}{6} = 28\text{kN}$$

$$\therefore \quad R_A = \text{Total load} - R_B = (48+40) - 28 = 60\text{kN}$$



Consider the section X in the last part of the beam (i.e. in length DB) at a distance x from the left support A. The B.M. at this section is given by,

$$EI\frac{d^2y}{dx^2} = RA.x \quad -48(x-1) \quad -40(x-3)$$

$$= 60x \quad -48(x-1) \quad -40(x-3)$$

Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{60x^2}{2} + C_1 \quad -48\frac{(x-1)^2}{2} \quad -48\frac{(x-3)^2}{2}$$

$$= 30x^2 + C_1 \quad -24(x-1)^2 \quad -20(x-3)^2 \quad .....(i)$$

Integrating the above equation again, we get

$$EI.y = \frac{30x^2}{3} + C_1x + C_2 \quad \frac{-24(x-1)^3}{3} \quad \frac{-20(x-3)^3}{3}$$

$$= 10x^3 + C_1x + C_2 \quad -8(x-1)^3 \quad \frac{-20}{3}(X-3)^3$$

To find the value of C_1 and C_2 use two boundary conditions. The boundary conditions are:

(i) at x = 0, y = 0, and

(ii) at x = 6m, Y = 0.

i) Substituting the first boundary condition i.e. at x = 0,Y = 0 in equation (ii) and considering the equation upto first dotted line (as x = 0 lines in the first part of the beam), we get

$$0 = 0 + 0 + C_2 \quad C_2 = 0$$

ii) Substituting the second boundary condition i.e. at x= 6m, y =0 in equation (ii) and considering the complete equation (as x= 6 lines in the part of the beam), we get

$$0 = 10\times 6^3 + C_1\times 6 + 0 - 8(6-1)^2 - \frac{20}{3}(6-3)^2 \qquad (\because C_2 = 0)$$

$$\text{or} \quad 0 = 2160 + 6C_1 - 8\times 5^3 - \frac{20}{3}\times 3^3$$

$$= 2160 + 6C_1 - 1000 - 180 = 980 + 6C_1$$

$$\therefore \qquad C_1 = \frac{-980}{6} = -163.32$$

Now substituting the values of C_1 and C_2 is equation (ii), we get

$$EI\,y = 10x^3 - 163.33x \qquad -8(x-1)^2 \qquad -\frac{20}{3}(x-3)^3 \qquad \text{........(iii)}$$

(i) (a) Definition under first load i.e. at point C. This is obtained by substituting x = 1 in equation (iii) upto the first dotted line (as the point C in the first part of the beam). Hence,

We get,

$$E\,I.\,y_c = 10\times I^3 - 163.33 = -153.33\,kN/m^3$$

$$= 10 - 163.33 = -153.33kNm^3$$

$$= -153.33\times 10^3\,Nm^2$$

$$= -153.33\times 10^3\times 10^3\ Nmm^3$$

$$= -153.33\times 10^{12}\ Nmm^2$$

$$\therefore \qquad y_c = \frac{-153.33\times 10^{12}}{EI} = \frac{-153.33\times 10^{12}}{2\times 10^5\times 85\times 10^6}mm$$

$$= -9.019\,mm. \qquad \text{Ans}$$

(Negative sign shows that deflection in downwards)

(b) Deflection under second load i.e. at point D. This is obtained substituting x = 3m in equation (iii) upto the second dotted lines (as the point D lines in the second part of the beams).

Hence, we get

$$EI.y_D = 10\times 3^3 - 163.33\times 3 - 8(3-1)^3$$

$$= 270 - 489.99 - 64 = -283.99kNm^2$$

$$= -283.99\times 10^{12}\,Nmm^2$$

$$\therefore \qquad Y_D = \frac{-283.99\times 10^{12}}{2\times 10^5\times 85\times 10^6} = -16.7mm. \qquad \text{Ans.}$$

(ii) Maximum deflection, the deflection is likely to be maximum at a section between C and D. For maximum deflection $\frac{dy}{dx}$ should be zero. Hence equate the equation (i) equal to zero upto the second dotted line.

$$\therefore \qquad 30x^2 + C_1 - 24(x-1)^2 = 0$$

or $$30x^2 - 163.33 - 24(x^2 + 1 - 2x) = 0 \qquad (\therefore C_1 = -163.33)$$

or $$6x^2 + 48x - 187.33 = 0$$

The above equation is a quadrature equation. Hence its solution is

$$x = \frac{-48 = \sqrt{48^2 + 4 \times 6 \times 187.33}}{2 \times 6} = 2.87\text{m.}$$

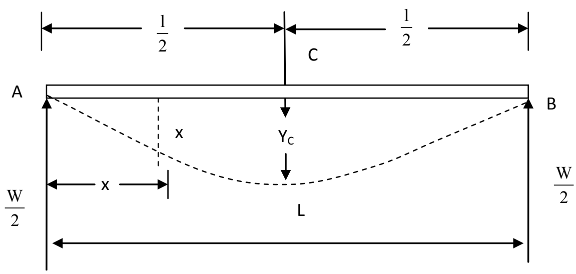
$$(\text{Negative} - \text{ve root})$$

Now substituting x = 2.87m is in equation (iii) upto the second dotted line, We get maximum deflection as

$$EI\,y_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87-1)^3$$
$$= 236.39 - 468.75 - 52.31$$
$$= 284.67\,\text{kNm}^3 = -284.67 \times 10^{12}\,\text{Nmm}^3$$

$$\therefore \qquad y_{max} = \frac{-284.67 \times 10^{13}}{2 \times 10^5 \times 85 \times 10^6} = -16.745\,\text{mm} \qquad \text{Ans.}$$

4. Derive the slope and deflection equations for a simply supported beam carrying point load at the centre.



Now

$$R_A = R_B = \frac{W}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_2 = R_A \times x$$
$$= \frac{W}{2} \times x \qquad (\text{Plus sign is as B.M for left portion at X is clockwise})$$

But B.M at any section is also given by equation (12.3) as

$$M = EI\frac{d^2y}{dx^2}$$

Equating the two values of B.M, we get

$$EI\frac{d^2y}{dx^2} = \frac{W}{2} \times x$$

On integration, we get

$$EI\frac{dy}{dx} - \frac{W}{2} \times \frac{x^2}{2} + C_1$$

Where C_1 is the constant of integration And its value is obtained from boundary conditions.

The boundary conditions is that at $\frac{f}{2}$ slope $\left(\frac{dy}{dx}\right) = 0$ (As the maximum deflection is at the centre, hence slope at the centre will be zero). Substituting this boundary condition in equation (iii), we get

$$O = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_2$$

or $$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in equation (ii), we get

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \qquad \text{.......(iii)}$$

The above equation is known the slope equation. We can find the slope at any point on

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_{at\,A} = \frac{W}{4} \times 0 - \frac{WL^2}{16}$$

$$\left[\left(\frac{dy}{dx}\right)_{at\,A} \text{ is the slope at A and is represented by } \theta_A\right]$$

or $$EI \times \theta_A = -\frac{WL^2}{16}$$

$$\therefore \quad \theta_A = -\frac{WL^2}{16EI}$$

The slope at point B will be equal to θ_A, since the load is symmetrically applied.

$$\therefore \qquad \theta_B = \theta_A = \frac{WL^2}{16EI} \qquad .....(12.6)$$

Equation (12.6)gives the slope in radians.

Deflection at any point

Deflection at any point is obtained bu integrating the slope equation (iii), Hence integrating equation (iii), we get

$$EI \times y = \frac{W}{4} \cdot \frac{x^3}{3} - \frac{WL^2}{16} x + C_2 \qquad .....(iv)$$

Where C_2 is another constant of integration. At A, x = 0 and the deflection (y) is zero.

Hence substituting these values in equation (iv), we get

EI × 0 = 0 -0 + C_2

Or **C_2 = 0**

Substituting the values of C_2 in equation (iv), we get

$$EI \times y = \frac{Wx^2}{12} - \frac{WL^2.x}{16} \qquad ....(V)$$

The above equation is known as the deflection equation. We can find the deflection at any point on the beam by substituting the values of x. The deflection is maximum at centre Point C, where $x = \frac{L}{2}$. Let y; represents the deflection at C. Then substituting $x = \frac{L}{2}$ and y = y_c in equation (v), we get

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^2 - \frac{WL^2}{16} \times \left(\frac{L}{2}\right)$$

$$= \frac{WL^2}{96} - \frac{WL^3}{32} = \frac{WL^3 - 3WL^3}{96}$$

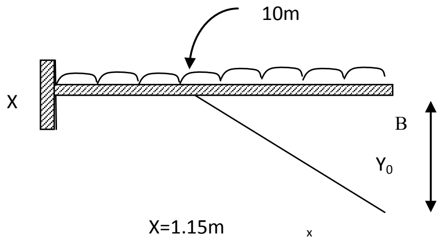
$$= -\frac{2WL^3}{96} = -\frac{WL^3}{48}$$

$$\therefore \qquad y_c = -\frac{WL^3}{48EI}$$

(Negative sign shows that deflection is downwards)

5. A cantilever 1.5m long carries a uniformly distributed load over the entire length find the deflection at the free end if the slope at the free end is 1.5°.

Solution:-



Length of cantilevers ℓ = 1.5m

Slope at the free end, = 1.5°

$$= 1.5^\circ \times \frac{\pi}{180} \text{radians.}$$

Deflection @ B:

$$\text{Slope at the free end} = \frac{\omega\ell^2}{6EI}$$

$$= 1.5 \times \frac{\pi}{180}$$

$$\frac{\omega\ell^3}{6EI} = 1.5 \times \frac{\pi}{180}$$

$$\frac{\omega\ell^3}{EI} = \frac{1.5 \times \pi \times 6}{180}$$

$$\frac{\omega\ell^3}{EI} = \frac{\pi}{20}$$

Deflection at the free end

$$y_B = \frac{\omega\ell h4}{8EI} = \frac{\omega\ell^3}{EI} = \frac{\ell}{8}$$

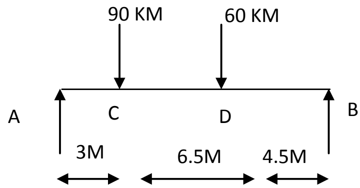
$$= \frac{\pi}{20} \times \frac{\ell}{8} = \frac{\pi}{20} \times \frac{1.5}{8}$$

6. **A steel girder 4m long is simply supported at the ends, it carries two concentrated loads of 90kN and 60kN at 3m and 4.5m from the two ends respectively. Calculate**

   i)The deflection of the girder at the points under the two loads

   ii)The maximum deflection

   Take I =64 × 10^{-4} m^4 and E = 210 × 10^6 kN/m^2



Given:

ℓ = 12m = 14000mm

loads, 90kN & 60kN.

I = 64 × 10^{-4} m^4

E = 210 × 10^{-6} kN/m^2

Taking moment about A:-

$\sum MA = 0$

$$-R_B \times 14 + 60 \times 9.5 + 90 \times 3 = 0$$

$$14R_B = 840$$

$$\boxed{R_B = 60KN.}$$

Total Load, = 150KN

$$R_A + R_B = 150Kn$$

$$R_A = 150 - 60$$

$$\boxed{R_A = 90KN}$$

Consider any section xx at a distance x from the end A, following Macaulay's method.

The bending moment is given by:-

M_x =Ex – d^2y – 90x – 90 (x-3) – 60 (x-9.5).

Integrating again we get,

$$EI.y = 45.\frac{x^3}{3} + C_1x + C_2 - 45\frac{(x-3)^2}{3} - 30\frac{(x-9.5)^2}{3}$$

$$EI.y = 15x^3 + C_1x + C_3 - 15(x-3)^2 - 10(x-9.5)^3 \quad ....(2)$$

At End A :- In equation (2) x = 0; y = 0.

$$E.I(0) = 15(0)^3 + C_1(0) + C_3 - 15(0-3)^2 - 10(0-9.5)^2$$

$$0 = C_3 + 405 + 8573.75$$

$$\boxed{C_3 = 0}$$

At End B:- In equation (2) x = 14; Y = 0.

$$EI.(0) = 15(14)^3 + C_1(14) + C_2 - 15(14-3)^3 - 10(14-9.5)^3$$

$$0 = 4116.0 + 14C_1 + 0 - 19965 - 911.25$$

$$0 = 20283.75 + 14C_1$$

$$\boxed{C_1 = -1448.84}$$

Hence the deflection at any section is given by :-

$$\boxed{EI.Y = 15x^3 - 1448.84x - 15(x-3)^3 - 10(x-9.5)^3}$$

i) Y_C and Y_D :-

Deflection at C, Y_C

Putting, x =3m in deflection equation, we get,

$y_c = -0.0029m.$

$\boxed{y_c = -2.93\,mm}$ $(\therefore \text{sign indicates downward deflection})$

Putting this value of x in the deflection we get,

(ii) Determination at d, Y_D

We get, Putting x= 9.5m in deflection we get,

$EI\ Y_{max} = 15\ x^3 - 1448.84x - 15(x\text{-}3)^3$

$EI\ Y_C = 15\ x^3 - 1448.84\ x - 15\ (x\text{-}3)^3$

$13344000\ Y_C = 15\ x^3 \times 6.87 - 1448.84 \times 6.87 - 15\ (6.87 - 3)^3$

$13344000\ Y_C = 12860.625 - 13763.98 - 4119.375$

$= 4863.6 - 9953.5 - 869.4$

$13344000\ Y_C = -5022.73$

$13344000\ Y_{max} = -5959.3$

$$Y_c = \frac{-5022.73}{1344000}$$

$$Y_{max} = -0.0044m$$

$$\boxed{Y_c = -3.744\,mm}$$

$$\boxed{Y_{max} = -4.4\,mm}$$

iii) Maximum deflection:- (Y_{max}):

Let as assume that the deflection will be maximum at a section between C and D. Equating the slope at the section is zero, we get,

$$Ex.\frac{dy}{dx} = 45x^2 - 1448.84 - 45(x-3)^2 = 0$$

$$45x^2 - 1448.84 - 45(x-3)^2 = 0$$

Result:-

(i) Y_C = 2.93 mm (downward)

Y_D = 3.74mm (downward)

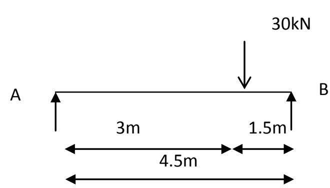
(ii) Y_{max} = 4.4mm (downward)

7. A beam with a span of 4.5cm carries a point load of 30kN at 3m from the left support. If for the section, Ixx = 54.97 × 10^{-6} m^4 and E =200GN/m^2. Find

i) The deflection under the load

ii) The position and amount of maximum deflection

Given:



W = 30kN;

a =3m; b =1.5m

I = 54.97 × 10^{-6} m^4

$E = 200GN/m^2$

i) The deflection under the load, Y_C:

$$Y_C = (-)\frac{Wa^2b^2}{3EI.\ell} = (-)\frac{30\times3^2\times1.5^2}{3\times300\times10^6\times54.97\times10^{-6}\times4.5}$$

$$Y_C = (-)0.00409m = (-)4.09mm.$$

ii) The position (x) and amount of maximum deflection (Y_{max}):

$$x = \sqrt{\frac{\ell^2 - b^2}{3}} = \sqrt{\frac{4.5^2 - 1.5^2}{3}} = 2.45m \text{ from left end.}$$

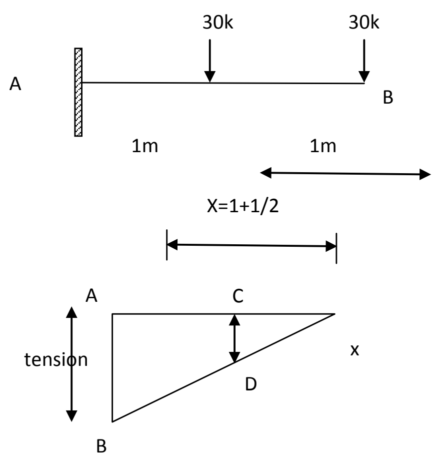
Maximum deflection,

$$Y_{max} = (-)\frac{Wb\left(\ell^2 - b^2\right)^{\frac{2}{2}}}{9\sqrt{3}\,Ex.\ell}$$

$$= (-)4.456\,mm\,\ell.$$

8. Find the slope and deflection for cantilever beam as shown in figure.

Solution:



i) Reaction:

Total load = Total reaction

$$\boxed{R_A = 40kN}$$

ii) Moment:

$M_B = 0$

M_C= - 20 × 1 =- 20kN.m

M_A= (-20 × 2) + (-20 × 1) = -60kN.m

iii) Area 0f BMD:

Area of section (1) ⟹ A_1 ⟹ ½ × b × h

⟹ ½ × 1 × 20

$\boxed{A_1 = 10}$ $kN.m^2$

Total Area A:-

$A = A_1 + A_2 + A_3$

A = 10 + 20 + 20

$\boxed{A = 50KN\,m^2}$

Area of section (2) ⟹ A_2 ⟹ b × h

⟹ 1 × 20

$\boxed{A_2 = 20}$ $KN.m^2$

Area of section (3) ⟹ A_3 ⟹ ½ × b × h

⟹ ½ × 1 × (60 – 20)

$\boxed{A_3 = 20}$ $KN.m^2$

$$\theta_{max} = \frac{\text{Area of BMD}}{E\,I}$$

$$\theta_{max} = \frac{50 \times 10^3 \times 10^6}{10^5 \times 10^6}$$

$$\boxed{\theta_{max} = 0.005\,\text{radians}}$$

(v) Deflection:-

$$Y_{max} = \frac{\text{Area of BMD}}{EI} \times \bar{x}$$

$$= \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{E\,x}$$

$$= \frac{(10 \times 0.667) + (20 \times 1.5) + (20 \times 1.667)}{10^5 \times 10^8} = \frac{70}{10^5 \times 10^8}$$

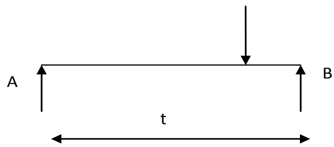
$$= \frac{70 \times 10^3 \times 10^9}{10^5 \times 10^8}$$

$$= \frac{70 \times 10^{12}}{10^5 \times 10^8}$$

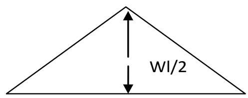
$$\boxed{y_{max} = 7mm}$$

9. Find the slope and deflection for a simply supported beam carrying load W at the centre by conjugate beam method.

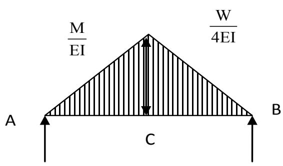
Solution:



REAL BEAM



BMD for REAL BEAM



CONTINUING BEAM WITH RMD

$R_A = R_B = w/2$

$M_A = M_B = 0$

$M_c = w\ell/4.$

Total load =

$$\frac{1}{2} \times b \times h$$

$$\frac{1}{2} \times \ell \times \frac{w\ell}{4}$$

$$T.L = \frac{w\ell^2}{8EI}$$

$$\therefore R_A = R_B = \frac{\frac{w\ell^2}{8EI}}{2}$$

$$\boxed{R_A = \frac{w\ell^2}{16EI}} \qquad \therefore \boxed{R_B = \frac{w\ell^2}{16EI}}$$

Slope:-

$$\theta = \frac{S.F.@C_1}{EI} = \frac{R_{B1}}{EI}$$

$$\boxed{\theta = \frac{w\ell^2}{16EI}}$$

Slope:

$$\theta_A = S.F.@A$$

$$= -R_B + \frac{w\ell}{4EI} \times \ell \times \frac{1}{2}$$

$$= -\frac{w\ell^2}{16EI} + \frac{w\ell^2}{8EI}$$

$$\boxed{\theta_A = (-)\frac{w\ell^2}{16EI}}$$

Deflection:-

$$y = \frac{B.M@C_1}{EI} = \frac{M_{C1}}{EI} = \frac{R_B \times \frac{\ell}{2}}{EI}$$

$$y = \frac{\frac{w\ell^2}{T} \times \frac{\ell}{2}}{EI} = \frac{w\ell^2 \times \ell}{16 \times 2 \times EI}$$

Deflection:-

$$y_c = Y_{max} = B.M@C$$

$$= R_B \times \frac{\ell}{2} - \left(\frac{1}{2} \times \frac{\ell}{2} \times \frac{w\ell}{2}\right) \times \frac{1}{2} \times \frac{\ell}{2}$$

10. Find the slope and Deflection for a cantilever beams carrying point load at the free end by conjugate method

(i) Support Reaction:-

Total load = Total reaction

$$\boxed{R_A = W}$$

ii) Shear Force:

S.F. @ B = 0

S.F. @ A = W.

(iii) Bending Moment:

B.M @ B = 0

B.M @ A = - w l

(iv) Slope:

Θ_B = S.F @ B

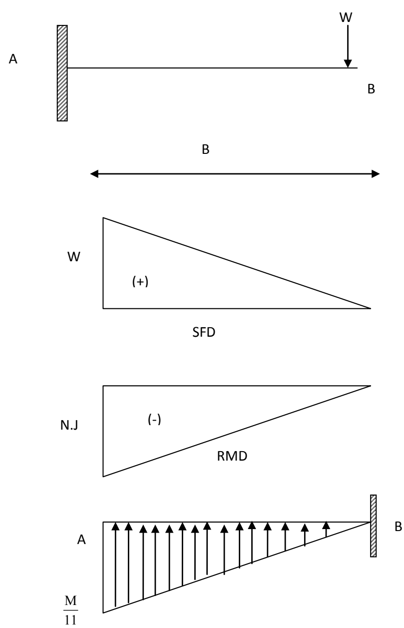
S.F. @B = Total load = $\frac{1}{2} \times \ell \times \frac{w\ell}{EI}$

$$\theta_B = \frac{w\ell^2}{2EI}$$

(v) Deflection:

$$Y_B = MB = \frac{M}{EI} \times \ell$$

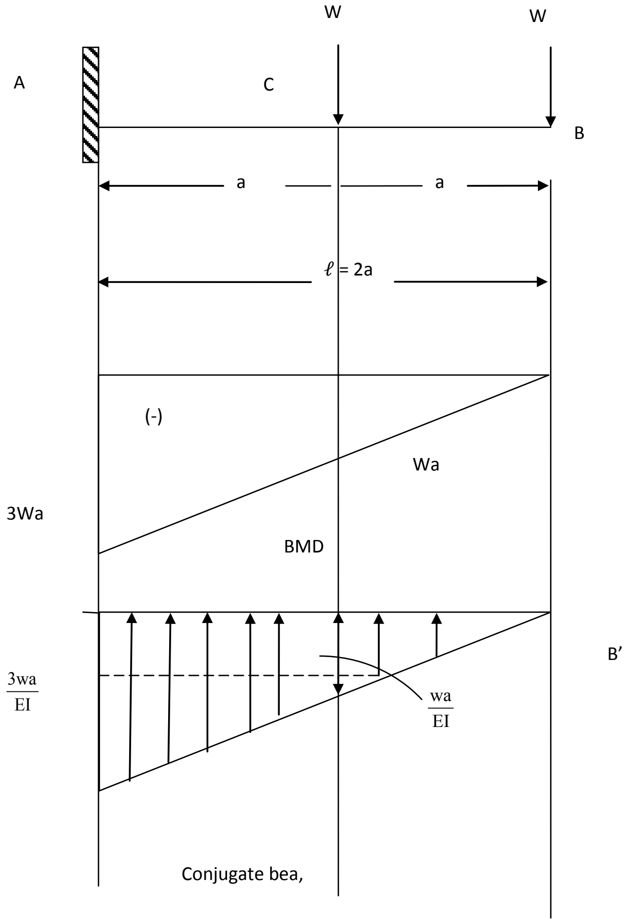
$$= \left(\frac{W\ell}{EI} \times \frac{1}{2} \times \ell \right) \times \frac{1}{2} \ell.$$



11. A cantilever of length 2a is carrying a load of W at the free end and another load of W at its center determine the deflection of the cantilever at the free end, using conjugate beam method.

(Nov/Dec 2014)

Given Data:



BMD

BM @ 'B' = 0

BM @ 'C' = -Wa

BM @ 'A' = - 2wa – wa = -3 wa

Deflection at the free end = The B.M at 'B' of conjucated Beam

$$y_B = -\left[\left(\frac{wa}{EI}\times a\right)\times\left(a+\frac{a}{2}\right)+\left(\frac{1}{\cancel{2}}\times a\times\frac{2wa}{EI}\right)\times\left(a+\frac{2}{3}a\right)+\left(\frac{1}{2}\times a\times\frac{wa}{EI}\right)\left(\frac{2}{3}\times a\right)\right]$$

$$y_B = -\left[\left(\frac{wa^2}{EI}\times\frac{3a}{2}\right)+\left(\frac{Wa^2}{EI}\times\frac{5a}{3}\right)+\left(\frac{wa^2}{\cancel{2}EI}\times\frac{\cancel{2}}{3}a\right)\right]$$

$$y_B = -\left[\frac{3Wa^3}{2EI}+\frac{5Wa^3}{5EI}+\frac{\cancel{2}Wa^3}{\underset{3}{\cancel{6}}EI}\right]$$

$$y_B = -\frac{Wa^3}{EI}\left[\frac{3\times 3}{2\times 3}+\frac{5}{3}+\frac{1}{3}\right] = -\frac{5Wa^3}{EI}$$